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Viscous Modified Chaplygin in Classical and Loop Quantum Cosmology

Presented by

Dalel Aberkane

Members of jury:

President	M. A. Benslama	Professor	University of the Mentouri Brothers
Supervisor	N. Mebarki	Professor	University of the Mentouri Brothers
Examiners	H. Auissaoui	Professor	University of the Mentouri Brothers
	M. Boussahel	Professor	University of M'sila
	S. Zaim Batna	Professor	University of Batna

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Introduction

One of the most intriguing questions in modern physics is the fundamental machinery behind the accelerated expansion of the universe. With recent observational data coming from Type Ia Supernovae [1,3], cosmic microwave background anisotropies[4−6] and large galaxy surveys[7,8] , the misleading conception of a static universe is abandoned for a universe that is in an accelerated motion where stuff are constantly taking away from each other. This discovery has shed light on a new research era in aim to explain the physics behind this motion. Different answers was brought to the arena and structured mainly into two different approaches: the first one treats general relativity as incomplete theory that needs to include modifications in aim to predict the accelerated expansion. In different words, the accelerated expansion should be inherently related, at large scale limit, to the geometry of a modified theory of gravity. The second one takes more seriously the Einstein's theory in a way that it should be fully trusted, so the accelerated motion is derived by adding a new exotic component of the universe with a negative pressure called dark energy [9,10]. If we follow the second path, especially that the theory is well tested at large scale, the existence of a new physical object filling the universe will rise several questions about its fundamental structure and how it couples with other stuffs in the universe. Although, different models were proposed involving baryonic and non-baryonic candidates [11,12], we still don't know which one describes the reality the best, models are subject to verifications through recent observations.

One of the well accepted models are the Chaplygin gas models, they were widely scrutinized and modified since they were first proposed. They are combined models that unify both dark energy and dark matter and give a suitable negative pressure that drives the acceleration of the universe. The Chaplygin gas was first proposed as a dark energy model in [13]. Although, its equation of state was generalized [14] to fit with observational data, at high energy density the Generalized Chalpygin Gas (GCG) mimics dust with ($p = 0$) and suffers from instabilities at the perturbative level [15]. Therefore, Modified Chaplygin Gas (MCG) was proposed by adding a further modification to the GCG. Its EoS parameters were also constrained using different observational data [16-19].
Similarly, viscous modified Chaplygin gas (VMCG) with the generalized EoS was investigated in Ref. [20], as it is possible to assume that the expansion process is a collection of states out of thermal equilibrium that gives rise to bulk viscosity. A variety of bulk cosmological models have been explored by several researchers [21-26].
Unfortunately, classical models suffer from early and late time singularities. Those can be avoided in the framework of loop quantum cosmology (LQC) [27-31] which is a non-perturbative and background-independent type of quantization of gravity [32,33] used to probe some cosmological problems. In addition of predicting an inflationary phase of the early universe [34-37] and late time cosmic acceleration[38], the semi-classical approximation in LQC formalism can be validly used at late time and at large scale [39].

As the MCG was found to be consistent with the evolution of the universe over a wide range of epochs [40] and it is preferred by recent observational data because of its small minimum χ^2 value [16] and as the universe throughout its evolution might gave rise at its beginning to bulk viscosity, we chose VMCG with a specific bulk viscosity pressure to model the dark content of the universe and explore the model's behavior at present time when fitted to recent observational data, its fate at late time and whether it suffers from singularities or not.

First, we constrained its Eos parameters using Union 2.1 data for a suitable model that describes the current universe. We also evaluate cosmological key parameters at present and early universe and determine their present values to deduce if the model is consistent or not with observational data and theoretical predictions. The values are compared to those of other well accepted models. Then, we probe the dynamical behavior of the model at early and late time in the LQC framework especially as the model suffers from the Big Bang singularity.

This thesis is organized as follows:

In the first chapter we explore the FRW cosmology along with some cosmological models derived from it. The bulk viscosity is also introduced when fluids describing stuff in the universe are no longer perfect. This is followed by exposing some flaws within the classical cosmology. Then, we define some important parameters used for cosmological measurements.

In the second chapter we begin by giving an overview of the motivation behind a quantum theory of gravity and loop quantum gravity as a special case. Then, the mathematical formalism inherent to the theory is introduced and the main ideas behind every step toward this formalism are explained. The modifications on Friedmann equations due to loop quantum cosmology corrections are illustrated.

In the last chapter, some of the Chaplygin gas models are explored briefly before introducing the VMCG model and the formula chosen for the bulk pressure. Then, we started by solving analytically the conservation equation of a VMCG dominated universe to check the behavior of the solution at early, present and late time. The model is constrained using recent observational data, we used Mathematica to calculate the best fit parameters and draw the contour plots of some confidence levels. The behavior of the model is then probed at small and present scale using the time evolution of cosmological parameters. Finally, in LQC framework, the dynamical analysis is conducted using Maple and Mathematica.

Chapter one:

Classical Cosmology

"We cannot solve our problems with the same

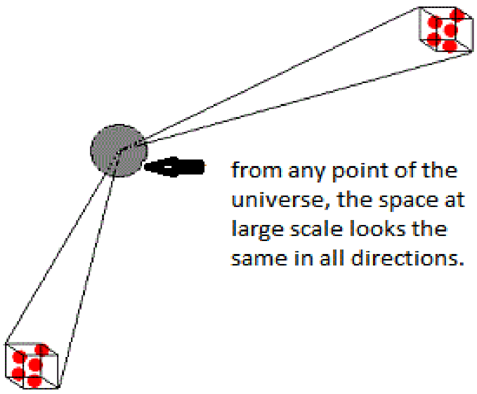
thinking we used when we created them"

Albert Einstein

1 FRW Cosmology

1.1 The cosmological principle

It states that the universe is large enough (level of clusters of galaxies) to assume that all points of the universe are equivalent which means that the universe is assumed to be homogeneous and isotropic around any point.



Homogeneous: there is an isometry (a transformation that preserves distance relationships) that can carry any vector to a nearby point, so the geometry is the same at any point as it is at another.

Isotropic: if you rotate around a point the space looks the same without any preferred direction.

1.2 The Einstein field equation

According to Newtonian gravity, gravitational mass is the only source of the gravitational field ($\nabla^2\varphi \propto \rho$) but with both energy-mass equivalence and equivalence principle, gravitational field couples in the same way mass and energy and they are both described by the same mathematical entity called the energy-momentum tensor T_{ab} . The space used to describe the theory of general relativity is not the 3-dimensional Euclidean space of Newton mechanics and the second derivative in the Euclidean space is replaced by the curvature (Ricci) tensor in the Riemannian space so

Newtonian Gravity	General Relativity
$\boldsymbol{\nabla^2}$	$the\ Ricci\ tensor\ R_{ab}$
$\boldsymbol{\rho}$	T_{ab}
$\boldsymbol{\nabla^2\varphi \propto \rho}$	$R_{ab} \propto T_{ab}$

In its field equation, Einstein established a relationship between the energy density content of the universe and the curvature of the space time

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi G\, T_{ab} \tag{1.1}$$

where R is the Ricci scalar, g_{ab} the metric and G the gravitational constant. This equation states that the space-time geometry is dictated by the distribution of energy filling the universe.

The vacuum equation used to study the gravitational field outside the source is called Vacuum Einstein equation and is given by

$$R_{ab} = 0 \tag{1.2}$$

The problem with Eq. (1.1) is that if the gravitational force (an attractive force) is the only active force at the present scale, the universe will eventually shrink and collapse. As Einstein disliked the idea of a dynamic universe, he added a fudge factor to the equation to completely balance the attractive force and made the universe closed, homogeneous and static.

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda\, g_{ab} = 8\pi G\, T_{ab} \tag{1.3}$$

Where Λ is the cosmological constant.

In 1920s, Wirtz and K. Lundmark showed that Siphers's red shifts increased with the distance of the nebulae, and in 1929, Hubble established a linear relation between distances and velocities so the furthest objects are the fastest [41]. Therefore, the universe is not static and rather in an accelerated motion, this fact forced Einstein to admit that the added factor Λ is his biggest mistake. The infinite structure of the universe is no longer a problem if we assume that the cosmological constant is the vacuum energy.

1.3 The Robertson-Walker metric

As the universe is assumed to be homogeneous and isotropic, the metric describing how lengths are measured in this space should include those two conditions.

The geometry of such a space is spherically symmetric about a point and can be described using the Schwarzschild metric for a static gravitational field

$$ds^2 = e^{2\vartheta(r)}dt^2 - e^{2\gamma(r)}dr^2 - r^2(d\theta^2 + sin^2\theta\ d\varphi^2) \qquad (1.4)$$

The metric has no preferred angular direction and is time-independent (no mixed terms). Taking into account the fact that an observer at a fixed point moves only forward in time along a geodesic which is parallel to the time coordinate line we have

$$ds^2 = g_{tt}\ dt^2 = dt^2 \qquad (1.5)$$

As the universe is dynamic the metric can be written as

$$ds^2 = dt^2 - a^2(t).\left(e^{2\gamma(r)}dr^2 + r^2(d\theta^2 + sin^2\theta\ d\varphi^2)\right) \qquad (1.6)$$

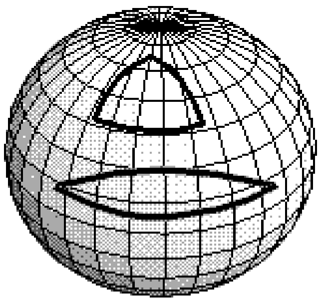
Where $a(t)$ is the scale factor that describes how the size of the universe evolves in time.

Using the fact that in spatially isotropic and homogeneous space the curvature of the space is constant and is related to the Riemann tensor [42] we find the R-W metric

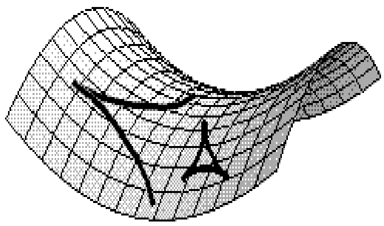
$$ds^2 = dt^2 - a^2(t).\left(\frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + sin^2\theta\ d\varphi^2)\right) \qquad (1.7)$$

Where k is the normalized curvature constant.

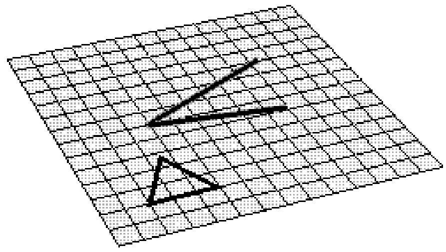
Positive curvature($k = +1$) : the surface is a two sphere (a closed space) where the sum of the angles of a triangle on this surface is greater than 180^0.



Negative curvature($k = -1$) : the surface has an infinite volume (an open space) where the sum of the angles of a triangle on this surface is less than 180^0.



Zero curvature$(k = 0)$: a flat space where the sum of the angles of a triangle on this surface is equal to 180^0.



1.4 The energy-momentum tensor

According to the general covariance principle, all invariant laws in physics under coordinate transformation should be stated in tonsorial form. Similarly, any distribution of matter or energy as a source of the gravitational field should be stated in terms of the energy-momentum tensor as energy is an invariant quantity under coordinate transformation.

We consider a momentum $dp^i, (i = 1,2,3)$ and an energy dp^0 that are contained in an infinitesimal volume $dS_b, (b = 0,1,2,3)$ so the momentum 4-vector $dp^a(dp^0, dp^i)$ is proportional to the volume [42] by

$$dp^a = T^{ab}\, dS_b \tag{1.8}$$

Where T^{ab} is the factor of proportionality, called the energy-momentum tensor. It is the flux of the "a" component of the momentum 4-vector across the surface defined by a constant"x^b".

T^{00}: is the energy density ρ defined as the flow of the energy through a surface of a constant time.

T^{0i}: is the energy flux defined as the flow of the energy through a surface of a constant "x^i".

T^{i0}: is the momentum density defined as the flow of the momentum through a surface of a constant time.

T^{ij}: is the stress defined as the flow of the momentum through a surface of a constant "x^i"which is the flux of force per unit area.

The conservation equation:

The conservation equation of the energy momentum tensor is given by

$$\nabla_b T^{ab} = 0 \tag{1.9}$$

Where ∇_b is the covariant derivative.

Energy-momentum tensor of dust:

Dust is defined as a perfect fluid with non-interacting and non-relativistic particles with no pressure, moving together with some velocity so they carry energy and momentum as a source of a gravitational field.

The energy-momentum tensor for dust is given by

$$T^{ab} = \rho\, u^a u^b \tag{1.10}$$

Where ρ is the energy density of dust particles, u^a is the velocity 4-vector of dust particles in the chosen frame.

Energy-momentum tensor of a perfect fluid:

A perfect fluid is defined as a fluid where any region nearby a co-moving observer with the fluid is seen to be homogeneous and isotropic so all the directions for a co-moving frame are equivalent. In such a fluid there is no heat flow or viscosity and the changes within the fluid are only adiabatic. The energy-momentum tensor of such a fluid is given by

$$T^{ab} = -p\, g^{ab} + (\rho + p)\, u^a u^b \tag{1.11}$$

Where p is the pressure of the perfect fluid, ρ is the energy density of the fluid, g^{ab} is the metric of the space.

Energy-momentum tensor of a generalized fluid:

The general form of energy-momentum tensor of a more complicated fluid [42] is given by

$$T^{ab} = \rho(1+\varepsilon)u^a a^b + (p - \zeta\theta)h^{ab} - 2\eta(\sigma^{ab} + q^a\, u^b + q^b\, u^a) \tag{1.12}$$

Where ε: specific energy density of fluid in its rest frame, $h^{ab} = u^a u^b + g^{ab}$: the spatial projection tensor, ζ: bulk viscosity, $\theta = \nabla_a u^a$:expansion or the divergence of the fluid world lines, a^b:the acceleration tensor, η: shear viscosity, $\sigma^{ab}=\frac{1}{2}(\nabla_c u^a h^{cb}+\nabla_c u^b h^{ac})-\frac{1}{3}\theta h^{ab}$: shear tensor, q^a: the energy flux tensor.

1.5 Bulk and shear viscosity

Shear viscosity of a fluid measures how strong is the couple between different layers of the fluid of the same velocity under a shear stress (the friction between two layers with different velocities)

$$\sigma_{ij}=\eta(\nabla_i u_j+\nabla_j u_i) \tag{1.13}$$

Where $\sigma_{ij}$is the stress tensor, η shear viscosity and $\nabla_i u_j$ the velocity gradients. The stress in this case is not provoked by velocity but by the change of velocity from point to another. In case of incompressible fluids (flow velocities << the speed of sound) the divergence of the velocity vanishes.

Bulk viscosity deals with compressible fluids (flow velocities ≈ the speed of sound) where the divergence of the velocity is non-vanishing and induces an extra dynamic pressure

$$p=-\zeta\,\nabla.v \tag{1.14}$$

Where ζ is the bulk viscosity. As it is noticed, the dynamic pressure is negative in regions where the fluid expands $\nabla.v>0$. The general form of the stress tensor will be given by

$$\sigma_{ij}=-p_0\,\delta_{ij}+\eta\big(\nabla_i u_j+\nabla_j u_i\big)+a\,\nabla.v\,\delta_{ij} \tag{1.15}$$

Where a is a constant, p_0 is the pressure of the fluid taken as the average of the three normal stresses $p_0=-\sum_i \sigma_{ii}/3$ defined by

$$p_0=p-\zeta\,\nabla.v \tag{1.16}$$

Where p is the equilibrium pressure given by the state equation $p=p(\rho,T)$, so the stress tensor can be written as

$$\sigma_{ij}=-(p-\zeta\,\nabla.v)\delta_{ij}+\eta\big(\nabla_i u_j+\nabla_j u_i\big)+\frac{2}{3}\,\eta\nabla.v\,\delta_{ij} \tag{1.17}$$

Viscosity may arises from a number of dissipative processes in the early universe such as the decoupling of matter and radiation era, the inflationary phase, formation of galaxies,..etc.

Universes that are assumed to be isotropic and homogeneous are shearless and only bulk viscosity is taken into account. Bulk stress at late time may induce a negative pressure that drives the acceleration of the universe. This viscosity might be attributed to a fluid describing matter or dark energy. Eckart [43] made the first attempt to describe a relativistic theory of viscosity with the bulk pressure

$$p = -3\zeta H \tag{1.18}$$

in which ζ is the bulk viscosity and H the Hubble parameter.

1.6 Cosmological models in standard cosmology

Standard cosmology is a class of dynamical cosmological models characterized by a homogeneous and isotropic distribution of stuff in the universe, such models have a universal time which is non-common in relativity.

We model matter and energy by a perfect fluid energy-momentum tensor then we solve Eq. (1.1) using the RW metric and find the Freidmann equations [44]

$$\frac{3}{a^2}(k + \dot{a}^2) = 8\pi G\rho \tag{1.19}$$

$$2\frac{\ddot{a}}{a} + \frac{1}{a^2}(k + \dot{a}^2) = -8\pi G p \tag{1.20}$$

Solving the conservation equation of the energy-momentum tensor $\nabla_b T^{ab} = 0$ gives

$$\frac{\partial\rho}{\partial t} = -3\frac{\dot{a}}{a}(\rho + p) \tag{1.21}$$

This result corresponds to the first law of thermodynamics $dE + pdV = 0$.

Friedmann equations gives rise to different possible models, we only state some of them as the following:

<u>Radiation dominated universe:</u>

The early universe was dominated by radiation or an extremely relativistic gas with non-interacting particles, radiations are modeled as a perfect fluid where the state equation is given by

$$\rho_R = 3p_R \tag{1.22}$$

Where ρ_R and p_R are respectively the density and pressure of radiations, replacing the state equation into the Eq. (1.21) we find

$$\rho_R \propto a^{-4} \tag{1.23}$$

So as the universe is expanding, the radiation density drops faster than matter because of the redshift effect on photons.

Matter dominated universe:

Matter represents all the non-relativistic stuff of the universe considered as a source of the gravitational field, it is modeled by dust($p_M = 0$). With the expansion of the universe, matter density decreases with a factor of

$$\rho_M \propto a^{-3} \tag{1.24}$$

As the density of radiation drops faster than matter within the expansion process the universe gets colder and becomes dominated by matter.

In the above two models, we have a decelerated expansion but it is more rapid in a universe dominated by radiation with $a(t) \propto \sqrt{t}$ as it is in a universe dominated by matter with $a(t) \propto t^{2/3}$, this is due to the pressure of the radiation.

