PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA
MINISTRY OF HIGHER EDUCATION
AND SCIENTIFIC RESEARCH

FRERES MENTOURI CONSTANTINE 1 UNIVERSITY
FACULTY OF SCIENCES EXACTES
PHYSICS DEPARTMENT

Order No:

Series:

THESIS
SUBMITTED IN CANDIDACY FOR THE DEGREE OF DOCTORATE IN PHYSICAL SCIENCES

SPECIALITY: THEORETICAL PHYSICS

THEME

STUDY OF SOME PHYSICAL MODELS IN THE FRAMEWORK OF NONCOMMUTATIVE GEOMETRY

By

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DEFENDED ON: 13 / 07 / 2016

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Acknowledgements

First of all, I would like to address my sincere gratitude to all the jury member Mr. Noureddine Mébarki, Professor at Constantine 1 University, Mr. Karim Aït Moussa, Professor at Constantine 1 University, Mr. Mounir Boussahel, Doctor at M'Sila University, Mr. Slimane Zaim, Professor at Batna University and Mr. Mustafa Moumni, Doctor at Biskra University, for taking the effort for reading my thesis and offering detailed evaluation of it.

I am particularly grateful to Prof. Noureddine Mebarki for accepting to be part in this committee as Jury chairman.

I would like to thank my supervisor Mr. Achour Benslama Professor at Constantine 1 University, for accepting me as his student, giving me an interesting topic and also for many useful suggestions during this research.

I would like to express my gratitude to Mr. Thomas Hahn, Professor at Max-Planck-Institute for Physics in Munich, Germany, for his patience for answering all my questions and for his help concerning FeynArts and FormCalc. I also have to thank Mr. Peter Schupp, Professor at Jacobs University in Bremen, Germany and Dr. Michael Wohlgenannt for their useful clarifications and comments.

Last but not least, I am especially grateful to my friend Leïla and her husband Mourad and I really appreciate their help and support during my stay in Germany. A special thanks to Abou Mayssara (Mr. Fouad) from Egypt and his wife Khadija from Morocco, also Dr. Jamal Ramadan from Lebanon, Dr. Fatma Fetoh, Dr. Gehad Elshenawi from Egypt and Mr. Mouhieddine Abdeli, for their help, kindness and for everything.

“If you have found the God

so what have you lost

and if you have lost the God

so what have you found”

Thanks God…………….

I owe a great debt

To my Parents for their invaluable love

To the spirit of my Father

To my life my Mother

To the spirit of my Brothers

To my Sisters and my Brothers.

Contents

Chapter 1

Introduction

On July 4^{th} 2012 the ATLAS and CMS experiments at CERN presented their results in the search for the Higgs boson. The data collected at the Large Hadron Collider (LHC) during the first run clearly indicated that a new particle had been observed. The search for this particle was one of the main reasons the LHC was constructed as the Higgs boson, it is not *just* a new particle in particle physics, but really forms one of the foundations of the electroweak sector of the Standard Model (SM), it allows to give masses to both fermions and gauge bosons in a local gauge invariance theory, it is at the heart of electroweak unification, quark mixing etc.

The importance discovery was clear a bit more than a year later when, on October 8^{th} 2013, François Englert and Peter Higgs were awarded the Nobel prize in physics: "*for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERNs Large*

Hadron Collider".

Despite the remarkable experimental confirmation of the Standard Model, even with the Higgs boson present, it is not able to explain several observations like dark matter, the special role of gravity and the expansion of the universe. It is these *irritating* open questions that make particle physicists believe that the Standard Model is only a simplification of a more complex underlying structure.

There are various motivations for studying noncommutative space-time (NC). The idea of space-time noncommutativity is in fact very old. It is usually attributed to Werner Heisenberg who proposed it in the late 1930's as a means of regulating the ultraviolet divergences which plague quantum field theory (GFT). Heisenberg suggested this idea in a letter to his doctoral student Rudolf Peierls [1], who actually applied it in a non-relativistic context of electronic systems in external magnetic fields. He also passed the idea to Wolfgang Pauli who then involved Robert Oppenheimer in the discussion [2]. Oppenheimer carried it to his student Hartland Snyder, who published the first concrete example in 1947 [3, 4]. It was a period where ideas about renormalization also born and the success of the renormalization theory took over the ideas about noncommutative coordinates. Thus, the idea of noncommutative space-time was abandoned for the time being. On the other hand, noncommutativity was pursued on the mathematical side, where especially the work of Alain Connes on noncommutative geometry in the 1980's stands out, providing the mathematical tools for further studies on noncommutative space-time [5]. In particle physics the interest in quantum field theories on noncommutative space-time declined, though not entirely, and was renewed only in 1999 by the work of Seiberg and Witten on string theory [6]. They showed that the dynamics of the endpoints of an open string on a D-brane in the presence of a magnetic background field can be described by a Yang-Mills theory on noncommutative space-time. Since string theory is nowadays the most popular ansatz for an ultimate theory capturing all laws of nature up to the Planck scale, the impact of this result on the particle physics community resulted in an outburst of publications on theories on noncommutative space-time within the last decade.

Nevertheless, the motivation for studying physics on noncommutative space-time can also be provided independently of string theory. Whatever the theory describing physics at the Planck scale is, we know that it is certainly not the Standard Model (SM) of particle physics, even though its predictive power has been experimentally verified to astonishing accuracy within the past decades. One of its major drawbacks is its incompatibility with general relativity. Thus, QFT and the SM have to be altered on the road towards the Planck scale in order to incorporate gravity. Since gravity alters the geometry of ordinary space-time, we expect that its quantization occurs at or before the Planck scale. Doplicher et al. show that space-time noncommutativity prevents the gravitational collapse allowing thus to incorporate space-time fluctuations into quantum field theory [7].