Vacuum dominated universe:

When we drain the universe from its content (all matter and radiation) we are left with a vacuum energy that can be modeled by the cosmological constant Λ with

$$\rho_V = -\, p_V \tag{1.25}$$

or by a perfect fluid with negative pressure with equation of state

$$\rho_V = -\omega\, p_V \tag{1.26}$$

where $\omega < -1/3$ is the required state parameter to drive an accelerated expansion of the vacuum dominated universe.

In FRW cosmology De-Sitter universe corresponds to a homogenous and isotropic vacuum dominated universe with a positive cosmological constant and a positive curvature. The accelerated expansion in this universe is exponential

$$a(t) \propto e^{Ht} \tag{1.27}$$

1.7 The Big Bang singularity and Inflation

When the early universe was dominated by radiation with$(\rho > 0, p > 0)$, it was found to be in a decelerated expansion which means that if we keep going back in time the universe will be shrinking till will reach a singularity point at $a = 0$ called the "initial" or the "Big Bang" singularity.

The idea of a singular origin of the universe was firstly proposed by Lemaitre, a catholic priest who worked on the theory of general relativity and the origin of the universe. According to his ethnic beliefs, the universe was created from a "cosmic egg". Nevertheless, as he was not capable to develop further this idea, he has not been taken seriously. In the 1940s, R. Alpher and G. Gamow assumed that the universe at its beginning was hot and dense enough to allow the creation of helium, lithium, deuterium and later hydrogen, and in 1960, the astronomer Fred Hoyle came up with the name "Big Bang".

From this singularity point the universe is assumed to be created, it can be predicted by the singularity theorems where every universe with $(\rho > 0, p \geq 0)$ must have begun at a singularity.

As $a \to 0$ we have a density that increases and a temperature that goes to infinity $T \to \infty$, in this case classical theory of relativity is not capable of describing the physics in the vicinity of this singularity. A quantum theory of gravity is needed to solve this problem.

Even if the Big Bang model gave successful predictions on Cosmic Microwave Background (CMB) radiations, the abundance of light elements and the Big Bang nucleosynthesis, several problems are embodied in this model, for example dark energy and dark matter are not described by this model, likewise, CMB radiations have been observed in different directions, for points that are not in causal contact, with surprisingly uniform results. This is called the Horizon problem.

Another problem arises where recent observations set the density parameter of the present universe to $\Omega = 1$, this result implies the flatness of the universe as $t = 0$, because any weak deviation from $\Omega = 1$ at $t = 0$ implies a great deviation from unity at the present time. How can we explain the steadiness of this equality along the evolution of the universe?

The prediction of the existence of magnetic monopoles created in the hot early universe is another problem of this model because no observational evidence of their existence has been made yet.

All these problems and others were solved in the context of an inflationary theory.

The first simple model describing an inflationary period was proposed by Alan Guth in 1981 called the "old inflation", it was based on an exponential expansion of the universe in a super-cooled false vacuum state (a state without any particles or fields but with large energy density). This model didn't work and was replaced by a new inflationary model in 1981-1982, however, both were considered as incomplete modifications of the Big Bang model. In 1983, a chaotic inflation scenario was proposed to solve problems of the old and new inflation.

We consider that the very early universe was filled with a scalar field φ called "inflaton" with a mass m and a potential energy density $V(\varphi) = \frac{m^2}{2}\varphi^2$. The Einstein equation for a homogeneous universe filled with the inflaton φ is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{6}(\dot{\varphi}^{\,2} + m^2\varphi^2) \tag{1.28}$$

Where k is the curvature constant, the dot stands for the derivative with respect to the cosmic time.

Because of the expansion of the universe the equation of motion of the scalar field coincides with equation for harmonic oscillator

$$\ddot{\varphi} + 3H\dot{\varphi} = -m^2\varphi \tag{1.29}$$

If φ is large initially then H from Eq. (1.27) and the friction term $3H\dot{\varphi}$ from Eq. (1.28) are large too. This means that the scalar field is moving slowly and maintains an almost constant energy density when the universe is expanding rapidly, so we have at the very beginning

$$H = \frac{m}{\sqrt{6}}\varphi \, , \dot{\varphi} = -\sqrt{\frac{2}{3}}m \tag{1.30}$$

This leads to a slow change in φ and an exponential expansion of the universe with

$$a \propto e^{\frac{m}{\sqrt{6}}\varphi t} \tag{1.31}$$

For small values of $V(\varphi)$ called also the slow-roll potential, the inflaton moves slowly down as a ball in a viscous liquid [45].

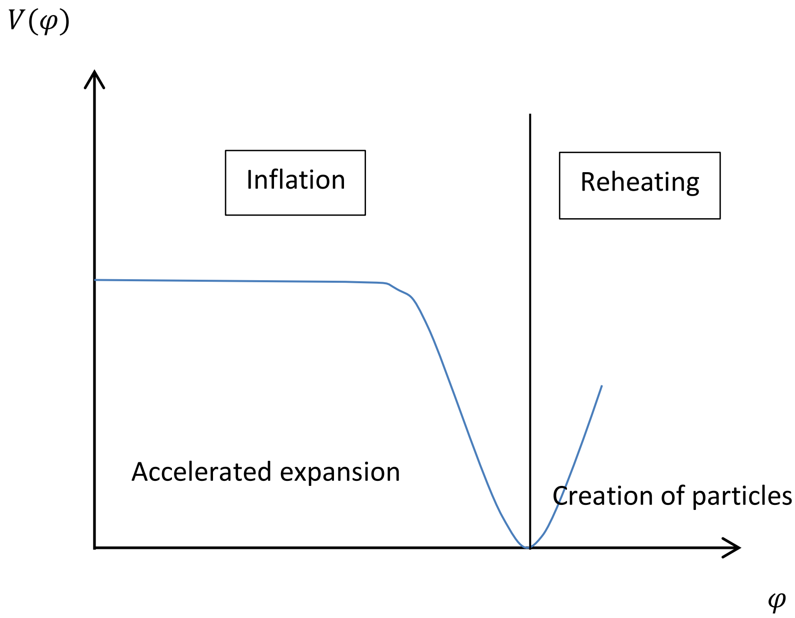


Figure 1.1: At the minimum of $V(\varphi)$, the inflationary period comes to an end and the scalar field rapidly oscillates, the universe enters a reheating period where pairs of elementary particles are created from the scalar field φ.

The inflation period is so rapid, for example, in one of the inflationary models it is approximately 10^{-30}s during this time the universe expands from 10^{-33}cm the Plank size to $10^{10^{10}}$cm. Those numbers are model-dependent but the size of the universe always gain in many orders of magnitude compared to its initial size and compared to the actual horizon size 10^{28}cm, that is the part of the universe that we can see now. In fact, this property is the key solution to both horizon and platitude problems, so even if the universe is initially closed after inflation the distance between its both poles is greater than 10^{28}cm which means that the visible universe looks flat. Similarly, neighboring points in causal contact before inflation will be driven apart in different directions during inflation with a speed greater than the speed of light which gives a plausible explanation to CMB observations and the horizon problem.

1.8 Modeling dark energy and dark matter

Recently, Type Ia Supernovae observational data[1−3] with cosmic microwave background anisotropies[4−6] and large galaxy surveys[7,8] have shown that the universe is undergoing an accelerated expansion phase. The mysterious force or energy leading to the accelerated expansion was attributed to:

1- Vacuum as a vacuum energy with some exotic properties called “dark energy”.

2- An asymptotic behavior of a modified theory of gravity at the cosmological scale. A theory of a modified Newtonian dynamic (MOND) that can solve the problem of the velocity anomalies without the need of a concept of dark energy.

3- Signature of extra-dimensions.

Following the first stream of ideas, the existence of an exotic kind of energy, called dark energy, with negative pressure that drives the universe to expand was proposed and is modeled by several candidates:

1- The cosmological constant Λ with $\rho_V = -\, p_V$
2- Dark energy as a perfect fluid with the equation of state $\rho_V = \omega p_V$, $\omega < -1/3$
3- Dynamical dark energy with $\omega \neq const$
4- Chaplygin gas models.

Astronomers have long known that galaxies and clusters would fly apart unless they were held together by the gravitational pull of much more material than we actually see. The argument that clusters of galaxies would be unbound without dark matter dates back to Zwicky (1937) and others in the 1930s. A wide range of different candidates for dark matter were considered. The first suggested were baryonic, consisting of three quarks , candidates in this category were ionized gas, very faint, low-mass stars and collapsed objects, like stellar black holes. Non-baryonic candidates were also proposed, like neutrinos. MOND is another alternative to dark matter in which the theory of gravitation requires modification without the need to postulate the existence of dark matter [46].

2 Cosmological measurements

2.1 The comoving coordinate system and the cosmic time:

Let us imagine in a region of space particles that are free falling carrying a coordinate system and a clock so the event line between two events of a moving particle will be simply seen as the proper time that collapses between the two events (purely temporal)

$$ds^2 = g_{tt}dt^2 = dt^2 \tag{1.33}$$

This means that $g_{tt} = 1$. We also, have a trajectory that satisfies the free falling equation

$$\frac{\partial^2 x^\nu}{\partial \tau^2} + \boldsymbol{\Gamma^\nu_{ij}} \frac{\partial x^i}{\partial \tau}\frac{\partial x^j}{\partial \tau} = \boldsymbol{\Gamma^\nu_{tt}} = 0 \tag{1.34}$$

which leaves us with $\frac{\partial g_{\sigma t}}{\partial \tau} = 0$.

The coordinate system that satisfies Eq. (1.33) and (1.34) is called Gaussian.

In the comoving coordinate system the observer is moving with the Hubble flow, in other terms the observer expands with the universe expansion, for such an observer the universe is isotopic.

The cosmic time is the proper time of a local observer for whom the local material of the universe is on the average at rest [42].

2. 2 The proper distance and particle horizon

The distance between dl two galaxies at r and $r + dr$ with same angle coordinate is the cosmic time it takes light to travel from r to $r + dr$

$$dl = dt = a(t)\frac{dr}{(1-kr^2)^{1/2}} \tag{1.35}$$

The proper distance between two galaxies at $r = 0$ and r at a fixed cosmic time is

$$d(t,r) = a(t)\int_0^r \frac{dr}{(1-kr^2)^{1/2}} \tag{1.36}$$

The partical horizon is the largest value of r from which we could have received at the present time t_0 a light signal emitted at the earliest possible time.

$$\int_0^{t_0} \frac{dt}{a(t)} = \int_0^{r_{ph}(t_0)} \frac{dr}{(1-kr^2)^{1/2}} \tag{1.37}$$

2.3 The redshift parameter

a signal emitted at t_1 from r arrived at t_0

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r} \frac{dr}{(1-kr^2)^{1/2}} \tag{1.38}$$

another signal emitted at $t_1 + \delta t_1$ from r arrived at $t_0 + \delta t_0$

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)} = \int_0^{r} \frac{dr}{(1-kr^2)^{1/2}} \tag{1.39}$$

As the change in the scale factor $a(t)$ is very insignificant during this period we find

$$\frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)} \tag{1.40}$$

Where δt_0 and δt_1 are the period of light received and emitted. Now, we can write

$$\frac{\lambda_0}{\lambda_1} = \frac{a(t_0)}{a(t_1)} \tag{1.41}$$

Where $\lambda_0$and λ_1 are the wave length of the first and the second signals. Depending on how space is evolving in time (expansion or contraction) during the transit of a signal of light we can have either a red shift result or a blue shift result.

Several astronomical observations in1920's showed that $a(t_0) > a(t_1)$ then $\lambda_0 > \lambda_1$ which means a redshift result and the fractional increase in the wave length is given by the redshift parameter z

$$z = \frac{\lambda_0-\lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1 \tag{1.42}$$

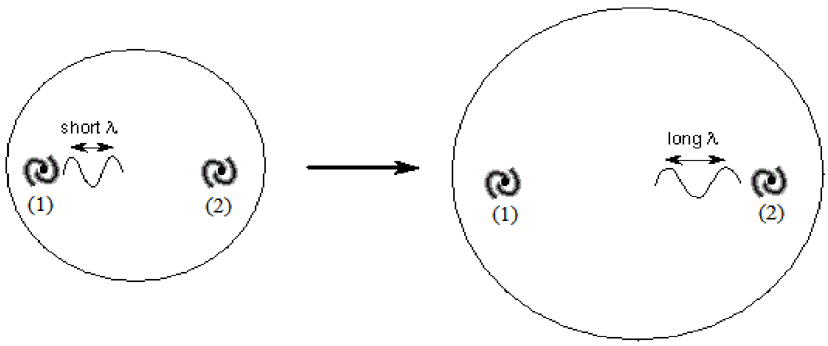


Figure 1.2: As the universe is expanding the wavelength of a signal of light travelling from (1) to (2) is stretched out.

For nearby galaxies where r and $t_0 - t_1$ are small and $k = 0$ we have

$$z = \frac{\dot{a}(t_0)(t_0 - t_1)}{a(t_0)} = r\,\dot{a}(t_0) = \dot{d}(t_0, r) \tag{1.43}$$

Where $d(t_0, r)$ is the proper distance and the dot stands for the derivative with respect the cosmic time. The redshift parameter z here is due to the Doppler shift for low relative velocities $\dot{d}(t_0, r)$ between the observer and the emitter [42].

2.4 The Hubble's law

The Hubble's law states that the distance d of a (nearby) galaxy from us is related to its velocity v [41]. For nearby galaxies, $d(t_0, r) = t_0 - t_1$ so we have

$$z = \frac{\dot{a}(t_0)(t_0 - t_1)}{a(t_0)} = \frac{\dot{a}(t_0)}{a(t_0)}\, d(t_0, r) = H_0 d(t_0, r) \tag{1.44}$$

From (1.40) we can write

$$v = \dot{d}(t_0, r) = H_0 d(t_0, r) \tag{1.45}$$

Where $H_0 = \frac{\dot{a}(t_0)}{a(t_0)}$ is the Hubble's constant.

The Hubble parameter is defined as the rate of expansion of the universe and is given by

$$H = \frac{\dot{a}(t)}{a(t)} \tag{1.46}$$

There is a great uncertainty on its present value H_0 called the Hubble constant and given by (sec^{-1}or Km/sec/Megaparsecs).

The Hubble time ${H_0}^{-1}$ is a time scale for the present universe and at a given Hubble time all galaxies in the universe are located at the same point.

2.5 Luminosity distance

The luminosity L of a galaxy is defined as the total power of radiation emitted per unit time and is related to the flux F by

$$F = \frac{L}{4\,\pi\,{d_L}^2} \tag{1.47}$$

Where d_L is the luminosity distance and is given by

$$d_L = \frac{a^2(t_0)}{a(t_1)}\,r = \frac{(1+z)}{H_0}\,r \tag{1.48}$$

Where $t_1$and t_0 are the time when the light signal is emitted and received from a galaxy at r, for nearby galaxies $d_L \approx d(t_0, r)$.

Since r is not an observable quantity and we need to replace it by an observable quantity, we begin by expanding the redshift parameter z in power series of $(t_0 - t_1)$ for galaxies not far away $z < 1$ we find

$$t_0 - t_1 = \frac{1}{H_0}(z - \left(1 + \frac{q_0}{2}\right) z^2 + \cdots) \tag{1.49}$$

Where q_0 is the present deceleration parameter, then we use Eq. (1.30) to expand r we find

$$r = \frac{1}{a(t_0)}((t_0 - t_1) + \frac{1}{2} H_0 (t_0 - t_1)^2 + \cdots .) \tag{1.50}$$

Hence the distance luminosity will be given by

$$d_L = \frac{1}{H_0}(z + \frac{1}{2}(1 - q_0) z^2 + \cdots) \tag{1.51}$$

Therefore, the measurement of the luminosity distance and the redshift of a sufficient number of galaxies we can determine both H_0 and q_0 in a good approximation [47].

Different methods are used to measure the distance luminosity and every method has its own limits in terms of precision, type of galaxies and the range of the distance scale. One of them consists of finding what astronomers call “Standard Bulbs”, objects with the same intrinsic brightness wherever they are. The distance luminosity in this case depends only on the apparent brightness; the furthest objects are the faintest. Several suggestions were made including “supergiant stars, planetary nebulae, giant ellipticals and brightest member of a galaxy cluster”. Another interesting candidate is type Ia Supernovae, those objects all reach nearly the same intrinsic brightness (4.5 10^9 L_{sun}) and they can be observed in all type of galaxies, even more, their range of distance scale is over the 8 billion light year.

We recall another useful method that is however used for a range of distance scale less than 110 million light year, in this method the astronomers use the variable stars “pulsating stars” to draw their light curves (the apparent magnitude as a function of time in days) and then deduce the luminosity and the distance luminosity. Candidates for variable stars are Cepheids and RR Lyrae stars. Cepheids are the brightest with a greater period (3-50 days) compared to RR Lyrae stars (less than a day). They were the object of measurements in 1920 by Hubble when discovering the expansion of the universe.

2.6 Distance modulus

The distance modulus "μ" is the difference between the apparent magnitude "m" (How bright a star appears in the sky) and the absolute magnitude "M" (How bright a star would appear at 10pc) given by

$$\mu = m - M \tag{1.52}$$

and is related to the luminosity distance d_L by

$$\mu = 5\log(\frac{d_L}{Mpc}) + 25 \tag{1.53}$$

where $Mpc = 3.26\ 10^6\ light\ years$ [47]

2.7 Critical density and the density parameter

As stated before the geometry of the space is determined by the density of things in the universe, and the critical density ρ_c is defined as the amount of density required to have a flat spatial geometry of the universe

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{1.54}$$

The density parameter is the ratio between the total density of stuff in the universe ρ and the critical density

$$\Omega = \frac{\rho}{\rho_c} \tag{1.55}$$

In case of $\Omega > 1$ the universe is closed and $k = +1$, in case of $\Omega = 1$ the universe is flat with $k = 0$ and in case of $\Omega < 1$ the universe is open with $k = -1$.

<u>Chapter two:</u>

Loop Quantum Cosmology

“Science never solves a problem without creating ten more”

George Bernard Shaw.

1 The motivation behind loop quantum gravity

1.1 Why we need to quantize gravity?

Seventy years ago, was the golden age of new ideas for physics and all the breakthroughs were the result of pushing boundaries and limits of the incomplete theories of that time. At the microscopic scale, a complete set of new ideas were proposed to describe the strange behavior of elementary brakes of matter giving birth to Quantum Mechanics. The new theory is background dependent, non-local and probabilistic, where particles are treated as quanta of fields and fields as quanta of particles and the dynamic of such fields is described through the time evolution of the Hilbert space functions with respect to a space background. Whereas, at the macroscopic scale, General relativity attempts to describe the gravitational force as the deformation of the space-time which means that space-time is not anymore an absolute web structure that witnesses the dynamic of other objects but it is treated as a dynamical object itself. The Einstein's new theory is then background independent, deterministic and local so both theories are giving us a schizophrenic understanding of the universe.

May be we need again to push both theories to their limits and explore what happens?

Quantum field theory suffers from UV or short distance divergences, the renormalization by introducing a short-cut off allows us to avoid infinities but also comes with price of ignoring the physics of extreme short distances.

General relativity also has its own divergences, Big Bang or black hole singularities where high energy density is confined in a singularity point results in a divergence of the curvature and a breakdown of the geometry.

In both cases, QFT and GR are pushed beyond their limits when describing extreme short distances of space filled with extreme high energy density. The fabric of space-time is no longer continuous and high energy density requires quantum effects which call for a theory of quantum gravity. In this new theory, QFT and GR can coherently coexist to solve the above inconsistencies [48].

1.2 Why loop quantum gravity?

The quantization of gravity can be treated in two different ways:

One way is to split the metric into a background Minkowsky metric and a perturbative metric to restore the background notion when quantizing the theory. This approach predicts the existence of extra-dimensions of the space-time along with new particles and may lead to a unified theory of all interactions (string theory, M theory). However, the splitting of the metric violates the background independence, the diffeomorphism covariance and leads to the non-renormalizability of the theory .

Another way to do the quantization without additional structure is the canonical quantization of GR where matter and geometry are unified in a non-standard sense making them both transform covariantly under the diffeomorphism group at the quantum level. This is a background independent, non perturbative type of quantization where the fundamental principles of general covariance and quantum mechanics are combined in a consistent mathematical way. Space-time is treated as a dynamical field, interacting with other fields and quantized like any matter field with no need to any background structure [48-50].

2 The Hamiltonian formalism of GR

2.1 The Hamiltonian formalism of a classical theory

The dynamic description of a system is defined by its evolution in time. This evolution is encrypted in the Hamiltonian function or density $H(\mathrm{q}^i, \mathrm{p}_i)$, where q^i are the generalized coordinates of the phase space, p_i the canonical momentum of q^i.

To determine the Hamiltonian density we need first to introduce the Lagrangian density $L(\mathrm{q}^i, \dot{\mathrm{q}}^i)$ as a function of the generalized coordinates q^i and their velocities $\dot{\mathrm{q}}^i$, it is defined at every point (generalized coordinate) of the trajectory of the system.

The canonical momentum p_i of q^i is then given by

$$\mathrm{p}_i = \frac{\partial L(\mathrm{q}^i, \dot{\mathrm{q}}^i)}{\partial \mathrm{q}_i} \tag{2.1}$$

The resulting equations are used to find $\dot{\mathrm{q}}^i = f(\mathrm{p}_i)$.

The Hamiltonian density is then defined by

$$H\left(\mathrm{q}^i, \mathrm{p}_i\right) = \sum_i \mathrm{p}_i \, \dot{\mathrm{q}}^i - L(\mathrm{q}^i, \dot{\mathrm{q}}^i) \tag{2.2}$$

which encodes the dynamic of the system and as a result the equations of motions are given by

$$Hamiltonian\ equations \begin{cases} \dot{\mathrm{q}}^i = \dfrac{\partial H\left(\mathrm{q}^i, \mathrm{p}_i\right)}{\partial \mathrm{p}_i} \\ \dot{\mathrm{p}}_i = -\dfrac{\partial H\left(\mathrm{q}^i, \mathrm{p}_i\right)}{\partial \mathrm{q}^i} \\ \dfrac{\partial H}{\partial t} = -\dfrac{\partial L}{\partial t} \end{cases} \tag{2.3}$$

For systems where $H \neq f(t)$, the Hamiltonian density is the total energy of the system.

However, if we have $H\left(\mathrm{q}^i, \mathrm{p}_i, t\right)$, the third equation of motion in (2.3) is added [51].

For a constrained system the resulting equation of motions are

$$\textit{Hamiltonian equations}\begin{cases} \dot{\mathrm{q}}^i = \frac{\partial H(\mathrm{q}^i,\mathrm{p}_i)}{\partial \mathrm{p}_i} + \lambda^k\{\mathrm{q}^i, C_k(\mathrm{q}^i,\mathrm{p}_i)\} \\ \dot{\mathrm{p}}_i = -\frac{\partial H(\mathrm{q}^i,\mathrm{p}_i)}{\partial \mathrm{q}^i} + \lambda^k\{\mathrm{p}_i, C_k(\mathrm{q}^i,\mathrm{p}_i)\} \end{cases} \tag{2.4}$$

where $C_k(\mathrm{q}^i,\mathrm{p}_i)$are the constraints of the system, λ^k are arbitrary variables called Lagrange multipliers and the Poisson brackets are defined by

$$\{g(\mathrm{q}^i,\mathrm{p}_i), f(\mathrm{q}^i,\mathrm{p}_i)\} = \sum_i \frac{\partial g}{\partial \mathrm{q}^i}\frac{\partial f}{\partial \mathrm{p}_i} - \frac{\partial g}{\partial \mathrm{q}^i}\frac{\partial f}{\partial \mathrm{p}_i} \tag{2.5}$$

2.2 The ADM formalism

In aim to describe the dynamics of the gravitational field using the Hamiltonian approach we need first to fix a proper time from which the evolution of the system is carried out. However, general relativity treats space and time on the same footing, and breaking off the space-time into space and time may break the general covariance (the diffeomorphism) of the theory.

Thanks to Geroch's theorem, a globally hyperbolic space-time M is diffeomorphic to a manifold $\Sigma_{x_0} \otimes R$

$$\varphi : M \to \Sigma_{x_0} \otimes R \tag{2.6}$$

where Σ_{x_0}a hypersurface of equal time $x_0$and φ is a diffeomorphism mapping the space-time manifold M to $\Sigma_{x_0} \otimes R$. We recall that diffeomorphism is an isomorphism on a differential manifold.

A globally hyperbolic space-time contains Cauchy surfaces (spatial-like surfaces) as submanifolds. This is a fundamental requirement because the causality of theory is encoded in Cauchy surfaces where a causal time-like curve intersects the spatial slice only once. A way to see this is to imagine the whole universe at a constant time x_0 as a Cauchy surface Σ_{x_0}.

This feature is very important when formulating the theory as it implies that the behavior of the universe at any time can be derived from initial data of the system. This is dictated by the change of the flow of the time-like curves through space-like surfaces in accordance with Einstein‘s field equations.

Now as we divide the space-time into space-like slices crossed by time-like curves, the metric of the space-time g_{ab} will be written in terms of the induced metric $h_{ab}$of the spatial slices in 4 dimensional indices (a, b) as

$$g_{ab} = h_{ab} - n_a \, n_b \tag{2.7}$$

Where n_a is the normal vector field to the hypersurface and

$$h_{ab} = h_{ij} b_a^i b_b^j \tag{2.8}$$

Where h_{ij} is the induced metric of the slices in 3 dimensional indices (i, j) and

$$b_a^i = \frac{\partial u^a(x^i, x^0)}{\partial x^i} \tag{2.9}$$

Are projectors from 4 dimensional representations to 3dimensional representations and $u^a(x^i, x^0)$ is the hypersurface identified by x^0.

Now, we need to introduce the inner product of the normal vectors which represents the time fixing gauge, the normal vectors are normalized and defined as time –like vectors

$$n^a n_a = -1 \tag{2.10}$$

The time-evolution vector or the deformation vector defined as the flow of time through space time is then decomposed into space and time components

$$t^a = N \;\; n^a + N^a \tag{2.11}$$

Where N is called the lapse function and is defined as the rate of flow of proper time τ with respect to coordinate time t as one moves along n^a

$$d\tau = N dt \tag{2.12}$$

N^a is the shift vector and it measures how much the local spatial coordinate system shifts tangential to Σ_{x^0} when moving from a hypersurface to another along n^a [51].

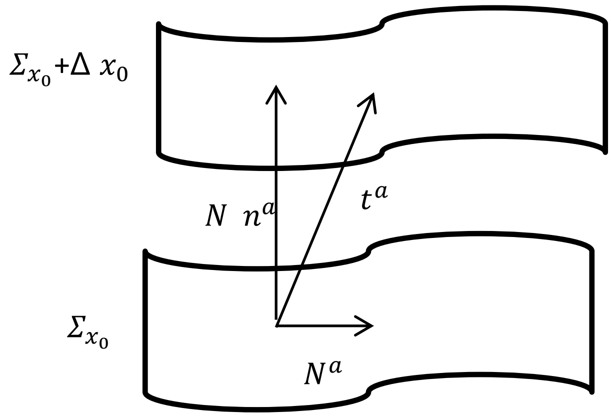


Figure 2.1: we recall that the time-evolution vector as its name indicates, links two points of the same coordinates of two neighboring slices which marks the evolution of time of this point.

From Eq. (2.11) we can write

$$g^{ab} = h^{ab} - N^{-2}(t^a - N^a)(t^b - N^b) \tag{2.13}$$

So the geometry of the space -time can be described by h^{ab}, N^a, N rather than g^{ab}.

2.3 The Hamiltonian formulation of a GR

Now, in aim to proceed with the Hamiltonian formulation of general relativity, we need first to rewrite the Lagrangian density in terms of the new variables (h^{ab}, N^a, N).

The Lagrangian density defined for vacuum space is given by

$$L = \frac{\sqrt{-g}}{16\pi G} R \tag{2.14}$$

where $g = \det(g_{ab})$ and R is the scalar curvature.

As the space-time is foliated into hyeprsurfaces crossed by a flow of time-like curves, we need to determine a new scalar curvature defined on the 3 dimensional hypersurfaces. This is possible by defining a 3 dimensional Riemannian tensor in the same way we define a 4 dimensional Riemannian tensor through covariant derivatives

$$\tilde{R}_{abc}{}^{d}\omega_d = D_a D_b \omega_c - D_b D_a \omega_c \tag{2.15}$$

Where $\tilde{R}_{abc}{}^{d}$ is called the 3 dimensional intrinsic curvature tensor in terms of 4 dimensional indices, D_a is the 3 dimensional covariant derivative in terms of 4-dim indices on Σ_{x_0} and ω_d a spatial 1-form. By using contractions with h^{ab}, we can find the intrinsic Ricci tensor and the intrinsic scalar curvature $\tilde{R}$. The intrinsic term is used to indicate that the variable describing the geometry of Σ_{x_0} doesn't depend on the embedding of Σ_{x_0}in M.

Another important variable called the extrinsic curvature K_{ab} defined by

$$K_{ab} = D_a n_b \tag{2.16}$$

which describes the curvature of the hypersurface Σ_{x_0}as seen by the 4 dimensional manifold, this means that it measures how the normal vector field n_b changes with the way neighboring hypersurfaces are bending that's why it is an extrinsic feature of the geometry of Σ_{x_0} [51,52]. Another way to define K_{ab} is

$$K_{ab} = \frac{1}{2}\mathcal{L}_n h_{ab} \tag{2.17}$$

in which $\mathcal{L}_n$ is the Lie derivative of h_{ab} with respect to the normal vector n_c. One can see that both (2.16) and (2.17) are equivalent weather you choose to see it as a change of n_b with respect to the embedding of hypersurfaces or the change of the geometry of the hypersurface with respect to a parallel transport along the normal vector n_b.

In aim to write the 4 dimensional scalar curvature in terms of the new variables (h^{ab}, N^a, N) we write it first in terms of $(K_{ab}, \tilde{R})$. Developing the Eq. (2.15) and using the projectors h_a^k (from 3 dimensional variables written in 4 dimensional indices a to the 4 dimensional space-time manifold indices k) we find the Gauss equation

$$h_a^k h_b^l h_c^m R_{klm}{}^{h} = \tilde{R}_{abc}{}^{d} + K_{ac}K_b^d - K_{bc}K_a^d \tag{2.18}$$

where $R_{klm}{}^{h}$is the 4 dimensional Riemannian tensor.

In the same way, using the definition of the 4-dim Riemannian tensor we find the Codazzi equation

$$h_a^k h_b^l h_c^m R_{klmh}\, n^h = D_a K_{bc} - D_b K_{ac} \tag{2.19}$$

The Ricci equation is expressed as

$$R_{klmh} n^l\, n^h = n^l (\nabla_k \nabla_l - \nabla_l \nabla_k) n_m \tag{2.20}$$

where $\nabla_k$is the space-time covariant derivative.

Using Eqs. (2.18), (2.19) and (2.20) the Gauss-Codazzi equation is given by

$$R = \tilde{R} + K_{ab}K^{ab} - K^2 - 2\partial_a ADM^a \tag{2.21}$$

Where $\partial_a ADM^a$ is the ADM boundary term defined by

$$\partial_a ADM^a = \partial_a (\, n^a K - a^a) \tag{2.22}$$

in which a^a is the normal acceleration.

The boundary term vanishes for because we assume a sufficiently large surface so the boundary effects are negligible and the final Lagrangian density in ADM formulation is given by

$$L = \frac{N\sqrt{h}}{16\pi G} (\tilde{R} + K_{ab}K^{ab} - K^2) \tag{2.23}$$

where $\sqrt{-g} = N\sqrt{h}$ is easily found when we write the line element of space-time in terms of (h^{ab}, N^a, N) as

$$ds^2 = h_{ij}\big(dx^i + N^i dt\big)\big(dx^j + N^j dt\big) - (Ndt)^2 \tag{2.24}$$

Equation (2.23) can be written as

$$L(h^{ab}, N^a, N) = \frac{N\sqrt{h}}{16\pi G} (\tilde{R} + (h^{ad}\, h^{be} - h^{ab} h^{de}) K_{ab} K_{de}) \tag{2.25}$$

The first thing to notice is that this Lagrangian density is free from terms with time derivatives of (N^a, N), which means that the canonical momentum of those variables vanishes

$$\begin{cases} \pi_c = \frac{\partial L(h^{ab},N^a,N)}{\partial \dot{N}^c} = 0 \\ \pi = \frac{\partial L(h^{ab},N^a,N)}{\partial \dot{N}} = 0 \end{cases} \tag{2.26}$$

Those two equations are the primary constraints of the system.

The canonical momentum of h^{ab} using Eq. (2.25) is given by

$$\pi^{ab} = \frac{\partial L}{\partial \dot{h}_{ab}} = \frac{\sqrt{h}}{16G\pi}(h^{ad}\, h^{be} - h^{ab}h^{de})K_{de} \tag{2.27}$$

Now we can define the Hamiltonian density of the system as

$$H = \pi^{ab}\dot{h}_{ab} + \gamma^C\pi_c + \gamma\pi - L \tag{2.28}$$

Where γ^C, γ are Lagrange multipliers and $q^i \rightarrow h_{ab}$, $p_i \rightarrow \pi^{ab}$.

From Eqs. (2.25), (2.27) and (2.28) the Hamiltonian density can be written as

$$H = NC_{grav} + N^a C_a^{grav} + \gamma^C\pi_c + \gamma\pi \tag{2.29}$$

Where C_{grav} is the Hamiltonian constraint given by

$$C_{grav} = \frac{16\pi G}{\sqrt{h}}\left(\pi^{ab}\,\pi_{ab} - \frac{1}{2}\,\pi^2\right) - \frac{\sqrt{h}}{16\pi G}\tilde{R} \tag{2.30}$$

And C_a^{grav} is the spatial Diffeomorphism constraint given by

$$C_a^{grav} = -2D_b\pi_a^b \tag{2.31}$$

From the primary constraints given by (2.26) we can deduce

$$\begin{cases} \dot{\pi} = \{\pi, H\} = -C_{grav} = 0 \\ \dot{\pi}_c = \{\pi_c, H\} = -C_c^{grav} = 0 \end{cases} \tag{2.32}$$

C_{grav} and C_c^{grav} are called secondary constraints and $N, N^a$are just Lagrange multipliers, those constraints are first class constraints so they generate gauge transformations that don’t change the physical information.

*Diffeomorphism constraint: let's suppose that the time-evolution vector is tangential to Σ_{x_0} ($N = 0, H \propto N^a C_a^{grav}$), this can be interpreted as a tangent translation of $\Sigma_{x_0}$to itself through a spatial diffeomorpic mapping.

*Hamiltonian constraint: in case where time-evolution vector has only a normal component ($N^a = 0, H \propto N C_{grav}$), this can be interpreted as a translation of Σ_{x_0}forwards in the normal direction.

Hence, to generate time-evolution with the Hamiltonian density we need to specify both $N^a$and N. In aim to do this we first can notice that the constrained surfaces represent the physical space in which the Hamiltonian density vanishes, in another word there is no time-evolution with respect to an absolute time which is in agreement with general relativity. This implies that the evolution of the system can be seen as a gauge flow that is arbitrarily parameterized [51,52].

3 The Platini formulation of GR

3.1 The tetrad formalism

The basis vectors of a coordinate basis in the tangential space-time are written as

$$e_a = \partial_a \tag{2.33}$$

Generally speaking, these basis vectors are not orthonormal.

Instead, orthonormal basis are of great interest in physics because working in such basis vectors means working in the local frame of the observer. The attempt to rewrite the theory of general relativity in terms of a new orthonormal coordinate basis called non-holonomic basis leads to the Platini action.

The non-holonomic basis vectors $\tilde{e}_I$ are defined by the inner product

$$\tilde{e}_I \tilde{e}_J = \eta_{IJ} \tag{2.34}$$

where $\eta_{IJ}$is the Minkowski metric and I, J are called 4-dim internal indices.

We have $e_a = e_a^I \tilde{e}_I$ then

$$g_{ab} = e_a e_b = e_a^I \tilde{e}_I e_b^J \, \tilde{e}_J = e_a^I \, e_b^J \, \eta_{IJ} \tag{2.35}$$

and equivalently for $\tilde{e}_I = e_I^a e_a$ we have

$$\eta_{IJ} = \tilde{e}_I \tilde{e}_J = e_I^a e_a e_J^b e_b = e_I^a \, e_J^b \, g_{ab} \tag{2.36}$$

Both $\tilde{e}_I$ and $e_I^a$are called tetrads, they hold all the information contained in g_{ab} and hence can describe the geometry of the space-time instead of the metric g_{ab}. It is important to state that under Lorentz transformations of the tetrad, the metric g_{ab} doesn’t change, this gauge freedom is called internal gauge.