Several physical models have been formulated in the framework of the noncommutative space-time such as noncommutative quantum electrodynamics (NCQED) and the noncommutative standard model of particle physics (NCSM). Regarding the latter model, there are two approaches that have been proposed; the first one is given by Chaichian et al. [8] which enlarges the gauge group of the Standard Model (SM) by introducing new gauge bosons in addition to those of the SM, where new Higgs scalars are then also required. The second approach [9] keeps the same gauge group of the SM, i.e. the same structure group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as in the original SM, no new particles are introduced but it links between commutative and noncommutative gauge theories via Seiberg-Witten (SW) maps [6]. This approach introduce corrections to the SM interactions which are given in [10] and [11]. The contributions of these corrections up to the second order in $\Theta^{\mu\nu}$ are also calculated in [12].

The starting point is that the space-time coordinates do not commute with each other. The noncommutative space-time can be characterized by Hermitian operators satisfying the following commutation relations:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu},$$

where here, and throughout the rest of the paper, hatted quantities indicate operators. A priori there is no reason to expect that $\Theta^{\mu\nu}$ is constant. There is no fundamental theoretical obstacle to formulate the theory also for non-constant $\Theta^{\mu\nu}(x)$, but we shall concentrate on the constant case in the following for simplicity of presentation. $\Theta^{\mu\nu}$ is a real and antisymmetric matrix which describes the noncommutativity and assumed here to be constant. This constant matrix can also be thought of as some background field relative to which the various space-time directions are distinguished. Furthermore, introducing a NC scale Λ_{NC} where the noncommutative effects of the space-time become relevant, we rewrite equation above as

$$[\hat{x}^\mu, \hat{x}^\nu] = \frac{i}{\Lambda_{NC}^2} c^{\mu\nu}.$$

where the matrix $c^{\mu\nu}$ are dimensionless coefficients of order unity and can be parametrized with two three-vectors $\vec{E}$ and $\vec{B}$, and denote the timelike components c^{0i} by

$\vec{E}$ and the spacelike components c^{ij} by $\vec{B}$.

Instead of constructing quantum field theories on NC space-time (NCQFT) directly in terms of the operators $\hat{x}$, the NCQFT can be phrased in terms of conventional commuting QFT through the application of the Weyl-Moyal (WM) correspondence, [13] i.e. an ordinary function can be used instead of the corresponding NC one, by replacing the ordinary product with the *star*-product.

The noncommutative extension of the SM considered within this work relies on two building blocks: the Moyal-Weyl $\star$-product of functions on ordinary space-time and the Seiberg-Witten maps. The latter relate the ordinary fields and parameters to their noncommutative counterparts such that ordinary gauge transformations induce noncommutative gauge transformations. This requirement is expressed by a set of inhomogeneous differential equations '*the gauge equivalence equations*' which are solved by the Seiberg-Witten maps order by order in the noncommutative parameter. Thus, by means of the Moyal-Weyl $\star$-product and the Seiberg-Witten maps a noncommutative extension of the SM as an effective theory as expansion in powers of can be achieved, providing the framework of our phenomenological studies.

There has been a lot of activity recently around physics on noncommutative space-time. Since the construction of the NC version of the SM, many studies have been done to explore its phenomenological consequences. The first limits on NCQED from an e^+e^- colider experiment, yielding 141 GeV at 95% confidence level was obtained at LEP by the OPAL collaboration [14] and several high energy processes have also been explored by many authors in order to obtain the limits on the scale of noncommutativity parameter Λ_{NC} such as $e^-e^- \to e^-e^-$ (Möller), $e^+e^- \to e^+e^-$ (Bhabha) [15] $e^+e^- \to \gamma\gamma$ [12], $e^+e^- \to \mu^+\mu^-$ [16], $e^+e^- \to HH$ [17] and neutrino-photon scattering [18] have been investigated in the context of mNCSM. The usual bounds on the NC scale obtained from the mentioned papers are about 1 TeV. On the other hand, there exist other works where the predicted a reach scale seem ambiguous such as $t \to bW$ [19].

Actually, there are no theoretical predictions on the scale of noncommutativity parameter Λ_{NC}, such that only experiments can determine or constraint it. The main purpose of this Theses is to estimate the bounds on the noncommutative scale Λ_{NC} in the context of NCSM, by following the approach of [9] and using the definition of the Θ matrix that we have assumed and we will adopt it throughout this work. This thesis is structured as follows. **Chapter** 2 is meant to provide the theoretical basis for the model we will study in the remainder of this work. We will define noncommutative space-time in the canonical case and we will see Moyal-Weyl formalism which will play a central role in this model. In the following chapter, **Chapter** 3, will be a presentation of the gauge theory on noncommutative space-time, and giving first a brief recall about classical gauge theory. In **Chapter** 4, we will discuss the basics of electroweak symmetry breaking and the role of the Higgs mechanism in the Standard Model (SM) in quite some detail. In **Chapter** 5, we will give an introductory overview of the NCSM. We will show different choices for representations of the gauge group and the expressions of the noncommutative electroweak interactions with Higgs and Yukawa sector. Additionally, in **Chapter** 5, we will list a number of selected Feynman rules for noncommutative electroweak sectors up to the first order in Θ. We are finally moving to the main part of this thesis. In **Chapter** 6, we will discuss the limits on the scale of the noncommutativity parameter Λ_{NC} via studying the top-quark pair production through electron-positron collision in the framework of the minimal noncommutative standard model (mNCSM), using the Seiberg-Witten (SW) maps up to the first order of the noncommutativity parameter $\Theta^{\mu\nu}$. The closing chapter, **Chapter** 7, contains a summary of this study with some remarks, while the **Appendix** A contains the complete source code of the program used to calculate the scattering cross-section at tree level of the process $e^-e^+ \to \gamma, Z \to t\bar{t}$. It also contains the instruction how one can modify FeynArts and FormCalc [20-23] in order to include NCSM.