In aim to reformulate the Largrangian density given by Eq. (2.14) using the tetrad formalism we need to define first the covariant derivative D_a in such a frame, which is given by

$$D_a v^I = \nabla_a v^I + \omega_{a\,J}^{\;I} \, v^J \tag{2.37}$$

in which v^I is a vector field in the orthonormal frame and $\omega_{a}{}_J^I$ is the connection 1-form given by

$$\omega_{a}{}_J^I = e^{bI}\nabla_a e_{bJ} \quad (2.38)$$

This can be seen as a parallel transport of the tetrad through the space-time manifold.

In aim to preserve η_{IJ} under the covariance derivation ($\eta_{IJ}$is everywhere the same when a parallel transport is conducted via 1-forms connections), connection 1-forms have to be anti-symmetric on their internal indices

$$\omega_{aIJ} = -\omega_{aJI} \quad (2.39)$$

This requirement implies that the metric $g_{ab}$is also preserved and the connection 1-form is called Lorentz connection.

Now, we need to define the internal Riemannian tensor

$$R_{IJKL} = e_I^a e_J^b e_K^c e_L^d R_{abcd} \quad (2.40)$$

Where R_{abcd} is the space-time Riemannian tensor defined on the tangent space, it is given by the covariant derivative as

$$R_{IJKL} = e_I^a e_J^b e_K^c (\nabla_a \nabla_b - \nabla_b \nabla_a)\, e_{cL} \quad (2.41)$$

By contractions we can find both internal Ricci tensor and the internal scalar curvature.

The curvature of the connection $\omega_{a}{}_J^I$ is defined by

$$F_{ab}^{IJ} = \partial_a \omega_b^{IJ} - \partial_b \omega_a^{IJ} + [\omega_a, \omega_b]^{IJ} \quad (2.42)$$

The imitation Riemannian on the tangent space then is given by

$$R_{ab}{}^{cd} = F_{ab}^{IJ} e_I^c e_J^d \quad (2.43)$$

By contractions we can find both the imitation Ricci tensor and the imitation scalar curvature, so the Platini action reads

$$S[e,\omega] = \frac{1}{16\pi G}\int_M dx^4 |e|\, F_{ab}^{IJ}(\omega) e_I^a e_J^b \quad (2.44)$$

This formalism is called “first order formalism”, where e is the determinant of the tetrad e_I^a.

When differentiating the action with respect to the tetrad we find Einstein equations in the vacuum with the imitation Ricci tensor and the imitation curvature scalar. However, when differentiating with respect to the connection we find the compatibility equation that shows the covariance of the tetrad with respect to a covariant derivative defined by the connection ω.

The constraints of this action are not closed under the Poisson brackets which may complicates the quantization of the theory, a solution for this problem is to modify the Platini action to

$$S[e,\omega] = \frac{1}{16\pi G}\int_M dx^4 |e|\, P^{IJ}_{KL}\, F^{KL}_{ab}(\omega) e^a_I e^b_J \tag{2.45}$$

this modified action is called Holst-Platini action, where $P^{IJ}_{KL} = \delta^{[I}_K \delta^{J]}_L - \frac{1}{2\gamma}\epsilon^{IJ}_{KL}$ with ϵ^{IJ}_{KL} the Levi-Cevita tensor and γ as the Immirzi-Barbero parameter.

3.2 The ADM formalism on the tetrad

As we did previously we split the tetrad into spatial and normal components then with a gauge fixing we define the normal component as the time component.

We define the spatial component ε^a_I as

$$\varepsilon^a_I = e^a_I + n^a n_I \tag{2.46}$$

Where n^a is the unit normal vector to the spatial surface and $n_I = e^a_I n_a$ the normal internal vector to the internal spatial surface with $\varepsilon^a_I n^I = 0$.

Now, we need to fix the time component which is called the time gauge, we define the n^I as the unit internal time-like vector $n^I = \delta^I_0$ and then $e^a_0 = e^a_I n^I = n^a$. This means that we are working in the local frame of an Eulerian observer. We should mention that this gauge fixing doesn't affect the symmetry of the theory.

3.3 The Ashtekar's variables

We define the spatial tensor called the densitized triad [53] by

$$P^a_i = \frac{\sqrt{h}}{8\pi\gamma G}\varepsilon^a_i \tag{2.47}$$

Introducing this new variable to the Holst-Platini action and using Eqs. (2.45), (2.46) and (2.11) one can find after some calculations and integration by part that the variable canonically conjugate to P_i^a called Ashtekar-Barbero connection is given by

$$A_a^i = \Gamma_a^i + y\, K_a^i \tag{2.48}$$

where Γ_a^i is the spin connection defined through the covariant derivative in the tetrad formalism over spatial vector fields, K_a^i is related to the exterior derivative by

$$K_{ab} = K_a^i \varepsilon_{ib} \tag{2.49}$$

A_a^i, P_i^a are called Ashtekar's variables, their Poisson brackets are given by

$$\left\{P_i^a(x^0, x^i), A_b^j(y^0, y^i)\right\} = 8\pi G \delta_b^a\ \delta_j^i\ \delta^3(\boldsymbol{x} - \boldsymbol{y}) \tag{2.50}$$

$$\left\{P_i^a(x^0, x^i), P_j^b(y^0, y^i)\right\} = \left\{A_a^i(x^0, x^i), A_b^j(y^0, y^i)\right\} = 0 \tag{2.51}$$

Now, we introduce the curvature of the A_a^i

$$F_{ab}^i = \partial_a A_b^i - \partial_b A_a^i - \epsilon_{kl}^i A_a^k A_b^l \tag{2.52}$$

This is called also the strength field of A_a^i.

The constraints in the tetrad formalism with the Ashtekra's variables of the Holst-platini action are then

$$\begin{cases} G_i = D_a P_i^a \qquad \mathit{Gauss\ Constraint} \\ C_a^{grav} = P_j^b F_{ab}^j \qquad \mathit{Diffeomorphism\ \ Constraint} \\ C^{grav} = -4\pi G \gamma^2 \frac{P_i^a P_j^b}{\sqrt{h}} \epsilon^{ij}{}_k \left(F_{ab}^k + (1+\gamma^2)\epsilon^k{}_{mn} K_a^m K_b^n\right) \quad \mathit{Hamiltonian\ constraint} \end{cases} \tag{2.53}$$

The Gauss constraint generates gauge transformations as it implies gauge invariance in phase space. This is means that this constraint underlies a gauge symmetry under SU (2) group, which allows to adopt quantization methods used for Yang-Mills models.

It is also important to mention that Ashtekar connections (with complex Immirzi parameter) are simpler to manipulate then the real ones. For instance, the second term in the Hamiltonian constraint vanishes which simplifies the constraint. Ashtekar connections are associated to *SU(2)* group of gauge transformations. This is not the case for real connections; they are

associated with a group of transformations that is not a subgroup of the Lorentz group which means that working with real connections under diffeomorphism is a difficult task.
However, in loop quantum gravity it is much easier to quantize real connections then complex ones especially from Eq. (2.47), which means that the geometry is determined by P_i^a and as a consequence the immirzi parameter needs to be real. Another problem arise from the fact that LQG works only with compact groups which is not the case for complexified *SU(2)* groups and it is much more complicated to find reality conditions for a quantized theory in a complex phase space [53].

4 Isotropic loop quantum cosmology

4.1 Why holonomies?

Wilson loops were first introduced in the study of the strong confinement between quarks, they can be used to describe quantum states with quarks at ending points where they represent lines of non-Abelian electric fluxes and they turn out to be Eigen states of the Hamiltonian in the strong coupling limit.

The canonical quantization of gravity can induce some anomalies when writing the canonical commutation relations of the canonically conjugate variables of the phase space, for instance (h_{ab}, π^{ab}), where we expect a commutation relation of the form

$$[\hat{q}, \hat{p}] = ih \qquad (2.54)$$

where $\hat{p}$ is seen as the generator of q-translation. However, the scalar product in the Hilbert space is not invariant under those translations; this implies that the commutation relations need to be replaced. As a consequence, another approach was proposed by Rovelli and Smolin based on the quantization of the holonomy-flux algebra.

To introduce the definition of the holonomy, we need first to remind that Ashtekar connections are elements of *SU(2)* group and the parallel transport of those connections along a curve α is called holonomy

$$U_\alpha(A) = Pexp(\int_\alpha \ A_a^i \tau_i \, dx^a) \qquad (2.55)$$

Where P is the path operator, $\tau_i = \frac{i}{2}\sigma_i$ are the generators of the algebra of *SU(2)* groups and σ_i are Pauli matrices [52].

4.2 Holonomy-flux algebra

The hypersurface $\Sigma_{x_0}$is divided into faces f delimited by edges e, the idea introduced by Rovelli and Smolin is to use the holonomies along all the possible edges e of the hypersurface Σ_{x_0} , and fluxes on all the possible faces f of the hypersurface Σ_{x_0} as the new variables of the phase-space. Holonomies are already defined in (2.55), fluxes can be defined in the same way by smearing P_i^a in two dimensions: given a 2-dim surface σ, $P_a(\sigma)$ is defined as

$$P_a(\sigma) = \int_\sigma \; n^i P_i^a du dv \tag{2.56}$$

being u and v coordinates on the surface σ and n^i the normal vector.

The Poisson brackets between holonomies and fluxes are

$$[U_{e'}(A), P_a(\sigma)] = i4\pi G\gamma \;\; o(e',\sigma)\; \tau_a U_{e'}(A) \tag{2.57}$$

$$[U_{e''}(A), P_a(\sigma)] = i4\pi G\gamma \; o(e'',\sigma)\; U_{e''}(A)\; \tau_a \tag{2.58}$$

Where $o(e',\sigma)$is the sign of the scalar product $n_i \frac{dx^i}{dt}$.

4.3 The modified Friedman equation

As the universe is not static, the geometry of the spatial hypersurface can be described by the metric

$$h_{ij} = a^2(t) q_{ij} \tag{2.59}$$

where $q_{ij}$is the spatial metric independent of time and $a(t)$is the scale factor.

Now, we write the triad associated to the metric h_{ij} as

$$e_i^a = a(t)\varepsilon_i^a \tag{2.60}$$

where ε_i^a is the triad associated to the metric q_{ij}.

The next step will be to write Friedmann equations in terms of the real connections and their densitized triads. First, from Eq. (2.60) the densitized triad [50] can be written as

$$P_i^a = \frac{\sqrt{q}}{8\pi\gamma G} p(t)\varepsilon_i^a \tag{2.61}$$

Where $|p| = a^2$. Similarly, we find the connection

$$A_a^i = c(t)\varepsilon_a^i \tag{2.62}$$

Where $c = \gamma\dot{a}$ and the dot denotes the derivative with respect to the cosmic time.

We can notice that the new variables c and p are canonically conjugate and can be taken as the new coordinates of the phase-space.

Their Poisson brackets are given by

$$\{c, p\} = \frac{8\pi G\gamma}{3V_0} \tag{2.63}$$

Where $V_0$is the coordinate volume of the spatial hypersurface defined as

$$V_0 = \int_\Sigma \ d^3x \tag{2.64}$$

Now, we rewrite the Hamiltonian density in terms of the new variables. We find that both diffeomorphism and Gauss constraints vanishe because they are both solved by Eqs. (2.61) and (2.62) and we are left with the Hamiltonian constraint. The total Hamiltonian density including matter is then reduced to

$$H = H_{matt} + H_{grav} = H_{matt} + \frac{1}{16\pi G}\int_\Sigma \ NC^{grav} \tag{2.65}$$

Where H_{matt} is the Hamiltonian density for matter defined by

$$H_{matt} = \int_\Sigma \ h\, N\rho\, d^3x = a^3 N\rho\, V_0 = N\rho\, p^{3/2}\, V_0 \tag{2.66}$$

in which ρ is the energy density.

The total Hamiltonian density then reads

$$H = NV_0(\rho\, p^{3/2} - \frac{3}{16\pi G\gamma^2} p^{1/2} c^2) \tag{2.67}$$

As the physical space is constrained by a vanishing Hamiltonian density $H = 0$ we find that

$$\rho = \frac{3}{16\pi G\gamma^2} p^{-1} c^2 \tag{2.68}$$

and when squaring the Hubble parameter, it gives

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{c^2}{\gamma^2 p} = \frac{16\pi G}{3}\rho \tag{2.69}$$

which is the Friedmann equation. This is expected since we are still working in the classical limit. Now, to introduce loop quantum corrections we need to work with holonomies and fluxes.

Substituting Eq. (2.62) in Eq. (2.55) we find

$$U_{\alpha_i}(A) = e^{|\alpha_i|c\tau_i} = \cos\left(\frac{1}{2}|\alpha_i|c\right) + 2\sin\left(\frac{1}{2}|\alpha_i|c\right)\tau_i \tag{2.70}$$

where $U_{\alpha_i}(A)$ is the holonomy along a curve α in the coordinate direction i . The simplest loop (the holonomy along a closed graph) can be constructed from a square as

$$U_{\square ab} = U_{\alpha_a}U_{\alpha_b}U_{\alpha_a}^{-1}U_{\alpha_b}^{-1} \tag{2.71}$$

Using Eq. (2.55) and the fact that the coordinate lengths of the sides of the square are equal we can write

$$U_{\square ab} = 1 - 2\sin\left(\frac{1}{2}|\alpha|c\right)^4 + \sin(|\alpha|c)^2\left(\tau_i\tau_j - \tau_j\tau_i\right) + 4\cos\left(\frac{1}{2}|\alpha|c\right)\sin\left(\frac{1}{2}|\alpha|c\right)^3(\tau_i - \tau_j) \tag{2.72}$$

The strength field of the connection is defined on a closed loop as

$$F_{ab}^i = -2\lim_{|\alpha|\to 0}\frac{tr\left[\tau_k(U_{\square ab}-1)\right]}{|\alpha|^2} \tag{2.73}$$

From Eq. (2.72) we find

$$F_{ab}^i = \lim_{|\alpha|\to 0}\frac{\sin(|\alpha|c)^2}{|\alpha|^2}\epsilon_{ab}^i \tag{2.74}$$

As the area operator has a non-vanishing minimum value in LQG, the coordinate length $|\alpha|$ has also a minimum value $|\alpha_{min}|$ so we can write

$$F_{ab}^i = \frac{\sin(|\alpha_{min}|c)^2}{|\alpha_{min}|^2}\epsilon_{ab}^i \tag{2.75}$$

The new modified Hamiltonian is given by

$$H = NV_0(\rho\, p^{3/2} - \frac{3}{8\pi G\gamma^2}\frac{\sin(|\alpha_{min}|c)^2}{|\alpha_{min}|^2}p^{1/2}) \tag{2.76}$$

The minimum coordinate length needs to be replaced by a fixed physical length l

$$|\alpha_{min}| = \frac{l}{a} = \frac{l}{\sqrt{p}} \tag{2.77}$$

This is due to the fact that the coordinate length depends on the coordinate choice which makes our Hamiltonian dependent on the coordinate choice. l is usually defined as the square root of the minimum area gap of LQG [54].

As the Hamiltonian vanishes we find

$$\sin\left(\frac{l}{\sqrt{p}}c\right)^2 = \frac{8\pi G\gamma^2 l^2}{3}\rho \tag{2.78}$$

To find the modified Friedmann equation we need to calculate the Hubble parameter, that is

$$H = \frac{\dot{a}}{a} = \frac{\dot{p}}{2p} \tag{2.79}$$

Then, we use the Hamiltonian equations of motion to get

$$\dot{p} = \frac{1}{N}\{p, H\} = \frac{2p}{\gamma l}\sin\left(\frac{l}{\sqrt{p}}c\right)\cos\left(\frac{l}{\sqrt{p}}c\right) \tag{2.80}$$

Now, we replace this result in Eq. (2.79) and square

$$H^2 = \frac{1}{\gamma^2 l^2}\sin\left(\frac{l}{\sqrt{p}}c\right)^2\cos\left(\frac{l}{\sqrt{p}}c\right)^2 \tag{2.81}$$

Using Eq. (2.78) we find the modified Friedmann equation expressed as

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{2.82}$$

In which

$$\rho_c = \frac{3}{8\pi G\gamma^2 l^2} \tag{2.83}$$

Chapter three:

Modeling Dark Energy

By VMCG

"Not only is the universe stranger than we think,

It is stranger than we can think"

Werner Heisenberg

1 Viscous Modified Chaplygin Gas Model

1.1 Chaplygin Gas Models

In 1904, S. Chaplygin proposed, in the context of an aerodynamic research [55], a gas model with the equation of state

$$p = -\frac{B}{\rho} \tag{3.1}$$

where B a positive constant, this model is also obtainable from Nambu-Goto action for d-branes moving in a (d+2) dimensional space-time. This fluid has the property of the only known fluid to admit a supersymmetric generalization.

From the conservation equation (1.21) of a universe filled with Chaplygin gas we find

$$\rho = \sqrt{B + \frac{A}{a^6}} \tag{3.2}$$

where A is an integration constant chosen to be positive and a is the scale factor.

At early times , we have a matter like behavior with

$$\rho \approx \sqrt{A}a^{-3},\ p \approx 0 \tag{3.3}$$

At large scale $a \gg 0$ and $\frac{A}{a^6} \ll B$, we have a mixture of cosmological constant and stiff matter ($\rho = p$) like behavior with

$$\rho \approx \sqrt{B} + \frac{A}{\sqrt{4B}}a^{-6}, \quad p \approx -\sqrt{B} + \frac{A}{B}a^{-6} \tag{3.4}$$

At late time $\frac{A}{a^6} \lll B$ and we have a cosmological constant like behavior with

$$\rho \approx \sqrt{B},\ p \approx -\sqrt{B} \tag{3.5}$$

The Chaplygin gas model behavior evolves in time from matter like behavior to a mixture of cosmological constant and stiff matter like behavior to finally a cosmological constant like behavior [13].

The generalized Chaplygin gas with equation of state

$$p = -\frac{B}{\rho^{\alpha}} \tag{3.6}$$

in which $0 \leq \alpha \leq 1$ is a constant, is obtainable from Born-Infeld action [15].

In a universe filled with generalized Chaplygin gas, the conservation equation (1.21) gives

$$\rho = (B + \frac{A}{a^{3(\alpha+1)}})^{\frac{1}{1+\alpha}} \tag{3.7}$$

in which A is an integration constant.

At early times $a \to 0$, we have a matter like behavior with

$$\rho \propto a^{-3},\ p \approx 0 \tag{3.8}$$

At large scale, we have a mixture of cosmological constant and perfect fluid ($p = \alpha\ \rho$) like behavior with

$$\rho = B^{\frac{1}{1+\alpha}} + Ka^{-3(\alpha+1)}, \quad p = -B^{\frac{1}{1+\alpha}} + \alpha\ Ka^{-3(\alpha+1)} \tag{3.9}$$

where $K = \frac{A}{1+\alpha} B^{\frac{-\alpha}{1+\alpha}}$.

At late time, we have a cosmological constant like behavior with

$$\rho = B^{\frac{1}{1+\alpha}}, \quad p = -B^{\frac{1}{1+\alpha}} \tag{3.10}$$

The generalized Chaplygin gas model behavior evolves in time from matter like behavior to a mixture of cosmological constant and perfect fluid like behavior to finally a cosmological constant like behavior.

The modified Chaplygin gas with the equation of state is given by

$$p = A\rho - \frac{B}{\rho^{\alpha}} \tag{3.11}$$

where A is a constant..

In a universe filled with modified Chaplygin gas, the conservation equation (1.21) gives

$$\rho = (\frac{B}{1+A} + \frac{k}{a^{3(\alpha+1)(A+1)}})^{\frac{1}{(1+\alpha)}} \tag{3.12}$$

where k is an integration constant.

At early times, we have a perfect fluid ($p = A\rho$) like behavior with

$$\rho \propto a^{-3(A+1)} \tag{3.14}$$

At large scale, we have a mixture of cosmological constant and perfect fluid of state equation ($p = (A + \alpha + A\alpha)\rho$) like behavior with

$$\rho = \left(\frac{B}{1+A}\right)^{\frac{1}{(1+\alpha)}} + \left(\frac{B}{1+A}\right)^{\frac{-\alpha}{(1+\alpha)}} \frac{k}{1+\alpha} a^{-3(\alpha+1)(A+1)} \tag{3.15}$$

$$p = -\left(\frac{B}{1+A}\right)^{\frac{1}{(1+\alpha)}} + (A + \alpha + A\alpha)\left(\frac{B}{1+A}\right)^{\frac{-\alpha}{(1+\alpha)}} \frac{k}{1+\alpha} a^{-3(\alpha+1)(A+1)} \tag{3.16}$$

At late time, we have a cosmological constant like behavior with

$$\rho = (\frac{B}{1+A})^{\frac{1}{(1+\alpha)}}, \quad p = -(\frac{B}{1+A})^{\frac{1}{(1+\alpha)}} \tag{3.17}$$

1.2 A VMCG dominated universe

The VMCG model is investigated in the framework of the standard cosmology, first we solve the conservation equation of a universe dominated by Viscous MCG to find the energy density in terms of the scale factor and the EoS parameters, then we probe its stability condition at large scale. At small scale, the energy density can describe a matter dominated universe with a specific choice of the EoS parameters.

The EoS of the Viscous MCG is given by

$$P_{eff} = A\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H \rho_{mcg}^{1/2} \tag{3.18}$$

where ρ_{mcg} is the energy density of MCG, ε_0 is a positive bulk viscosity coefficient and $H = \frac{\dot{a}}{a}$ is the Hubble expansion parameter. The dot stands for the derivative with respect to the cosmic time.

We consider a flat Robertson-Walker universe filled with VMCG, the conservation equation and the Friedmann equation are given by

$$\dot{\rho}_{mcg} + 3H\left(\rho_{mcg} + P_{eff}\right) = 0 \tag{3.19}$$

$$H^2 = \frac{\rho_{mcg}}{3} \tag{3.20}$$

The effective pressure of the VMCG is given by

$$P_{eff} = A\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H \rho_{mcg}^{1/2} \tag{3.21}$$

Using the above definition of the effective pressure P_{eff}, the conservation equation can be written as

$$\dot{\rho}_{mcg} + 3H\left(\rho_{mcg} + A\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H \rho_{mcg}^{1/2}\right) = 0 \tag{3.22}$$

We substitute by $H = \frac{\dot{a}}{a}$ and $H = \frac{\rho_{mcg}^{1/2}}{\sqrt{3}}$ to find

$$\dot{\rho}_{mcg} + 3\frac{\dot{a}}{a}\left((1+A)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - \sqrt{3}\varepsilon_0\rho_{mcg}\right) = 0 \tag{3.23}$$

$$\dot{\rho}_{mcg} + 3\frac{\dot{a}}{a}\left((1+A-\sqrt{3}\varepsilon_0)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}}\right) = 0 \tag{3.24}$$

We move all the terms that depend on the scale factor to the left-hand side and those depending on the energy density at the right-hand side of the equality

$$\frac{d\rho_{mcg}}{(1+A-\sqrt{3}\varepsilon_0)\rho_{mcg} - B\rho_{mcg}^{-\alpha}} = -3\frac{da}{a} \tag{3.25}$$

$$\frac{\rho_{mcg}^{\alpha}\, d\rho_{mcg}}{(1+A-\sqrt{3}\varepsilon_0)\rho_{mcg}^{\alpha+1} - B} = -3\frac{da}{a} \tag{3.26}$$

We multiply by $\left(1+A-\sqrt{3}\varepsilon_0\right)(1+\alpha)$ and integrate

$$\int \frac{\left(1+A-\sqrt{3}\varepsilon_0\right)(1+\alpha)\rho_{mcg}^{\alpha}\, d\rho_{mcg}}{(1+A-\sqrt{3}\varepsilon_0)\rho_{mcg}^{\alpha+1} - B} = -3\int\left(1+A-\sqrt{3}\varepsilon_0\right)(1+\alpha)\frac{da}{a} \tag{3.27}$$

We find

$$\ln[(1+A-\sqrt{3}\varepsilon_0)\rho_{mcg}^{\alpha+1} - B] = -3\left(1+A-\sqrt{3}\varepsilon_0\right)(1+\alpha)\ln a + C \tag{3.28}$$

where C an integration constant. Finally we obtain the energy density of the Viscous MCG in term of the scale factor a

$$\rho_{mcg} = \left(\frac{K}{a^{3(\alpha+1)(1+A-\sqrt{3}\varepsilon_0)}} + \frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{1}{\alpha+1}} \tag{3.29}$$

with K an integration positive constant. As the energy density varies with its parameters, we consider the qualitative behavior of the solution of Eq. (3.19) as the parameters $(A, B, \alpha, \varepsilon_0)$ vary using the bifurcation theorem. The dynamical stability of Eq. (3.19) depends on its equilibria and their stability.

Using both Eqs. (3.19) and (3.21) of a universe filled with viscous modified Chaplygin gas, we can write

$$\dot{\rho}_{mcg} = -3H\left(\rho_{mcg} + A\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H\rho_{mcg}^{1/2}\right) \tag{3.30}$$

Then, the equilibria point $\rho_{(mcg)eq}$ is found by solving the equation

$$f(\rho_{mcg}) = \rho_{mcg} + A\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H\rho_{mcg}^{1/2} = 0 \tag{3.31}$$

and we obtain

$$\rho_{(mcg)eq} = \left(\frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{1}{1+\alpha}} \tag{3.32}$$

To probe the stability of this point we solve

$$\left.\frac{d(Hf(\rho_{mcg}))}{d\rho_{mcg}}\right|_{\rho_{(mcg)eq}} = 0 \tag{3.33}$$

We find that the equilibria point is only stable if ($\alpha > -1$ and $1 + A - \sqrt{3}\varepsilon_0 > 0$) which means that at late time the energy density is only stable for a positive choice of B and is given by $\rho_{mcg} = \left(\frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{1}{\alpha+1}}$ corresponding to dark energy dominated universe.

At the early universe, we have a perfect fluid ($p = (A - \sqrt{3}\varepsilon_0)\rho$) like behavior with

$$\rho_{mcg} \propto a^{-3(1+A-\sqrt{3}\varepsilon_0)} \tag{3.34}$$

If we take $A = \sqrt{3}\varepsilon_0$, the energy density becomes $\rho_{mcg} \propto a^{-3}$ which corresponds to matter-dominated universe.

At large scale, we have

$$\rho = \left(\frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{1}{(1+\alpha)}} + \left(\frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{-\alpha}{(1+\alpha)}} \frac{k}{1+\alpha} a^{-3(\alpha+1)\left(A-\sqrt{3}\varepsilon_0+1\right)} \tag{3.35}$$

$$p = -\left(\frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{1}{(1+\alpha)}} + \left(\left(A-\sqrt{3}\varepsilon_0\right)(1+\alpha)+\alpha\right)\left(\frac{B}{1+A-\sqrt{3}\varepsilon_0}\right)^{\frac{-\alpha}{(1+\alpha)}} \frac{k}{1+\alpha} a^{-3(\alpha+1)\left(A-\sqrt{3}\varepsilon_0+1\right)} \tag{3.36}$$

a mixture of cosmological constant and perfect fluid $(p = (\left(A-\sqrt{3}\varepsilon_0\right)(\alpha+1)+\alpha)\rho)$ like behavior. The latter also corresponds to stiff matter used in cosmology to model dark matter when it is made of relativistic self-gravitating Bose-Einstein condensates (BECs) [56].

The deceleration parameter is given by

$$q = -1 - \frac{\dot{H}}{H^2} \tag{3.37}$$

Now, using Eq. (3.20) we can write

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(\left(1+A-\sqrt{3}\varepsilon_0\right) - \frac{B}{\rho_{mcg}^{\alpha+1}}) \tag{3.38}$$

Then, the deceleration parameter of the VMCG can be written as

$$q = -1 + \frac{1}{2}[\left(1+A-\sqrt{3}\varepsilon_0\right) - \frac{B}{\rho_{mcg}^{\alpha+1}}] \tag{3.39}$$

The effective state parameter is given by

$$\omega_{eff} = \frac{P_{eff}}{\rho_{mcg}} = A - \sqrt{3}\varepsilon_0 - \frac{B}{\rho_{mcg}{}^{\alpha+1}} \tag{3.40}$$

A small non-negative sound speed ($c^2 \leq 1$) for matter component is necessary for forming the large scale structure of our Universe, c^2reduces to

$$c^2 = \frac{dP_{eff}}{d\rho_{mcg}} = A - \sqrt{3}\varepsilon_0 + \alpha\frac{B}{\rho_{mcg}{}^{\alpha+1}} \tag{3.41}$$

At large scale the values of deceleration parameter, effective state parameter and the sound speed are the following

$$q = -1, \omega_{eff} = -1, c^2 = (\alpha+1)\left(A-\sqrt{3}\varepsilon_0\right) + \alpha \tag{3.42}$$

2 Constraining VMCG in Standard Cosmology

2.1 The χ^2test

The collected observational data in cosmology are getting larger and more accurate. This requires statistical tools to combine and analyze those data in a way where "accuracy" and "precision" are achieved to derive plausible cosmological models and conclusions that are in the best way consistent with those data.

When dealing with statistics we are dealing with probabilities and cosmologists interpret probabilities as the measure of the degree of belief in hypothesis. The probability that a random variable x can take a specific value is the probability distribution $P(x)$, and it has the following properties:

1- It is a positive defined real value $P(x) \geq 0$.
2- It is normalized over the spectrum of all possible values of the random variable x such as $\int P(x)\, dx = 1$ for continuous values of x or $\sum P(x) = 1$ for discrete values of x.
3- For two independent events a and b, the probability that a or b to happen $P(a+b)$ is the sum of individual probabilities $P(a)$ and $P(b)$.

$$P(a+b) = P(a) + P(b) \tag{3.43}$$

4- The probability of a and b to happen $P(a,b)$ is the probability of a times the conditional probability $P(a|b)$ of a given b

$$P(a,b) = P(a).P(a|b) \tag{3.44}$$

In cosmology, every proposed cosmological model has to predict at least most of the collected observational data with the minimum bar of errors. This means that the parameters describing the cosmological model need to be fitted to the observational data, and statistically speaking those parameters are the hypothesis that we need to measure the degree of belief (their consistency with the data) so we define

$$P(H|D) = \frac{P(H)P(D|H)}{P(D)} \tag{3.45}$$

where $P(H|D)$ is the posterior defined as the conditional probability of hypothesis H given a set of data D , $P(H)$ the prior, $P(D|H)$ the likelihood and $P(D) = 1$ (we assume that the data are already collected).

Given a set of observational data we want to define a function that can measure the agreement between the cosmological model and the data and then maximize it. As the model is described by a set of parameters, if we set $P(D) = 1$ and ignore the prior we can maximize the likelihood $P(D|H)$, the probability that the observational data D occurred given a set of parameters H, by adjusting the parameters to find the most likely hypothesis. The problem with this procedure is that we ignore both $P(D)$ and the prior which does not provide a goodness of fit or an absolute probability of the model and because of that cosmologist adopted another way to fit their models using the least squares χ^2 analyses [57].

We assume that we have a set of observational data D_i and a model to provide those data $f(x, b_i)$ that depends on a set of parameters b_i the least squares χ^2 is defined as

$$\chi^2 = \sum_i \frac{(D_i - f(x,b_i))^2}{{\sigma_i}^2} \tag{3.46}$$

in which $\sigma_i$is the error on the data D_i. The best fit parameters are those that minimize the χ^2.

The Chi-by-eye rule states that the minimum value of χ^2 should be roughly equal to the degree of freedom (number of data - number of the fitted parameters). From this,

$$\chi^2{}_{min}/d.o.f \approx 1. \tag{3.47}$$

Errors should be associated to the best fit parameters, if we move away from the best fit values the χ^2 will increase with $\Delta\chi^2$. The contour of a constant $\Delta\chi^2$ draws the boundary of a confidence region where the values of the parameters (a phase space defined by the values of just two parameters) are defined within a certain interval of error called the “confidence interval “ [57].

p	ν=1	ν=2	ν=3
68.27%	1	2.3	3.53
90%	2.71	4.61	6.25
95%	4	6.18	8.02

Table 3.1: $\Delta\chi^2$as a function of the confidence level p and the number of the fitted parameters ν.

If the number of the fitted parameters is large and if we know the probability distribution of all the parameters $P(b_i, k)$ regardless the values of one of them "k" then we can marginalize over this parameter and we have

$$P(b_i) = \int P(b_i, k)\, dk \tag{3.48}$$

2.2 The Best Fit values of the EoS parameters of VMCG model

we constrain the EoS parameters of the VMCG model using Supernovae Type Ia observational data that consists of 580 data points and belong to Union 2.1 (2012) data [58], the best fit values of the parameters are obtained by the minimization of the χ^2 function.

The luminosity distance $d_L(z)$ determines the dark energy density and is defined in a flat universe by

$$d_L\left(z, H_0, A, \acute{B}, \alpha, \varepsilon_0\right) = c\ (1+z) \int_0^z \frac{d\grave{z}}{H(\grave{z}, H_0, A, \acute{B}, \alpha, \varepsilon_0)} \tag{3.49}$$

Where z is the redshift parameter, c the speed of light and $H_0$is the present Hubble parameter. The distance modulus for Supernovas is given by

$$\mu\left(z, H_0, A, \acute{B}, \alpha, \varepsilon_0\right) = 5\log_{10}[\frac{d_L(z, H_0, A, \acute{B}, \alpha, \varepsilon_0)}{1MPc}] + 25 \tag{3.50}$$

The χ-square function χ^2 measures the goodness-of-fit of the model to the data and is defined by

$$\chi^2(H_0, A, \acute{B}, \alpha, \varepsilon_0) = \sum \frac{(\mu(z, H_0, A, \acute{B}, \alpha, \varepsilon_0) - \mu_{obs}(z, H_0, A, \acute{B}, \alpha, \varepsilon_0))^2}{\sigma_z^2} \tag{3.51}$$

Where $\mu_{obs}(z)$ is the observed distance modulus at redshift z and σ_z^2 its variance.

Using the field Eq. (3.2) in the presence of a baryonic matter

$$\dot{\rho}_{mcg} + 3H\left(\rho_{mcg} + A\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H \rho_{mcg}^{1/2}\right) = 0 \tag{3.52}$$

We substitute by $H = \frac{\dot{a}}{a}$

$$\dot{\rho}_{mcg} + 3\frac{\dot{a}}{a}\left((1+A)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H \rho_{mcg}^{1/2}\right) = 0 \tag{3.53}$$

$$d\rho_{mcg} + 3\frac{da}{a}\left((1+A)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H\rho_{mcg}^{1/2}\right) = 0 \tag{3.54}$$

We divide by da

$$\frac{d\rho_{mcg}}{da} + \frac{3}{a}\left((1+A)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H\rho_{mcg}^{1/2}\right) = 0 \tag{3.55}$$

We substitute by $a = \frac{1}{1+z}$ and $da = -\frac{dz}{(1+z)^2}$

$$-(1+z)^2\frac{d\rho_{mcg}}{dz} + 3(1+z)\left((1+A)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H\rho_{mcg}^{1/2}\right) = 0 \tag{3.56}$$

Then we have

$$(1+z)\frac{d\rho_{mcg}}{dz} = 3\left((1+A)\rho_{mcg} - \frac{B}{\rho_{mcg}^{\alpha}} - 3\varepsilon_0 H\rho_{mcg}^{\frac{1}{2}}\right) \tag{3.57}$$

We divide by $3H_0^2$ and substitute $\Omega_{mcg} = \frac{\rho_{mcg}}{3H_0^2}$

$$(1+z)\frac{d\Omega_{mcg}}{dz} = 3\left((1+A)\Omega_{mcg} - \frac{B}{3H_0^2\rho_{mcg}^{\alpha}} - \frac{3\varepsilon_0 H\Omega_{mcg}^{\frac{1}{2}}}{\sqrt{3}H_0}\right) \tag{3.58}$$

$$(1+z)\frac{d\Omega_{mcg}}{dz} = 3\left((1+A)\Omega_{mcg} - \frac{B}{(3H_0^2)^{\alpha+1}\Omega_{mcg}^{\alpha}} - \frac{\sqrt{3}\varepsilon_0 H\Omega_{mcg}^{\frac{1}{2}}}{H_0}\right) \tag{3.59}$$

$$(1+z)\frac{d\Omega_{mcg}}{dz} = 3\left((1+A)\Omega_{mcg} - \grave{B}\Omega_{mcg}^{-\alpha} - \frac{\sqrt{3}\varepsilon_0 H\Omega_{mcg}^{\frac{1}{2}}}{H_0}\right) \tag{3.60}$$

Where $\grave{B} = \frac{B}{(3H_0^2)^{\alpha+1}}$ and the Hubble parameter is defined by the field equation in the presence of a baryonic matter as

$$H^2 = \frac{\rho_{tot}}{3} = \frac{1}{3}(\rho_m + \rho_{mcg}) \tag{3.61}$$

We divide by H_0^2

$$\frac{H^2}{H_0^2} = \left(\Omega_m + \Omega_{mcg}\right) = (\Omega_{0m}(1+z)^3 + \Omega_{mcg}) \tag{3.62}$$

where Ω_{0m} is the present value of the baryonic matter density.