Chapter 2

Noncommutative Space-Time (NC)

This chapter is meant to provide the theoretical basis for the model we will study in the remainder of this work. We present noncommutative space-time in the canonical case with Moyal-Weyl formalism which will play a central role in the construction of the noncommutative standard model.

2.1 Noncommutative Space-Time (Canonical case)

Noncommutative space-time is a deformation of space-time that can be realized by representing ordinary space-time coordinates x^μ by Hermitian operators $\hat{x}^\mu$ that do not commute:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu}. \tag{2.1}$$

In this work, we assume for simplicity that

$$[\Theta^{\mu\nu}, \hat{x}^\rho] = 0. \tag{2.2}$$

A priori Θ has an arbitrary complicated dependency on $\hat{x}^\mu$. Nevertheless, we can assume a constant Θ. In the literature two other cases have also been studied, where depends linearly and quadratically on $\hat{x}^\mu$. Thus, noncommutativity with a Lie algebra structure

$$[\hat{x}^\mu, \hat{x}^\nu] = i\lambda^{\mu\nu}_\rho \hat{x}^\rho \tag{2.3}$$

and noncommutative space-time with quantum group structure

$$[\hat{x}^\mu, \hat{x}^\nu] = \left(\frac{1}{q}\hat{R}^{\mu\nu}_{\kappa\rho} - \delta^\mu_\rho \delta^\nu_\kappa\right) \hat{x}^\kappa \hat{x}^\rho \tag{2.4}$$

can be defined. We assume that the canonical noncommutativity (2.1) is a reasonable approximation. Thus, we will introduce the following parametrization

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} = \frac{i}{\Lambda^2_{NC}} c^{\mu\nu} = \frac{i}{\Lambda^2_{NC}} \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}, \tag{2.5}$$

with the constant symmetric 4×4 matrix $c^{\mu\nu}$. In analogy to the electromagnetic field strength tensor we have denoted the time-like components of $c^{\mu\nu}$ by $\overrightarrow{E}$ and the space-like components by $\overrightarrow{B}$. $\overrightarrow{E}$ and $\overrightarrow{B}$ will play different rôles, theoretically as well as phenomenologically.

Building quantum field theories on the noncommutative space-time (2.1) starting from the noncommuting operators $\hat{x}^\mu$ is a bold venture. The construction of quantum field theories on noncommutative space-time can be done more straightforwardly, if we take into account that experiments do not measure space-time coordinates themselves, but particles and fields, and that in the corresponding mathematical framework providing the calculation of observables we only encounter functions of the space-time coordinates and not the coordinates themselves. Therefore, we may seek for a way to express the commutator (2.1) of the noncommuting objects $\hat{x}^\mu$ by means of ordinary coordinates x^μ and a deformed product. Thus, we are looking for a homomorphism between the associative algebra $(\hat{A}, \cdot)$ generated by $\hat{x}^\mu$ which defines the noncommutative space-time and the algebra $(A, \star)$ of functions of the ordinary space-time coordinates and a deformed product $\star$, just like noncommutative geometry is constructed in algebraic geometry.

2.2 Moyal-Weyl $\star$-Product

The framework of Weyl's quantization procedure [24] provides a formalism for associating with the algebra of noncommuting coordinates $(\hat{A}, \cdot)$ an algebra of functions of commuting variables with deformed product $(A, \star)$. We define a map $W : A \to \hat{A}$ by which an element from $\hat{A}$ is assigned to a function $f(x^0, ..., x^{n-1}) \equiv f(x)$ from A:

$$W\left(f\right) = \hat{f} = \frac{1}{(2\pi)^{\frac{n}{2}}} \int d^n\kappa e^{i\kappa_\nu \hat{x}^\nu} \tilde{f}\left(\kappa\right), \tag{2.6}$$

with $\tilde{f}\left(\kappa\right)$ the Fourier transform of $f(x)$:

$$\tilde{f}\left(\kappa\right) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int d^n x e^{-i\kappa_\nu x^\nu} f\left(x\right). \tag{2.7}$$

The multiplication of two operators $W(f)$ and $W(g)$ obtained from (2.6) yields another operator $W(f \star g)$:

$$W\left(f\right) \cdot W\left(g\right) = \hat{f} \cdot \hat{g} = W\left(f \star g\right), \tag{2.8}$$

with $f \star g \in (A, \star)$, a classical function which is well defined, as we will now show. Inserting (2.6) in (2.8) we obtain:

$$W\left(f \star g\right) = W\left(f\right) \cdot W\left(g\right) = \frac{1}{\left(2\pi\right)^{n}} \int d^{n}\kappa d^{n} p e^{i\kappa_{\mu}\hat{x}^{\mu}} e^{ip_{\nu}\hat{x}^{\nu}} \tilde{f}\left(\kappa\right) \tilde{g}\left(p\right). \tag{2.9}$$

In the case of canonical noncommutativity (2.1), the product of the two exponentials in the above formula will give an exponential of a linear combination of the $\hat{x}^{\mu}$ after applying the Baker-Campbell-Hausdorff formula

$$e^{\hat{A}} e^{\hat{B}} = e^{\hat{A}+\hat{B}+\frac{1}{2}[\hat{A},\hat{B}]+\frac{1}{12}([\hat{A},[\hat{A},\hat{B}]]+[[\hat{A},\hat{B}],\hat{B}])+\ldots} \tag{2.10}$$

and considering the commutator relation (2.2), which thus makes all terms including more than one commutator in (2.10) vanish:

$$e^{i\kappa_{\mu}\hat{x}^{\mu}} e^{ip_{\nu}\hat{x}^{\nu}} = e^{i(\kappa_{\nu}+p_{\nu})\hat{x}^{\nu} - \frac{i}{2}\kappa_{\mu}p_{\nu}\Theta^{\mu\nu}}. \tag{2.11}$$

We obtain $f\star g$ by comparing (2.9) with (2.6) and replacing the operator $\hat{x}^{\mu}$ by the coordinate x^{μ}:

$$\left(f \star g\right)\left(x\right) = \frac{1}{\left(2\pi\right)^{n}} \int d^{n}\kappa d^{n} p e^{i(\kappa_{\nu}+p_{\nu})x^{\nu} - \frac{i}{2}\kappa_{\mu}\Theta^{\mu\nu}p_{\nu}} \tilde{f}\left(\kappa\right) \tilde{g}\left(p\right). \tag{2.12}$$

Thus, the Moyal-Weyl $\star$-product [25] is obtained:

$$(f \star g)(x) = \exp\left(\frac{i}{2}\Theta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}\right) f(x)g(y)\mid_{y\to x}. \tag{2.13}$$

Using this prescription for the $\star$-product, we now calculate the $\star$-commutator of the ordinary coordinate functions $[x^{\mu} \overset{\star}{,} x^{\nu}]$ and obtain, remembering the antisymmetry of $\Theta^{\mu\nu}$:

$$[x^{\mu} \overset{\star}{,} x^{\nu}] = x^{\mu} \star x^{\nu} - x^{\nu} \star x^{\mu} = x^{\mu}x^{\nu} + \frac{i}{2}\Theta^{\mu\nu} - x^{\nu}x^{\mu} - \frac{i}{2}\Theta^{\nu\mu} = i\Theta^{\mu\nu}. \tag{2.14}$$

This reproduces exactly the commutator (2.1):

$$[x^{\mu} \overset{\star}{,} x^{\nu}] = [\hat{x}^{\mu}, \hat{x}^{\nu}] = i\Theta^{\mu\nu}, \tag{2.15}$$

and shows how the noncommutativity encoded in the operators $\hat{x}^\mu$ is shifted into the $\star$-product of functions on ordinary space-time. Thus, we are now able to start the construction of SM on noncommutative space-time still dealing with ordinary space-time coordinates or more precisely, with functions on the ordinary space-time, but with a deformed product instead of the ordinary one. Before going on in doing so, we need to give some important properties of the $\star$-product. Under the integral the $\star$-product of *two* functions is equivalent to the ordinary product

$$\int d^4x (f \star g)(x) = \int d^4x (g \star f)(x) = \int d^4x f(x) g(x), \tag{2.16}$$

but this is not the case for the $\star$-product of *three or more* functions, where only one $\star$-product can be replaced by the usual $\cdot$-product:

$$\begin{aligned} \int d^4x (f \star g \star h)(x) &= \int d^4x ((f \star g) \cdot h)(x) = \\ &= \int d^4x (f \cdot (g \star h))(x) \\ &\neq \int d^4x f(x) g(x) h(x). \end{aligned} \tag{2.17}$$

Furthermore, we have invariance under cyclical permutation of the functions under the integral:

$$\begin{aligned} \int d^4x (f \star g \star h)(x) &= \int d^4x ((f \star g) \cdot h)(x) \\ &= \int d^4x (h \cdot (f \star g))(x) \\ &= \int d^4x (h \star f \star g)(x). \end{aligned} \tag{2.18}$$

Chapter 3

Gauge Theory on Noncommutative Space-Time

We will now concentrate on physics. We want to discuss the Standard Model on a canonically deformed space-time in **Chapter** 5. Before we can do so, we have to think about gauge theory on noncommutative space-time. Let us first briefly recall classical gauge theory. We will discuss in some detail the features that are essential for the noncommutative generalization.

3.1 Gauge Theory on Classical Spaces

Internal symmetries are described by Lie groups or Lie algebras, respectively.

$$[T^a, T^b] = f_c^{ab} T^c \tag{3.1}$$

The elements T^a are generators of the Lie algebra, where f_c^{ab} are its structure constants. Fields are given by n-dimensional vectors carrying a irreducible representation of the gauge group. Elements of the symmetry algebra are represented by $n \times n$ matrices. The free action of the field ψ is given by

$$S = \int d^4x \mathcal{L} = \int d^4x \partial_\mu \psi \partial^\mu \psi. \tag{3.2}$$

Requiring the gauge invariance of the action S, one has to introduce additional fields, gauge fields and to replace the usual derivatives by covariant derivatives D_μ.