The Hubble parameter then can be defined in term of the redshift parameter by

$$H(z) = H_0[\Omega_{0m}(1+z)^3 + \Omega_{mcg}]^{1/2} \tag{3.63}$$

We substitute the above result in Eq. (3.63) we obtain

$$(1+z)\frac{d\Omega_{mcg}(z)}{dz} = 3\left((1+A)\Omega_{mcg} - \grave{B}\Omega_{mcg}^{-\alpha} - \sqrt{3}\varepsilon_0\Omega_{mcg}^{\frac{1}{2}}(z)(\Omega_{0m}(1+z)^3 + \Omega_{mcg}(z))^{1/2}\right) \tag{3.64}$$

This equation need to be solved numerically in order to minimize the Chi-square function.

To reduce the number of the free parameters we marginalize assuming a constant prior over H_0; we construct a probability density function for the only parameters $(A, \acute{B}, \alpha, \varepsilon_0)$

$$\boldsymbol{P}(A, \acute{B}, \alpha, \varepsilon_0) = \int cte.\, e^{-\chi^2/2}.\ P(H_0)\, dH_0 \tag{3.65}$$

where $P(H_0)$ is the prior probability density function of the present Hubble constant H_0.

First, we write the dimensionless luminosity distance $\tilde{d}_L$ that doesn't depend on H_0

$$\tilde{d}_L(z, A, \acute{B}, \alpha, \varepsilon_0) = H_0 d_L(z, H_0, A, \acute{B}, \alpha, \varepsilon_0)/c = (1+z)\int_0^z \frac{d\grave{z}}{O(\grave{z}, H_0, A, \acute{B}, \alpha, \varepsilon_0)} \tag{3.66}$$

where $O(\grave{z}, H_0, A, \acute{B}, \alpha, \varepsilon_0) = H(\grave{z}, H_0, A, \acute{\acute{B}}, \alpha, \varepsilon_0)/H_0$.

Then, the distance modulus can be written as

$$\mu(z, H_0, A, \acute{B}, \alpha, \varepsilon_0) = 5\log_{10}[\frac{\tilde{d}_L(z, A, \acute{\acute{B}}, \alpha, \varepsilon_0).c}{1MPc.H_0}] + 25 \tag{3.67}$$

We define the dimensionless Hubble parameter $\widetilde{H_0} = H_0.\frac{MPc}{c}$, the Eq. (3.70) becomes

$$\mu(z, H_0, A, \acute{B}, \alpha, \varepsilon_0) = (5\log_{10}[\tilde{d}_L\left(z, A, \acute{\acute{B}}, \alpha, \varepsilon_0\right)] + 25) - 5\log_{10}[\,\widetilde{H_0}\,] \tag{3.68}$$

Where we put $\tilde{\mu}(z, A, \acute{B}, \alpha, \varepsilon_0) = 5\log_{10}[\tilde{d}_L\left(z, A, \acute{\acute{B}}, \alpha, \varepsilon_0\right)] + 25$ as the distance modulus free from $\widetilde{H_0}$. The new χ^2 function is given by

$$\chi^2(H_0, A, \acute{B}, \alpha, \varepsilon_0) = \sum \frac{(\tilde{\mu}(z, A, \acute{B}, \alpha, \varepsilon_0) - \mu_{obs} - 5\log_{10}[\,\widetilde{H_0}\,])^2}{\sigma_z^2} \tag{3.69}$$

This can be rewritten as

$$\chi^2\left(H_0, A, \acute{B}, \alpha, \varepsilon_0\right) = \sum \frac{(\widetilde{\mu}(z,A,\acute{B},\alpha,\varepsilon_0)-\mu_{obs})^2}{\sigma_z^2} - 2.\,(5\log_{10}[\,\widetilde{H_0}\,])\sum \frac{(\widetilde{\mu}(z,A,\acute{B},\alpha,\varepsilon_0)-\mu_{obs})}{\sigma_z} + (5\log_{10}[\,\widetilde{H_0}\,])^2 \sum \frac{1}{\sigma_z^2} \tag{3.70}$$

We put $F = \sum \frac{(\widetilde{\mu}(z,A,\acute{B},\alpha,\varepsilon_0)-\mu_{obs})^2}{\sigma_z^2}$, $G = \sum \frac{(\widetilde{\mu}(z,A,\acute{B},\alpha,\varepsilon_0)-\mu_{obs})}{\sigma_z}$, $C = \sum \frac{1}{\sigma_z^2}$ and $x = 5\log_{10}[\,\widetilde{H_0}\,]$

We have then

$$\chi^2\left(H_0, A, \acute{B}, \alpha, \varepsilon_0\right) = C\,x^2 - 2G.x + F \tag{3.71}$$

We substitute this result in Eq. (3.65) we find

$$\boldsymbol{P}\left(A, \acute{B}, \alpha, \varepsilon_0\right) = \int_{-\infty}^{+\infty} cte.\,e^{-C\,x^2+2G.x-F/2}.\ P\left(\widetilde{H_0}\right) d\widetilde{H_0} \tag{3.72}$$

The integration is over all the possible values of $\widetilde{H_0}$. We choose the prior to be constant which means that any value for $\widetilde{H_0}$ has the same probability of being. After changing variables in Eq. (3.75) we obtain

$$\boldsymbol{P}\left(A, \acute{B}, \alpha, \varepsilon_0\right) = \int_{-\infty}^{+\infty} cte.\frac{\ln(10)}{5}.\,e^{-C\,x^2+2\left(G+\frac{\ln(10)}{5}\right).x-F/2}\,dx \tag{3.73}$$

This can be developed to

$$\boldsymbol{P}\left(A, \acute{B}, \alpha, \varepsilon_0\right) = cte.\frac{\ln(10)}{5}.\,e^{(\frac{\acute{G}^2}{C}-F)/2} \int_{-\infty}^{+\infty} e^{-C(x-\frac{\acute{G}}{C})^2/2}\,dx \tag{3.74}$$

Where $\acute{G} = G + \frac{\ln(10)}{5}$. The integral has the form of a Gaussian distribution so we find

$$\boldsymbol{P}\left(A, \acute{B}, \alpha, \varepsilon_0\right) = cte.\sqrt{\frac{2\pi}{C}}.\frac{\ln(10)}{5}.\,e^{(\frac{\acute{G}^2}{C}-F)/2} = cte.\,e^{-\widetilde{\chi}^2/2} \tag{3.75}$$

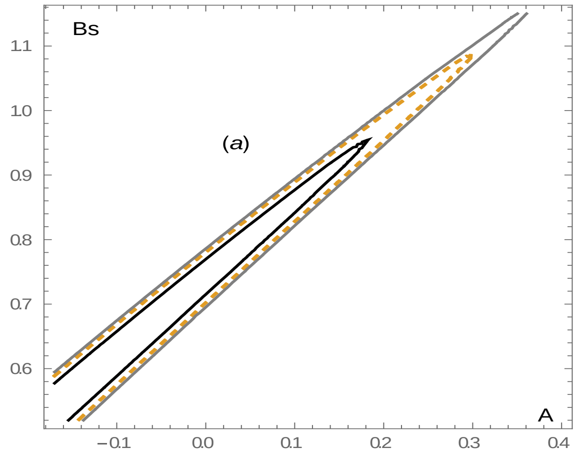
where $\tilde{\chi}^2$ is the new χ^2 function free from H_0 given by

$$\tilde{\chi}^2\,(A, \acute{B}, \alpha, \varepsilon_0) = F\left(A, \acute{B}, \alpha, \varepsilon_0\right) - \frac{\left(G(A,\acute{B},\alpha,\varepsilon_0)+\ln 10/5\right)^2}{C} \tag{3.76}$$

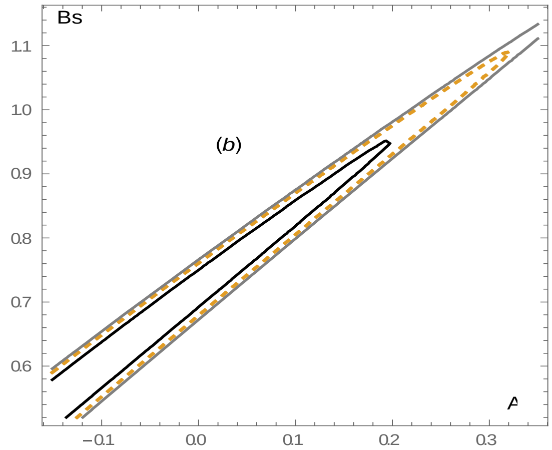
As the number of free parameters is still large we first fix the viscous coefficient that is assumed to be positive, then we constrain the EoS parameters$(A, \acute{B}, \alpha)$. We find that only small values of ε_0 corresponding to $\omega \approx -1$ are consistent with the observational data. The best fit values of EoS parameters in this case are listed in Table 3.2 where we find that $\acute{B}$ and α have approximately the same values for different choices of ε_0. The counter plot of the best fit values of both $(A\ , \acute{B})$ corresponding to the confidence levels 68.27%, 90% and 95.45% are shown in Figure 3.1.

.EoS parameters	ε_0	α	A	$\acute{B}$	χ^2_{cp}	$\chi^2_{cp}/d.o.f$
Best fit values	0.01	0.551	-0.167	0.543	562.191	0.974
	0.02	0.548	-0.149	0.543	562.191	0.974
	0.0001	0.549	-0.186	0.543	562.191	0.974

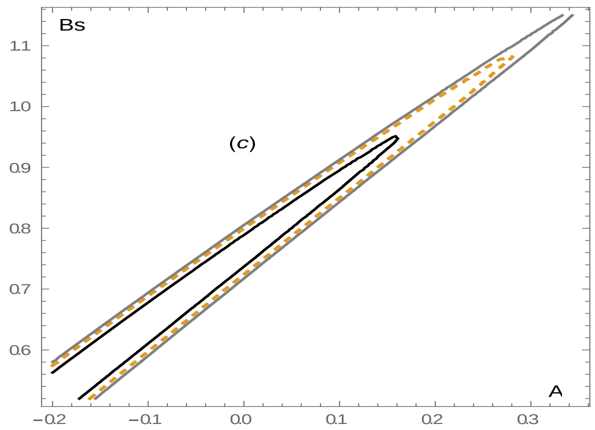
Table 3.2: Summary of the best estimates of the EoS parameters for the Viscous MCG. The best fit values are computed using Union 2.1 SNe Ia data, *d.o.f* denotes the degrees of freedom.



(a)



(b)



(c)

Figure 3.1: Contour plot of 68.27 % CL (black), 90% CL (dashed) and 95.45 % CL (gray) regions for VMCG parameters A and $\acute{B}$ when (a) $\varepsilon_0 = 0.01$, (b) $\varepsilon_0 = 0.02$ and (c) $\varepsilon_0 = 0.0001$.

2.3 Cosmological parameters in terms of the Best Fit values of the EoS parameters

We explore the behavior of the deceleration parameter, the state parameter, the adiabatic sound speed and the curvature scalar as functions of the redshift parameter till the present epoch at the best fit values of VMCG EoS parameters summarized in Table 1.

The Hubble parameter is defined by the field equation

$$H^2 = \frac{\rho_{tot}}{3} \tag{3.77}$$

we differentiate H^2 with respect to the cosmic time then we divide by $2H^3$

$$\frac{\dot{H}}{H^2} = \frac{\dot{\rho}_{tot}}{6H^3} = -\frac{1}{2H^2}\left(\left(\rho_m + \rho_{mcg}\right) + P_{eff}\right) = -\frac{1}{2}(3 + \frac{P_{eff}}{H^2}) \tag{3.78}$$

by substituting $\frac{\dot{H}}{H^2}$ in q given in Eq. (3.37)we find

$$q(z) = -1 + \frac{1}{2}[3 + \frac{(\omega_{eff}\rho_{mcg})}{H^2(z)}] \tag{3.79}$$

The effective state parameter is given by

$$\omega_{eff} = \frac{P_{eff}}{\rho_{mcg}} = A - 3\varepsilon_0 \frac{H(z)}{\sqrt{\rho_{mcg}}} - \frac{B}{{\rho_{mcg}}^{\alpha+1}} \tag{3.80}$$

The sound speed parameter is given by

$$c^2 = \frac{dP_{eff}}{d\rho_{mcg}} = A + \alpha\frac{B}{\rho_{mcg}^{\alpha+1}} + \frac{d}{d\rho_{mcg}}(-3\varepsilon_0 H \rho_{mcg}^{\frac{1}{2}})$$

$$= A + \alpha\frac{B}{\rho_{mcg}^{\alpha+1}} - 3\varepsilon_0\left(\frac{1}{2}H\rho_{mcg}^{-\frac{1}{2}}\right) - 3\varepsilon_0\rho_{mcg}^{1/2}\frac{dH}{d\rho_{mcg}} \tag{3.81}$$

We differentiate the Hubble parameter with respect to ρ_{mcg} we find

$$\frac{dH}{d\rho_{mcg}} = \frac{1}{\sqrt{3}}\frac{d}{d\rho_{mcg}}(\rho_{mcg} + \rho_m)^{1/2} = \frac{1}{2\sqrt{3}}(\rho_{mcg} + \rho_m)^{-1/2} = \frac{1}{6}H^{-1} \tag{3.82}$$

Then, the sound speed parameter will be given by

$$c^2 = A - \frac{3}{2}\varepsilon_0\frac{H(z)}{\sqrt{\rho_{mcg}}} - \frac{1}{2}\varepsilon_0\frac{\sqrt{\rho_{mcg}}}{H(z)} + \alpha\frac{B}{\rho_{mcg}^{\alpha+1}} \tag{3.83}$$

The energy density ρ_{mcg} is solved numerically and used to plot the above cosmological parameters at the best fit values of the EoS parameters shown in Table 3.2. The curves are plotted using Mathematica software.

In figure 3.2, the sound speed is plotted in terms of the redshift parameter using the best fit data listed in Table 3.2. In the early universe, the sound speed has negative values introducing fast exponential growth of instabilities, this anomaly can be explained by the fact that VMCG is an effective coupled dark energy/ dark matter fluid, in such models instabilities can occur when the coupling strength is strong compared to gravitational strength [59]. Moreover, when the coupling becomes moderate in the transition from a matter dominated universe to a dark energy dominated universe, the sound speed c^2 changes sign to take positive values and the perturbations grow much slower until the universe is dominated by dark energy. At large scale, the sound speed takes a positive value near zero leading to stable oscillating perturbations and structure predictions consistent with observations.

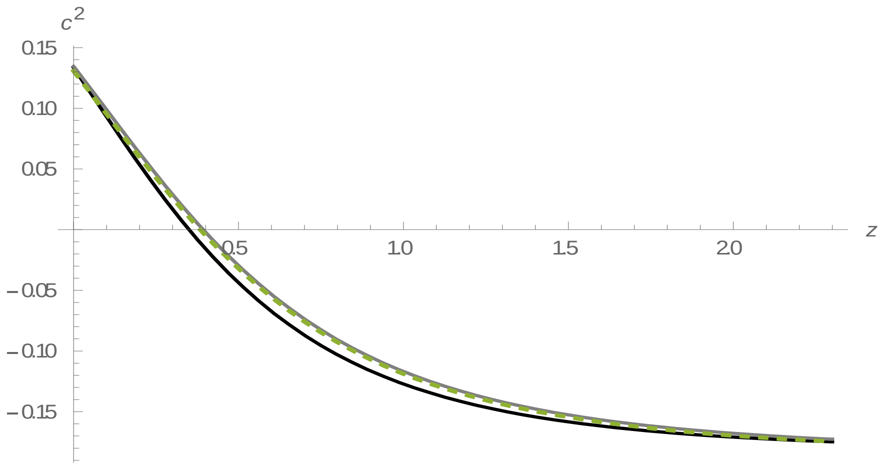


Figure 3.2: The sound speed c^2 as a function of the redshift z at best fit values of Table 3.2 for $\varepsilon_0 = 0.01$ (gray line), $\varepsilon_0 = 0.02$ (black line) and $\varepsilon_0 = 0.0001$ (dashed line).

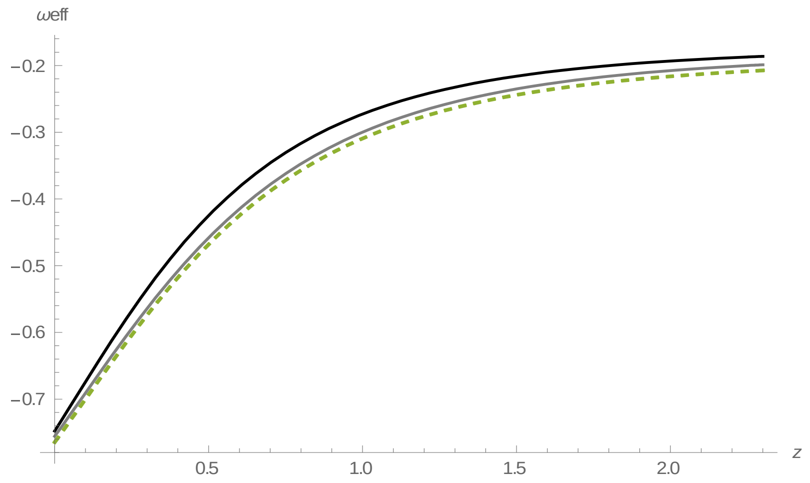


Figure 3.3: The evolution of the effective state parameter ω_{eff} at best fit values of Table 3.2 $\varepsilon_0 = 0.01$ (gray line), $\varepsilon_0 = 0.02$ (black line) and $\varepsilon_0 = 0.0001$ (dashed line).