Let us start with the field ψ building an irreducible representation of the gauge group, i.e.,

$$\delta\psi(x) = i\alpha(x)\psi(x), \tag{3.3}$$

where α is Lie algebra valued,

$$\alpha(x) = \alpha_a(x) T^a.$$

Observe that the derivative of a field ψ does not transform covariantly, i.e.,

$$\delta\partial_\mu\psi(x) \neq i\alpha(x)\partial_\mu\psi(x). \tag{3.4}$$

Replacing the usual derivatives ∂_μ by covariant derivatives D_μ and demanding that $D_\mu\psi$ transforms covariantly, one has to introduce a gauge potential $A_\mu(x)$,

$$D_\mu = \partial_\mu - igA_\mu(x),$$

$$A_\mu(x) = A_{\mu a}(x) T^a,$$

$$\delta A_\mu(x) = \frac{1}{g}\partial_\mu \alpha\left(x\right) + \left[\alpha\left(x\right), A_\mu(x)\right].$$

As it is well known, the interaction fields are a consequence of the gauge invariance of the action. Interactions are gauge interactions. The modified action reads

$$S = \int d^4x D_\mu \psi D^\mu \psi, \tag{3.5}$$

including gauge Fields A_μ. Forgetting about mass terms, we still need a kinetic term for the gauge fields in our action. The only requirements are the gauge invariance of the kinetic term and renormalizablility of the theory. That fixes the kinetic term uniquely. This is a crucial point, and the situation will be different in the case of the NCSM. The action is given by

$$S = \int d^4x \left(D_\mu \psi D^\mu \psi + Tr F_{\mu\nu} F^{\mu\nu}\right), \tag{3.6}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ is the field strength. Considering abelian gauge symmetry, commutators in $F_{\mu\nu}$ and in δA_μ will vanish. Let us make one more important remark, that there is a sharp distinction between internal and external symmetry transformations. As we will see, that is not true in the case of noncommutative gauge theory.

3.2 Noncommutative Gauge Theory

Noncommutative gauge theory, as presented in [26, 27], is based on essentially three principles,

- Covariant coordinates,
- Locality and classical limit,

- Gauge equivalence conditions.

Let us first briefly recall our starting point. We have noncommutative coordinates

$$\begin{aligned} [\hat{x}^{\mu}, \hat{x}^{\nu}] &= i\Theta^{\mu\nu}, \\ \hat{A} &= \frac{\mathbb{C}\left\langle\left\langle \hat{x}^{1}, ..., \hat{x}^{n}\right\rangle\right\rangle}{I}, \end{aligned}$$

where I is the ideal generated by the commutation relations of the coordinate functions. The product of function $f, g \in A$ is given by the Weyl-Moyal product.

3.2.1 Covariant Coordinates

Let ψ be a noncommutative field, i.e., $\widehat{\psi} \in \oplus_{i=1}^{n} \hat{A}$,

$$\widehat{\delta}\ \widehat{\psi}\left(\hat{x}\right) = i\hat{\alpha}\hat{\psi}\left(\hat{x}\right) \tag{3.7}$$

or

$$\widehat{\delta}\ \psi\left(x\right) = i\alpha \star \psi\left(x\right), \tag{3.8}$$

in the $\star$ formalism, where $W(\alpha) = \widehat{\alpha}$. Now, a similar situation arises as in **Eq.**(3.4), only the derivatives are replaced by coordinates. The product of a field and a coordinate does not transform covariantly, since the $\star$-product is not commutative,

$$\widehat{\delta}\left(x \star \psi\left(x\right)\right) = ix \star \alpha\left(x\right) \star \psi\left(x\right) \neq i\alpha\left(x\right) \star x \star \psi\left(x\right) \tag{3.9}$$

The arguments are the same as before, and we introduce covariant coordinates

$$X^{\mu} \equiv x^{\mu} + A^{\mu}, \tag{3.10}$$

such that

$$\widehat{\delta}(X^\mu \star \psi) = i\alpha \star (X^\mu \star \psi). \tag{3.11}$$

The covariant coordinates and the gauge potential transform under a noncommutative gauge transformation in the following way

$$\widehat{\delta} X^\mu = i[\alpha \stackrel{\star}{,} X^\mu], \tag{3.12}$$

$$\widehat{\delta} A^\mu = i[\alpha \stackrel{\star}{,} x^\mu] + i[\alpha \stackrel{\star}{,} A^\mu]. \tag{3.13}$$

Other covariant objects can be constructed from covariant coordinates, such as a generalization of the field strength,

$$F^{\mu\nu} = [X^\mu \stackrel{\star}{,} X^\nu] - i\Theta^{\mu\nu}, \qquad \widehat{\delta} F^{\mu\nu} = i[\alpha \stackrel{\star}{,} F^{\mu\nu}]. \tag{3.14}$$

For non degenerate Θ, we can define another gauge potential V_μ

$$\begin{aligned} \widehat{\delta} V_\mu &= \partial_\mu \alpha + i[\alpha \stackrel{\star}{,} V_\mu], & (3.15) \\ F_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu \stackrel{\star}{,} V_\nu], & (3.16) \\ \widehat{\delta} F_{\mu\nu} &= i[\alpha \stackrel{\star}{,} F_{\mu\nu}], & (3.17) \end{aligned}$$

using

$$A^\mu = \Theta^{\mu\nu} V_\nu, \qquad F^{\mu\nu} = i\Theta^{\mu\sigma}\Theta^{\nu\tau} F_{\sigma\tau}, \tag{3.18}$$

$$i\Theta^{\mu\nu}\partial_\nu f = [x^\mu \stackrel{\star}{,} f].$$

And we get for the covariant derivatives

$$\begin{aligned} D_\mu \star \psi &= (\partial_\mu - iV_\mu) \star \psi, \\ \widehat{\delta}(D_\mu \star \psi) &= i\alpha \star D_\mu \psi. \end{aligned} \tag{3.19}$$

Even for abelian gauge groups, the $\star$-commutators in **Eq.**(3.15) and (3.16) do not vanish, and the theory has similarities to a non-abelian gauge theory on a commutative space-time.

Let us have a closer look at the gauge parameters and the gauge fields. In classical theory, the gauge parameter and the gauge field are Lie algebra valued, as we have mentioned before. Two subsequent noncommutative gauge transformations are again a gauge transformation,

$$\delta_\alpha \delta_\beta - \delta_\beta \delta_\alpha = \delta_{-i[\alpha,\beta]}, \tag{3.20}$$

where $-i[\alpha, \beta] = \alpha^a \beta_b f_c^{ab} T^c$. However, there is a remarkable difference to the noncommutative case. Let M^α be some matrix basis of the enveloping algebra of the internal symmetry algebra. We can expand the gauge parameters in terms of this basis, $\alpha = \alpha_a M^a$, $\beta = \beta_b M^b$. Two subsequent gauge transformations take again the form

$$\widehat{\delta}_\alpha \widehat{\delta}_\beta - \widehat{\delta}_\beta \widehat{\delta}_\alpha = \widehat{\delta}_{-i[\alpha,\beta]}, \tag{3.21}$$

but the commutator of two gauge transformations involves the $\star$-commutator of the gauge parameters, and

$$[\alpha \overset{\star}{,} \beta] = \frac{1}{2} \{\alpha_a \overset{\star}{,} \beta_b\} \left[M^a, M^b\right] + \frac{1}{2} [\alpha_a \overset{\star}{,} \beta_b] \left\{M^a, M^b\right\}, \tag{3.22}$$

where $\{M^a, M^b\} = M^a M^b + M^b M^a$ is the anti-commutator. The difference to (3.20) is the anti-commutator $\{M^a, M^b\}$, respectively the $\star$-commutator of the gauge parameters,

$[\alpha_a \stackrel{\star}{,} \beta_b]$. This term causes some problems. Let us assume that M^a are the Lie algebra generators. Does the relation (3.22) close? Or does (3.22) rule out Lie algebra valued gauge parameters? Clearly, the only crucial term is the anti-commutator. The anti-commutator of two Hermitian matrices is again Hermitian. But the anti-commutator of traceless matrices is in general not traceless. Relation (3.22) will be satisfied for the generators of the fundamental representation of $U(n)$. Therefore it has been argued [28] that $U(n)$ is the only gauge group that can be generalized to NC spaces. But in fact arbitrary gauge groups can be tackled. But the gauge parameters α, β and the gauge fields A_μ have to be enveloping algebra valued [26-29], in general. Gauge fields and parameters now depend on infinitely many parameters, since the enveloping algebra is infinite dimensional. Luckily, the infinitely many degrees of freedom can be reduced to a finite number, namely the classical parameters, by the so-called Seiberg-Witten maps we will discuss in the next paragraph.

3.2.2 Locality and Classical Limit

The noncommutative $\star$-product can be written as an expansion in a formal parameter h,

$$f \star g = f \cdot g + \sum_{n=1}^{\infty} h^n C_n \left(f, g\right).$$

In the commutative limit $h \to 0$, the $\star$-product reduces to the pointwise product of functions. One may ask, if there is a similar commutative limit for the fields? The solution to this question was given for abelian gauge groups by [6],

$$\widehat{A}_\mu[A] = A_\mu + \frac{1}{2}\Theta^{\sigma\tau}\left(A_\tau \partial_\sigma A_\mu + F_{\sigma\mu} A_\tau\right) + O\left(\Theta^2\right), \tag{3.23}$$

$$\hat{\psi}\left[\psi, A\right] = \psi + \frac{1}{2}\Theta^{\mu\nu} A_\nu \partial_\mu \psi + O\left(\Theta^2\right), \tag{3.24}$$

$$\widehat{\alpha} = \alpha + \frac{1}{2}\Theta^{\mu\nu} A_\nu \partial_\mu \alpha + O\left(\Theta^2\right). \tag{3.25}$$

In this case, Θ is the noncommutativity parameter. First of all, let me introduce an important *convention* to which we will stick from now on. Quantities with *hat* $(\widehat{\psi}, \widehat{A}, \widehat{\alpha}... \in (A, \star))$ refer to noncommutative fields and gauge parameters, respectively which can be expanded (cf. above) in terms of the ordinary commutative fields and gauge parameters, resp. (ψ, A, α). The Seiberg-Witten maps (3.23)-(3.25) reduce the infinitely many parameters of the enveloping algebra to the classical gauge parameters. The origins of this map are in string theory. It is there that gauge invariance depends on the regularization scheme applied [6]. Pauli-Villars regularization provides us with classical gauge invariance

$$\delta a_i = \partial_i \lambda, \tag{3.26}$$

whence point-splitting regularization comes up with noncommutative gauge invariance

$$\widehat{\delta}\widehat{A}_i = \partial_i \widehat{\Lambda} + i\left[\widehat{\Lambda} \stackrel{\star}{,} \widehat{A}_i\right]. \tag{3.27}$$

Seiberg and Witten argued that consequently there must be a local map from ordinary gauge theory to noncommutative gauge theory

$$\widehat{A}\left[a\right], \quad \widehat{\Lambda}\left[\lambda, a\right] \tag{3.28}$$

satisfying

$$\widehat{A}\left[a + \delta_\lambda a\right] = \widehat{A}\left[a\right] + \widehat{\delta}_\lambda \widehat{A}\left[a\right]. \tag{3.29}$$

The Seiberg-Witten maps are solutions of (3.29). By locality we mean that in each order in the noncommutativity parameter Θ there is only a finite number of derivatives.