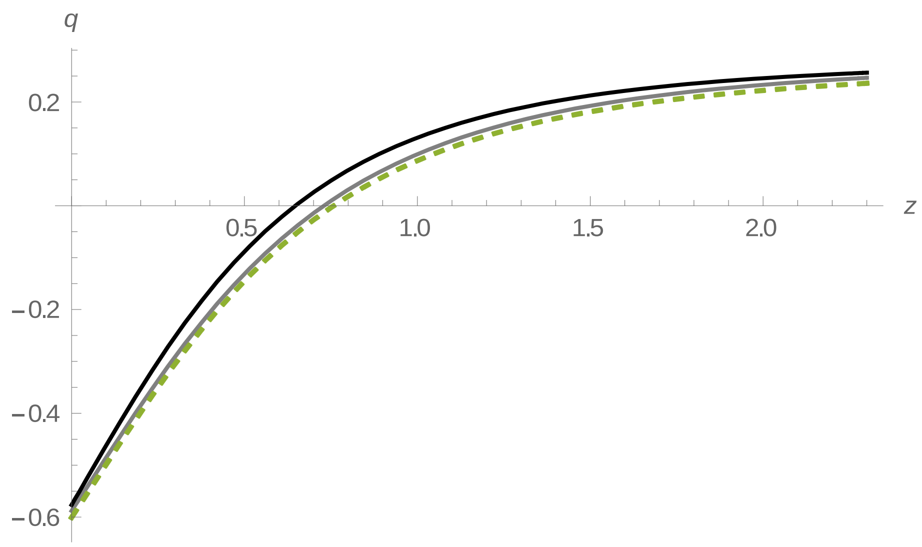


Figure 3.4: The variation of the deceleration parameter q at best fit values of Table 3.2 for $\varepsilon_0 = 0.01$ (gray line), $\varepsilon_0 = 0.02$ (black line) and $\varepsilon_0 = 0.0001$ (dashed line).

Figure 3.3 and figure 3.4, show respectively the variation of the effective state parameter and the deceleration parameter with redshift *z* at the best fit values of Table 3.2, it is obvious that the current value of ω_{eff}varies between -0.76 and -0.74 for different values of $\varepsilon_0$admitting an accelerated universe . At matter dominated era, ω_{eff} takes values of the range $\omega_{eff} > -1/3$ permitting a deceleration phase. The effective state parameter is slightly negative in this era due to the transition between the two epochs of matter dominated universe and dark energy dominated universe. When the deceleration parameter crosses the zero to negative values, the ω_{eff} takes values less than -0.33 and the VMCG behaves like quintessence scalar field.

In figure 3.4, for all best values of Table 3.2, the current deceleration parameter value varies between -0.60 and -0.57 which is consistent with $q_0 \in (-0.7, -0{,}4))$ given by the standard ΛCDM cosmology [60] . Moreover, a transition from decelerated $q < \frac{1}{2}$ to accelerated universe $q < 0$ is realized when q crosses the zero, and thus the universe passes from matter dominated universe to dark energy dominated universe. When the deceleration parameter crosses the zero (q=0) the universe passes from matter dominated universe where ($\rho_{DE} \approx \rho_{matter}$) and undergoes an accelerated phase. The crossing happened at $z = 0.75$ for both $\varepsilon_0 = 0.01$ and $\varepsilon_0 = 0.0001$ and at $z = 0.65$ for $\varepsilon_0 = 0.02$.

To probe the behavior of the model in the early universe, where $a \rightarrow 0$, we calculate the curvature scalar R in a flat universe, defined by

$$R = 6(\frac{\ddot{a}}{a} + H^2) \tag{3.84}$$

where the dot stands for the derivative with respect to the cosmic time and $\frac{\ddot{a}}{a} = \dot{H} + H^2$

$$R = 6(\dot{H} + 2H^2) \tag{3.85}$$

We substitute by $\dot{H} = \frac{\dot{\rho}_{tot}}{6H}$

$$R = 6(2H^2 - \frac{1}{2}(\rho_{tot} + p_{mcg} - 3\varepsilon_0 H \rho_{mcg}^{\frac{1}{2}})) \tag{3.86}$$

Where $H^2 = \frac{\rho_{tot}}{3}$

$$R = 6(2H^2 - \frac{3}{2}H^2 + \frac{1}{2}(-p_{mcg} + 3\varepsilon_0 H \rho_{mcg}^{\frac{1}{2}})) \tag{3.87}$$

Then

$$R = 3(H^2 - p_{mcg} + 3\varepsilon_0 H \rho_{mcg}^{\frac{1}{2}}) \tag{3.88}$$

In figure 3.5, the curvature scalar evolution is plotted in terms of the redshift parameter at the best values of Table 1. At $a \rightarrow 0, R \rightarrow \infty$ which corresponds to the Big Bang Singularity.

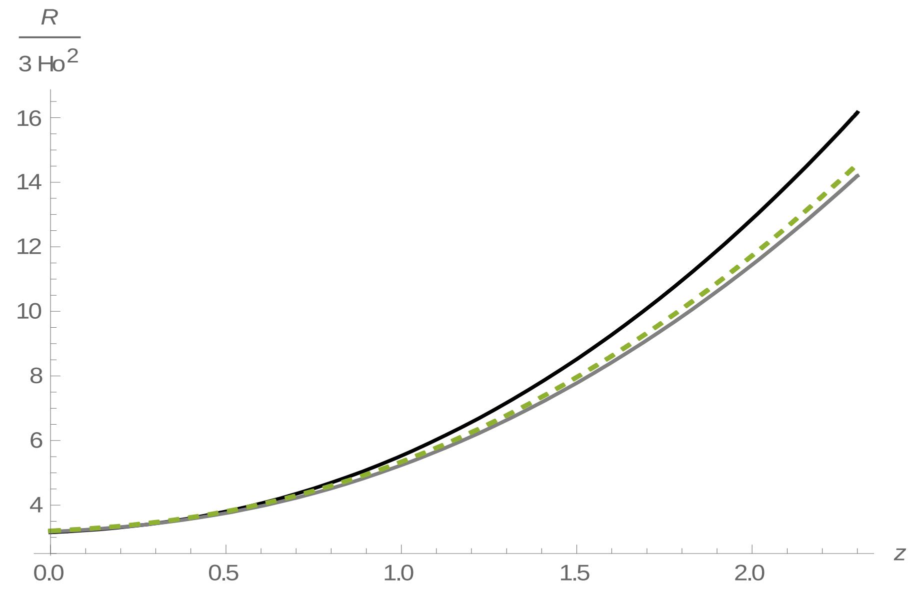


Figure 3.5: The evolution of the curvature scalar at best fit values of Table 3.2 for $\varepsilon_0 = 0.01$ (gray line), $\varepsilon_0 = 0.02$ (black line) and $\varepsilon_0 = 0.0001$ (dashed line).

3 Dynamical analysis of VMCG in LQC

3.1 Dynamical system

We call a dynamical system any system with variables evolving in time. Since most of the phenomena we observe in nature evolve in time it seems clear that the study of dynamical systems is of great interest. The evolution of the dynamical system [61] is very sensitive to the initial conditions of the system where a small variation on the initial state of the system can change significantly the evolution of the system.

Systems with equations of the form

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(x_1, x_2, .., x_n; a_1, a_2, .. a_k) \\ \frac{dx_2}{dt} &= f_2(x_1, x_2, .., x_n; a_1, a_2, .. a_k) \\ &\dots\dots\dots\dots\dots\dots \\ \frac{dx_n}{dt} &= f_n(x_1, x_2, .., x_n; a_1, a_2, .. a_k) \end{aligned} \tag{3.89}$$

Where x_i are *the dynamical variables* of the system, a_i are the *parameters* of the system that we keep constant and t the time as an independent variable. If the right hand side of the equations doesn't depend explicitly on time, the system is said to be *autonomous*. Whether the right hand side of the equations is linear or nonlinear the system is called *linear* or *nonlinear dynamical system*. The entire future course and the entire past of the system are uniquely determined by its state at the present instant of time.

\

The stability of a solution $x_i(t)$ depends on the behavior of nearby solutions at a given time and at late time. So if nearby solutions remains close to $x_i(t)$ at a given time and at late time the solution is said to be (*Lyapunov*) *stable*. If the solution is *stable* and the nearby solutions converge to $x_i(t)$ at late time then it is *asymptotically stable*.

The dynamical behavior of a dynamical system is determined through the stability of its fixed points or equilibrium points x_i (solutions that don't change in time)

$$f_i(x_1, x_2, .., x_n; a_1, a_2, .. a_k) = 0 \tag{3.90}$$

This means that the first step toward the dynamical analysis is to find the fixed points $(x_{1i}, x_{2i}, .., x_{ni})$ of the system by solving Eqs. (3.90).

Then, one computes the Jacobian matrix of the system given by

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \dots\dots\dots\dots\dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \dots\dots\dots\dots\dots & \frac{\partial f_2}{\partial x_n} \\ \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} \dots\dots\dots\dots\dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}_{(x_{1i}, x_{2i}, \dots, x_{ni})} \tag{3.91}$$

to find the eigenvalues corresponding to each fixed point and finally checks the stability of the fixed points.

The stability of the fixed points depends on the nature and the values of the eigenvalues:

Node: corresponds to a fixed point with distinct real eigenvalues with the same sign. The node is said to be stable if all eigenvalues are negative otherwise it is unstable.

Improper node: corresponds to a fixed point with a real eigenvalue with multiplicity 'n'. It is stable if the eigenvalue is negative and unstable if positive.

Saddle: corresponds to a fixed point with real eigenvalues with opposite signs and it is an unstable fixed point.

Focus: corresponds to a fixed point with complex eigenvalues. It is stable if all the real parts of the eigenvalues are negative and unstable if positive or with opposite signs.

Center: corresponds to a fixed point with pure imaginary eigenvalues.

3.2 The Autonomous System of VMCG in LQC

We investigate the model behavior at large scale within the LQC framework through the dynamical analysis of an autonomous system of equations. The system then is solved numerically using Mathematica software.

In the LQC frame work, the modified flat Friedmann equation [62] is given by

$$H^2 = \frac{\rho_{tot}}{3}\left(1 - \frac{\rho_{tot}}{\rho_c}\right) \tag{3.92}$$

Where ρ_{tot} is the total energy density, $\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3 G^2 h}$ is the critical density in LQG and γ is the dimensionless Barbero-Immirzi parameter. The quantum correction is negligible when $\rho_{tot} \ll \rho_c \sim \rho_{Pl}$ but it dominates dynamics when $\rho_{tot} \sim \rho_c$. We assume a universe filled with Viscous MCG and baryonic matter, the conservation equations for both are expressed as

$$\dot{\rho}_{mcg} + 3H\left(\rho_{mcg} + P_{eff}\right) = 0 \tag{3.93}$$

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{3.94}$$

To analyze the dynamical behavior of the above system, we introduce the following dimensionless variables:

$$x = \frac{\rho_{mcg}}{3H^2} \ ,y = \frac{p_{mcg}}{3H^2}\ ,z = \frac{\rho_{tot}}{\rho_c} \tag{3.95}$$

The phase space is bounded by $0 \leq x \leq 1, 0 \leq z \leq 1$ and a negative y (a negative pressure is needed to generate accelerated expansion), the modified Friedmann equation in term of the new variables is given by

$$\left(x + \frac{\rho_m}{3H^2}\right)(1 - z) = 1 \tag{3.96}$$

We differentiate Eq. (3.92) with respect to the cosmic time and we insert the new dimensionless variables given by Eq. (3.95) to obtain

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left(y - \sqrt{3}\varepsilon_0 x^{1/2} + \frac{1}{1-z}\right)(1 - 2z) \tag{3.97}$$

We differentiate x with respect to the cosmic time

$$\dot{x} = \frac{\dot{\rho}_{mcg}}{3H^2} - \frac{2}{3}\frac{\dot{H}}{H^3}\rho_{mcg} \tag{3.98}$$

Using the conservation equation of VMCG energy density we find

$$\dot{x} = -\frac{1}{H}(\rho_{mcg} + p_{mcg} - 3\varepsilon_0 H\rho_{mcg}^{1/2}) - 2\frac{\dot{H}}{H}\frac{\rho_{mcg}}{3H^2} \tag{3.99}$$

We differentiate x with respect to the e-folding number $N = \ln a$ and we denote the derivative as $\acute{x}$

$$\acute{x} = \frac{\dot{x}}{H} = -\frac{1}{H^2}(\rho_{mcg} + p_{mcg} - 3\varepsilon_0 H\rho_{mcg}^{1/2}) - 2\frac{\dot{H}}{H^2}\frac{\rho_{mcg}}{3H^2} \tag{3.100}$$

We substitute by the dimensionless variables

$$\acute{x} = -3(x + y - \sqrt{3}\varepsilon_0 x^{1/2}) - 2x\frac{\dot{H}}{H^2} \tag{3.101}$$

We differentiate H^2 with respect to the cosmic time

$$2H\dot{H} = \frac{1}{3}\dot{\rho}_{tot}\left(1 - \frac{\rho_{tot}}{\rho_c}\right) - \frac{1}{3}\dot{\rho}_{tot}\frac{\rho_{tot}}{\rho_c} \tag{3.102}$$

$$2H\dot{H} = \frac{1}{3}\dot{\rho}_{tot}\left(1 - 2\frac{\rho_{tot}}{\rho_c}\right) \tag{3.103}$$

We substitute by the conservation equation of the total energy density and we divide by $2H^3$

$$\frac{\dot{H}}{H^2} = -\frac{1}{2H^2}(\rho_{mcg} + \rho_m + p_{mcg} - 3\varepsilon_0 H\rho_{mcg}^{1/2})\left(1 - 2\frac{\rho_{tot}}{\rho_c}\right) \tag{3.104}$$

Using the dimensionless variables the above equation can be written as

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(y - 3\varepsilon_0 x^{\frac{1}{2}} + \frac{1}{1-z})(1 - 2z) \tag{3.105}$$

We substitute this result in $\acute{x}$ we obtain

$$\acute{x} = 3(x(1-2z) - 1)\left(y - \sqrt{3}\varepsilon_0 x^{1/2}\right) - 3x(\frac{z}{1-z}) \tag{3.106}$$

Now we differentiate y with respect to the cosmic time

$$\dot{y} = \frac{\dot{p}_{mcg}}{3H^2} - \frac{2}{3}\frac{\dot{H}}{H^3}p_{mcg} \tag{3.107}$$

We have

$$\dot{p}_{mcg} = A\dot{\rho}_{mcg} + \alpha\frac{B}{\rho_{mcg}^{\alpha+1}}\dot{\rho}_{mcg} \tag{3.108}$$

$$\dot{p}_{mcg} = (A + \alpha\frac{B}{\rho_{mcg}^{\alpha+1}})\dot{\rho}_{mcg} \tag{3.109}$$

We add and abstract αA

$$\dot{p}_{mcg} = (A + \alpha A - \alpha A + \alpha\frac{B}{\rho_{mcg}^{\alpha+1}})\dot{\rho}_{mcg} \tag{3.110}$$

We obtain

$$\dot{p}_{mcg} = ((1+\alpha)A - \alpha\frac{y}{x})\dot{\rho}_{mcg} \tag{3.111}$$

Then

$$\dot{y} = \left((1+\alpha)A - \alpha\frac{y}{x}\right)\frac{\dot{\rho}_{mcg}}{3H^2} - 2\frac{\dot{H}}{H}\frac{p_{mcg}}{3H^2} \tag{3.112}$$

Now, we differentiate y with respect to the e-folding number $N = \ln a$ and we denote the derivative as $\acute{y}$

$$\acute{y} = -3(x + y - \sqrt{3}\varepsilon_0 x^{1/2})\left((1+\alpha)A - \alpha\frac{y}{x}\right) - 2y\frac{\dot{H}}{H^2} \tag{3.113}$$

We substitute by $\frac{\dot{H}}{H^2}$ found earlier

$$\acute{y} = -3\left(x + y - \sqrt{3}\varepsilon_0 x^{\frac{1}{2}}\right)\left((1+\alpha)A - \alpha\frac{y}{x}\right) + 3y(y - 3\varepsilon_0 x^{\frac{1}{2}} + \frac{1}{1-z})(1-2z) \tag{3.114}$$

We obtain

$$\acute{y} = -3(A(1+\alpha)x - \alpha y) - 3\left(A(1+\alpha) - y\left(1 - 2z + \frac{\alpha}{x}\right)\right)\left(y - \sqrt{3}\varepsilon_0 x^{\frac{1}{2}}\right) + 3y(\frac{1-2z}{1+z}) \tag{3.115}$$

Now, we differentiate z with respect to the cosmic time

$$\dot{z} = \frac{\dot{\rho}_{tot}}{\rho_c} \tag{3.116}$$

Using the conservation equation of the total energy density we find

$$\dot{z} = -3\frac{H}{\rho_c}(\rho_{tot} + p_{mcg} - 3\varepsilon_0 H \rho_{mcg}^{1/2}) \tag{3.117}$$

We differentiate z with respect to the e-folding number $N = \ln a$ and we denote the derivative as $\acute{z}$ to obtain

$$\acute{z} = -3z - \frac{3}{\rho_c}(p_{mcg} - 3\varepsilon_0 H \rho_{mcg}^{1/2}) \tag{3.118}$$

We have

$$H^2 = \frac{\rho_c}{3}\frac{\rho_{tot}}{\rho_c}(1 - \frac{\rho_{tot}}{\rho_c}) \tag{3.119}$$

We substitute by the dimensionless variable z

$$H^2 = \frac{\rho_c}{3}z(1-z) \tag{3.120}$$

then

$$\frac{3}{\rho_c} = \frac{z(1-z)}{H^2} \tag{3.121}$$

We substitute the last result in $\acute{z}$ we find

$$\acute{z} = -3z - 3z(1-z)(y - \sqrt{3}\varepsilon_0 x^{1/2}) \tag{3.122}$$

the autonomous system then is defined as

$$\acute{x} = 3(x(1-2z)-1)\left(y - \sqrt{3}\varepsilon_0 x^{1/2}\right) - 3x\left(\frac{z}{1-z}\right)$$

$$\acute{y} = -3(A(\alpha+1)x - \alpha y) - 3(A(1+\alpha) - y\left(1 - 2z + \frac{\alpha}{x}\right))\left(y - \sqrt{3}\varepsilon_0 x^{1/2}\right) + 3y\left(\frac{1-2z}{1+z}\right)$$

$$\acute{z} = -3z - 3z(1-z)(y - \sqrt{3}\varepsilon_0 x^{1/2}) \tag{3.123}$$

3.3 Numerical Analysis

This autonomous system does not depend on the EoS parameter B, and its critical points $P_i(x_c, y_c, z_c)$ are found numerically at the best values of Table 3.2. Their properties are determined by the sign and nature of the eigenvalues $\nu_i (i = 1{,}3)$ of the Jacobi matrix J,

$$J = \begin{pmatrix} \frac{\partial \acute{x}}{\partial x} & \frac{\partial \acute{x}}{\partial y} & \frac{\partial \acute{x}}{\partial z} \\ \frac{\partial \acute{y}}{\partial x} & \frac{\partial \acute{y}}{\partial y} & \frac{\partial \acute{y}}{\partial z} \\ \frac{\partial \acute{z}}{\partial x} & \frac{\partial \acute{z}}{\partial y} & \frac{\partial \acute{z}}{\partial z} \end{pmatrix}_{(x_c, y_c, z_c)} \tag{3.124}$$

When we fix the values of both $\varepsilon_0$and A, the critical points are the same and independent of the choice of α as listed in Table 3.3. For $(\varepsilon_0 = 0.01, A = -0.167, \alpha = 0.551)$ and $(\varepsilon_0 = 0.02, A = -0.149, \alpha = 0.548)$ the only physical and stable critical points P_1 with negative eigenvalues describe an accelerated VMCG dominated universe with $\omega_{eff} \approx -1$ exactly as predicted in the classical case. Moreover, the values of the critical points corresponding to an accelerated VMCG dominated universe change only with ε_0. However, those describing a decelerated matter universe and a decelerated VMCG dominated universe depend on both (ε_0, A). For $(\varepsilon_0 = 0.01, A = 1, \alpha)$ the critical points are $P_1(1, -0.98{,}0)$ a stable critical point because it has negative eigenvalues as $0 < \alpha < 1$ and it corresponds to an accelerated VMCG dominated universe, $P_2(0.0003,0.0003,0)$ and $P_3(1,1,0)$ unstable saddle points due to the opposite signs of their eigenvalues corresponding respectively to a decelerated matter dominated universe and a decelerated VMCG dominated universe.