3.2.3 Gauge Equivalence Conditions

Let us remember that we consider arbitrary gauge groups. Noncommutative gauge fields $\widehat{A}$ and gauge parameters $\widehat{\Lambda}$ are enveloping algebra valued. Let us choose a symmetric basis in the algebra, T^a, $\frac{1}{2}(T^aT^b + T^bT^a)$, . . ., such that

$$\widehat{\Lambda}\left(x\right) = \widehat{\Lambda}_a\left(x\right)T^a + \widehat{\Lambda}^1_{ab}\left(x\right) : T^aT^b : +..., \tag{3.30}$$

$$\widehat{A}_\mu\left(x\right) = \widehat{A}_{\mu a}\left(x\right)T^a + \widehat{A}_{\mu ab}\left(x\right) : T^aT^b : +..., \tag{3.31}$$

Eq.(3.29) defines the SW maps for the gauge field and the gauge parameter. However, it is more practical to find equations for the gauge parameter and the gauge field alone [26]. First we will concentrate on the gauge parameters $\widehat{\Lambda}$. We already encountered the consistency condition

$$\widehat{\delta}_\alpha\widehat{\delta}_\beta - \widehat{\delta}_\beta\widehat{\delta}_\alpha = \widehat{\delta}_{-i[\alpha,\beta]}.$$

More explicitly, it reads

$$i\widehat{\delta}_\alpha\widehat{\beta}\left[A\right] - i\widehat{\delta}_\beta\widehat{\alpha}\left[A\right] + \left[\widehat{\alpha}\left[A\right] \overset{\star}{,} \widehat{\beta}\left[A\right]\right] = \left(\widehat{[\alpha,\beta]}\right)\left[A\right]. \tag{3.32}$$

Keeping in mind the results from **Section** 3.2.2, we can expand $\widehat{\alpha}$ in terms of the noncommutativity Θ,

$$\widehat{\alpha}\left[A\right] = \alpha + \alpha^1\left[A\right] + \alpha^2\left[A\right] + ..., \tag{3.33}$$

where α^n is $O(\Theta^n)$. The consistency relation (3.32) can be solved order by order in Θ.

$$0^{th}\text{ order:}\qquad \alpha^0 = \alpha, \tag{3.34}$$

$$1^{st}\text{ order:}\qquad \alpha^1 = \frac{1}{4}\Theta^{\mu\nu}\left\{\partial_\mu\alpha, A_\nu\right\}, \tag{3.35}$$

$$= \frac{1}{2}\Theta^{\mu\nu}\partial_\mu\alpha_a A_{\mu b} : T^a T^b : . \tag{3.36}$$

For fields $\widehat{\psi}$ the condition

$$\delta_\alpha\widehat{\psi}\left[A\right] = \widehat{\delta}_\alpha\widehat{\psi}\left[A\right] = i\widehat{\alpha}\left[A\right]\star\widehat{\psi}\left[A\right] \tag{3.37}$$

has to be satisfied, keeping in mind that δ_α denotes an ordinary gauge transformation and $\widehat{\delta}_\alpha$ a noncommutative one. That means that the ordinary gauge transformation induces a NC gauge transformation. We expand the fields in terms of the noncommutativity

$$\widehat{\psi} = \psi^0 + \psi^1\left[A\right] + \psi^2\left[A\right] + ... \tag{3.38}$$

and solve **Eq**.(3.37) order by order in Θ. In first order, we have to find a solution to

$$\delta_\alpha\psi^1\left[A\right] = i\alpha\psi^1 + i\widehat{\alpha}\psi - \frac{1}{2}\Theta^{\mu\nu}\partial_\mu\alpha\partial_\nu\psi. \tag{3.39}$$

It is given by

$$0^{th}\text{ order:}\qquad \psi^0 = \psi, \tag{3.40}$$

$$1^{st}\text{ order:}\qquad \psi^1 = -\frac{1}{2}\Theta^{\mu\nu}A_\mu\partial_\nu\psi + \frac{i}{4}\Theta^{\mu\nu}A_\mu A_\nu\psi. \tag{3.41}$$

The gauge fields $\widehat{A}_\mu$ have to satisfy

$$\delta_\alpha\widehat{A}_\mu\left[A\right] = \partial_\mu\widehat{\alpha}\left[A\right] + i\left[\widehat{\alpha}\left[A\right] \stackrel{\star}{,} \widehat{A}_\mu\left[A\right]\right] \tag{3.42}$$

Using the expansion

$$\widehat{A}_\mu = A^0_\mu + A^1_\mu\left[A\right] + A^2_\mu\left[A\right] + ... \tag{3.43}$$

and solving (3.42) order by order, we end up with

$$0^{th} \text{ order} \quad : \quad A^0_\mu = A_\mu, \tag{3.44}$$

$$1^{st} \text{ order} \quad : \quad A^1_\mu = -\frac{1}{4}\Theta^{\tau\nu}\left\{A_\tau, \partial_\nu A_\mu + F_{\nu\mu}\right\}, \tag{3.45}$$

where $F_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu - i[A_\nu, A_\mu]$. Similarly, we have for the field strength $\widehat{F}_{\mu\nu}$

$$\delta_\alpha \widehat{F}_{\mu\nu} = i\left[\widehat{\alpha}, \widehat{F}_{\mu\nu}\right] \tag{3.46}$$

and

$$\widehat{F}_{\mu\nu} = F_{\mu\nu} + \frac{1}{2}\Theta^{\sigma\tau}\left\{F_{\mu\sigma}, F_{\nu\tau}\right\} - \frac{1}{4}\Theta^{\sigma\tau}\left\{A_\sigma, \left(\partial_\tau + D_\tau\right)F_{\mu\nu}\right\}, \tag{3.47}$$

where

$$D_\mu F_{\tau\nu} = \partial_\mu F_{\tau\nu} - i\left[A_\mu, F_{\tau\nu}\right].$$

Let us conclude this Section with some remarks and observations.