	Critical points	Eigenvalues	ω_{eff}
$\varepsilon_0 = 0.01, A = -0.167$	$P_1(1, -0.98,0)$	$(-2.99, -2.34(\alpha+1), -0.008)$	-1
	$P_2(1, -0.167,0)$	$(-0.55,2.44(\alpha+1), -2.44)$	-0.184
$\varepsilon_0 = 0.02, A = -0.149$	$P_1(1, -0.96,0)$	$(-2.98, -2.36(\alpha+1), -0.016)$	-1
	$P_2(1, -0.149,0)$	$(-0.55,2.5(\alpha+1), -2.44)$	-0.184
$\varepsilon_0 = 0.01, A = 1$	$P_1(1, -0.98,0)$	$(-2.99, -5.93(\alpha+1), -0.008)$	-1
	$P_2(0.0003,0.0003,0)$	$(-3,3(\alpha+1), -1.5)$	0
	$P_3(1,1,0)$	$(-2.49,5.94(\alpha+1), -5.94)$	0.99

Table 3.3: The eigenvalues of the Jacobian matrix around given critical points P_i for the autonomous system Eq. (3.123)

This means that according to the constrained VMCG model and before the decoupling the universe was dominated by a VMCG describing a mixture of dark energy and dark matter but where this latter dominates. The model in this era behaves mostly like a perfect fluid describing dark matter leading to a decelerated expansion. This phase is only transitive corresponding to an unstable fixed point so the universe will finally become dark energy dominated.

From Fig. (3.6) the universe undergoes an accelerated expansion till a final de Sitter universe.

In classical cosmology, the model suffers from the Big Bang singularity. This problem does not occur in the loop quantum cosmology scenario. From Fig. (3.7) and (3.8), when $\rho_{tot} \approx$

$\frac{1}{2}\rho_c$, the Hubble parameter takes a maximum value and when ρ_{tot} takes its maximum value ρ_c, the Hubble parameter vanishes, thus the universe undergoes a contraction then enters the bounce.

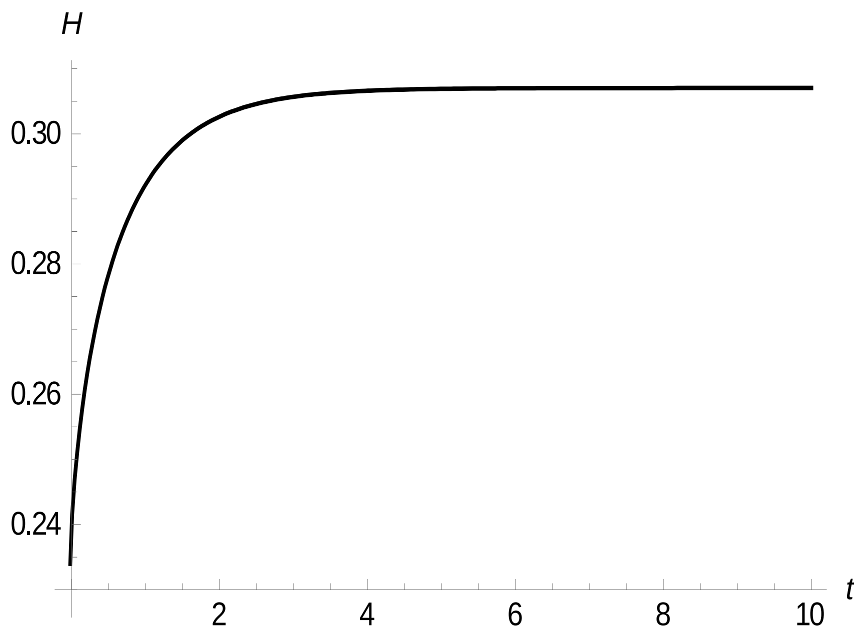


Figure 3.6: The evolution of H with time. Parameters are set at the best fit values of Table1 for $\varepsilon_0 = 0.01$ with $H_0 = 0.24, \rho_c = 1.5, \rho_{mcg0} = 0.01, \rho_{m0} = 0.0005$

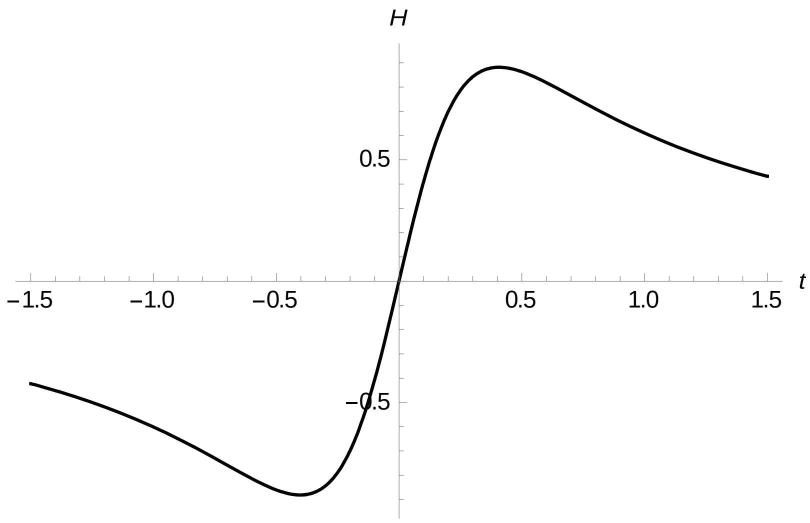


Figure 3.8: The evolution of the Hubble parameter H with time. Parameters are set at the best fit values of Table1 for $\varepsilon_0 = 0.01$ with $\rho_c = 10, \ \rho_{mcg0} + \rho_{m0} = 10$

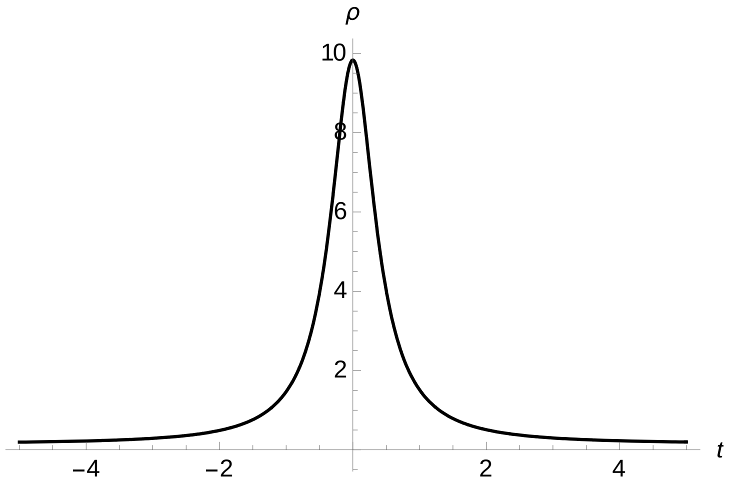


Figure 3.7: The evolution of the total energy density ρ with time. Parameters are set at the best fit values of Table1 for $\varepsilon_0 = 0.01$ with $\rho_c = 10$

Conclusion

In this thesis, we have investigated the model of VMCG. The observational data of Union2.1 constrained the viscous coefficient to $\varepsilon_0 \ll 1$, otherwise the perturbation instabilities at present time will grow exponentially leading to non-consistent model. With small values of ε_0, the model is found to be suitable to describe the current universe and gives good predictions at present time for both state and deceleration parameters $\omega_{eff_0} \in (-0.76, -0.74)$, $q_0 \in (-0.60, -0.57)$. The value of the state parameter is in agreement with $q_0 = -0.53^{-0.13}_{+0.17}$ at (68% C.L.; SN Ia+SALT2 fitter+ BAO/CMB) given by Ref. [63] and $q_0 = -0.54^{-0.07}_{+0.05}$ at (68% C.L.; SN Ia + BAO/CMB + H(z) + uniform prior with q_f = -1) given by Ref. [64]. The present value of the effective state parameter of VMCG is also consistent with $\omega_0 = -1.04^{-0.69}_{+0.72}$ at (95\% C.L.; Planck+WP+BAO) for dynamic state parameter estimated in Ref. [65], $\omega_0 = 0.91^{-0.20}_{+0.16}$ (SNLS3 team) of Refs. [66,67].

The perturbation instabilities, at the matter dominated era, are dropped down in present and late time as the coupling between dark energy and dark matter is decreasing. At large scale, the VMCG has no future singularities and its equation of state is nearly equivalent to cosmological constant (ω_{eff}= -1), while the sound speed takes a constant value different from zero as a difference between a dynamical fluid model and an inert cosmological constant model. The VMCG discussed here reproduces the main results of the standard model without assuming a priori the existence of cosmological constant, the problem related to fine-tuning is solved as the model is dynamical so it allows the energy density describing dark energy to decrease slowly to very small values and meets the observational data . In addition, the coincidence problem is solved too, according to our model the decoupling between dark energy and matter where ($\rho_{DE} \approx \rho_{matter}$) happened for both $\varepsilon_0 = 0.01$and $\varepsilon_0 = 0.0001$ at z = 0.75 which means a very recent past. This value is in agreement with $z_t = 0.64\ ^{-0.07}_{+0.13}$ given by Ref. [63] for models with final de Sitter phase, $z_t = 0.71 \pm 0.03$ of ΛCDM model of

Ref. [60], $z_t = 0.74 \pm 0.05$ given by Ref. [68] and z_t at (more than 68% C.L.;SN Ia + BAO/CMB(WMAP9) + H(z) + uniform prior with q_f = -1) of Ref. [64].

At LQC background and at large scale the results found are the same as those of classical background and at small scale the Big Bang singularity problem is solved and replaced by a bounce. The universe in its evolution will undergo a decelerated expansion when dominated by dark matter modeled by VMCG before the decoupling dark energy/dark matter, the model then changes to behave like dark energy and the universe enters a phase of an accelerated expansion. At large scale the stability of the model does not depend on the EoS parameter B and VMCG universe solutions depend only on ε_0.

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Abstract

In this thesis, we chose VMCG with a specific bulk viscosity pressure as a toy model for our universe to explore its behavior at present time, when fitted to recent observational data, and at late time to check whether it suffers from singularities or not.

The Eos parameters are constrained for a suitable model that describes the current universe. We also evaluate the evolution of the state and deceleration parameters at present and early universe and determine their present values to deduce if the model is consistent or not with observation data and theoretical predictions. The values are compared to those of other well accepted models. Then, we probe the dynamical behavior of our toy model at early and late time in the LQC framework especially as the model suffers from the Big Bang singularity.

The model is found suitable to describe the current universe with consistent present values of both state and deceleration parameters${\omega_{eff}}_0 \in (-0.76, -0.74)$, $q_0 \in (-0.60, -0.57)$. At large scale, the VMCG has no future singularities and its equation of state is nearly equivalent to cosmological constant (ω_{eff}= -1). The sound speed takes a constant value different from zero as a difference between a dynamical fluid model and an inert cosmological constant model. The VMCG discussed here reproduces the main results of the standard model without assuming a priori the existence of cosmological constant, the problems related to fine-tuning and coincidence problems are solved and the value of the redshift where ($\rho_{DE} \approx \rho_{matter}$) for both $\varepsilon_0 = 0.01$and $\varepsilon_0 = 0.0001$ is z = 0.75.

At LQC background and at large scale the results found are the same as those of classical background and at small scale the Big Bang singularity problem is solved and replaced by a bounce, at large scale the stability of the model does not depend on the EoS parameter B and VMCG universe solutions depend only on ε_0.

Key words: Loop Quantum Cosmology, Dark Energy, Chaplygin Gas, Dark Matter, Bulk Viscosity.

Le Gaz Modifié et Visqueux de Chaplygin dans la Cosmologie Classique et La Cosmologie Quantique des Boucles

Résumé

Dans cette thèse, on a choisi le gaz modifié et visqueux de Chaplygin avec une spécifique pression de seconde viscosité comme modèle pour notre univers afin d'explorer son comportement à l'heure actuelle, lorsqu'il est adapté aux données d'observation récentes, et à un moment tardif pour vérifier s'il souffre de singularités ou non.

Les paramètres de l'équation d'état sont estimés pour un modèle approprié qui décrit l'univers actuel. On évalue également l'évolution des paramètres d'état et de décélération au début de l'univers et au présent et on détermine leurs valeurs actuelles pour en déduire si le modèle est cohérent ou non avec les données d'observation et les prédictions théoriques. Les valeurs sont alors comparées à celles d'autres modèles bien acceptés. Ensuite, on étudie le comportement dynamique de notre modèle au début et à la fin de son évolution dans le cadre de la cosmologie quantique des boucles, surtout que le modèle souffre de la singularité du Big Bang.

Le modèle est jugé approprié pour décrire l'univers actuel avec des valeurs actuelles cohérentes des paramètres d'état et de décélération${\omega_{eff}}_0 \in (-0.76, -0.74)$, $q_0 \in (-0.60, -0.57)$. À grande échelle, le gaz modifié et visqueux de Chaplygin n'a pas de singularités au future et son équation d'état est presque équivalente à la constante cosmologique (ω_{eff}= -1). La vitesse du son prend une valeur constante différente de zéro ce qui marque la différence entre un modèle de fluide dynamique et un modèle de constante cosmologique inerte. Le gaz modifié et visqueux de Chaplygin discuté ici reproduit les principaux résultats du modèle standard sans supposer a priori l'existence d'une constante cosmologique, les problèmes liés aux réglage fin et coïncidence sont résolus et la valeur du décalage vers le rouge où ($\rho_{DE} \approx \rho_{matter}$) pour $\varepsilon_0 = 0.01$et $\varepsilon_0 = 0.0001$ est z = 0.75.

Dans le contexte de la cosmologie quantique des boucles et à grande échelle les résultats trouvés sont les mêmes que ceux trouvés dans le cas classique et à petite échelle le problème de la singularité de Big Bang est résolu et remplacé par un rebond, à grande échelle la stabilité du modèle ne dépend pas du paramètre B et les solutions qui correspondent à un univers de gaz modifié et visqueux de Chaplygin ne dépendent que de ε_0.

Mots clés: Cosmologie Quantique des Boucles, Energie Noir, Gaz De Chaplygin, Matière Noir, Seconde Viscosité.

غاز شابليجين المعدل و اللزج في الكوزمولوجيا الكلاسيكية و الكوزمولوجيا الكونتية الحلقية

ملخص

في هذه الأطروحة، تم اعتماد غاز شابليجين المعدل و اللزج مع عبارة ضغط محددة للزوجة السائبة كنموذج لوصف الكون بهدف معرفة سلوكه في الوقت الحاضر ، عند تكييفه مع بيانات الرصد الحديثة ، وفي وقت متأخر للتحقق مما إذا كان يعاني من نقاط شاذة أم لا.

تم تقييد معلمات معادلة الحالة للحصول على نموذج مناسب لوصف الكون الحالي. كما تم تقييم تطور متغيرات الحالة و التباطؤ في الوقت الحالي و في وقت مبكّر و تحديد قيمها الحالية لاستنتاج ما إذا كان النموذج متسقا ام لا مع بيانات الرصد الحديثة و التنبؤات النظرية. تمت مقارنة هذه القيم مع قيم لنماذج مقبولة جيدا. بعدها، تمت دراسة السلوك الديناميكي للنموذج في وقت متأخرو في وقت مبكّرا في إطار الكوزمولوجيا الكونتية الحلقية، خاصة و أن النموذج يعاني من تفرد الانفجار الأعظم.

وجد أن النموذج مناسب لوصف الكون الحالي مع قيم حالية مقبولة لكل من متغيرات الحالة و التباطؤ $q_0 \in (-0.60, -0.57)$ و $\omega_{eff_0} \in (-0.76, -0.74)$. على نطاق واسع، لا يعاني النموذج من نقاط شاذة مستقبلية و معادلة الحالة الخاصة به تعادل تقريبا معادلة الثابت الكوني (ω_{eff}= -1). إن سرعة الصوت تأخذ قيمة ثابتة مختلفة عن الصفر كفرق بين نموذج الموائع الديناميكية ونموذج ثابت كوني خامل.

إن نموذج الغاز المعدل واللزج لشابليجين المعتمد في الدراسة يعطي نتائج مقاربة لنموذج القياسي دون افتراض وجود ثابت كوني ، المشاكل المتعلقة بالمثالية والصدفة يتم حلها وقيمة الانحراف نحو الأحمر عند $(\rho_{DE} \approx \rho_{matter})$ لكل من $\varepsilon_0 = 0.01$ و $\varepsilon_0 = 0.0001$ هي z = 0.75.

في إطار الكوزمولوجيا الكونتية الحلقية وعلى نطاق واسع النتائج التي تم العثور عليها هي نفسها تلك الخلفية الكلاسيكية وعلى نطاق صغير يتم حل مشكلة تفرد الانفجار الكبير واستبدالها بارتداد، على نطاق واسع استقرار النموذج لا يعتمد على المعلمة B و الحلول الموافقة لكون يهيمن عليه الغاز المعدل واللزج لشابليجين تعتمد فقط على ε_0.

الكلمات المفتاحية: الكوزمولوجيا الكونتية الحلقية ، الطاقة المظلمة، غاز شابليجين ، المادة المظلمة، اللزوجة السائبة.