- SW maps provide solutions to the gauge equivalence relations.

- Gauge equivalence relations are not the only possible approach to SW maps. Another approach is via noncommutative Wilson lines, see e.g., [31].

- However, a certain ambiguity in the SW map remains. They are unique modulo classical field redefinition and noncommutative gauge transformation. We used these ambiguities in order to choose $\widehat{\Lambda}$, $\widehat{A}_\mu$ Hermitian. The freedom in SW map may also be essential for renormalization issues. There, parameters characterizing the freedom in the SW maps become running coupling constants [32]. Discussing tensor products of gauge groups, this freedom will also be of crucial importance in **Section** 5.2.

- Gauge groups in noncommutative spaces contain space-time translations. Since

$$\partial f = -i\Theta^{-1}_{ij}\left[x^j, f\right], \tag{3.48}$$

we can express the translation of the field A_i as

$$\delta A_i = \upsilon^j \partial_j A_i = i\left[\epsilon \stackrel{\star}{,} A_i\right],$$

where $\epsilon = -\upsilon^j \Theta^{-1}_{jk} x^k$ The gauge transformation of A_i with gauge parameter ϵ gives

$$\widehat{\delta}_\epsilon A_i = i\left[\epsilon \stackrel{\star}{,} A_i\right] - \upsilon^j \Theta^{-1}_{ji}$$

ignoring the overall constant, which has no physical effect [33].

- NC gauge theory allows the construction of realistic particle models on a noncommutative space-time with an arbitrary gauge group as internal symmetry group. Noncommutative gauge parameters and gauge fields are enveloping algebra valued, in general (e.g., for $SU(n)$), but via SW maps the infinite number of degrees of freedom is reduced to the classical gauge parameters. Therefore these models will have the proper number of degrees of freedom.

Chapter 4

Standard Model of Particle Physics (SM)

4.1 Introduction

The goal of particle physics is to explain the nature of the universe at its most fundamental level, including the basic constituents of matter and their interactions. The standard model of particle physics (SM) is a quantum field theory (QFT) based on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry group which describes the strong, weak, and electromagnetic interactions among fundamental particles. This theory has been the focus of intense scrutiny by experimental physicists, most notably at high energy particle colliders, over the last three decades. It has been demonstrated to accurately describe particles and their interactions up to $O(100)$ GeV. Despite the success of the SM, there are many reasons to believe that the SM is an effective theory which is only valid up 1 TeV. The Glashow-

Salam-Weinberg (GSW) model of the electroweak interaction was proposed by Glashow [34], Weinberg [35] and Salam [36] for leptons and extended to the hadronic degrees of freedom by Glashow, lIliopoulos and Maiani [37]. The GSW model is a Yang-Mills theory [38] based on the symmetry group $SU(2)_L \otimes U(1)_Y$. It describes the electromagnetic and weak interactions of the known 6 leptons and 6 quarks. The electromagnetic interaction is mediated by a massless gauge boson, the photon (γ). The short-range weak interaction is carried by 2 massive gauge bosons, Z and W. The strong interaction, mediated by the massless gluon, is also a Yang-Mills theory based on the gauge group $SU(3)_C$. This is known as Quantum chromodynamics (QCD) [39-42]. The Standard Model of particle physics is just a trivial combination of GSW model and QCD. The particle content of the SM is listed in **Fig**.4.1. There is an additional scalar field called the Higgs boson (H), the only remnant of the spontaneous symmetry breaking (SSB) mechanism invented by Brout, Englert, Guralnik, Hagen, Higgs and Kibble [43-47]. The SSB mechanism is responsible for explaining the mass spectrum of the SM.

This chapter gives a general overview of the SM of Particle Physics by taking a brief look at the definition of the symmetry group and showing how the spontaneous symmetry breaking is responsible for explaining the mass spectrum of the SM. In order to understand the field content of the SM, it is necessary to begin with the definition of the symmetry group.

4.2 Symmetries, Gauge Theory and Particle Content

Symmetries have always played an important role in physics. The creation of the SM has not followed a different path; it is a theory based on a local symmetry.

The first theorem of Noether assures that any differentiable symmetry of the action of a physical system leads to a corresponding conservation law [48]. Any conservation law of physics can be interpreted as resulting from the symmetries of a particular theory.

One example is the theory of Quantum Electrodynamics (QED). The invariance under local transformation implies the existence of gauge fields with specific properties. To demonstrate such remarkable consequence, let us begin by considering a free theory of fermions. The free Lagrangian can be written as

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi, \tag{4.1}$$

with ψ being the spin-1/2 field, $\bar{\psi} = \psi^\dagger \gamma^0$, and γ^μ the Dirac matrices. The lagrangian is invariant under a global gauge transformation $\psi \to e^{-i\alpha}\psi$, with α a constant phase, which implies the conservation of the Dirac current $j^\mu(x)$:

$$\partial_\mu j^\mu = 0; \qquad j^\mu = \bar{\psi}\gamma^\mu\psi. \tag{4.2}$$

However, **Eq.**(4.1) is not invariant under local gauge transformations, where now α depends on the space-time coordinates, and suggests that the derivative should be redefined as

$$\partial^\mu \to D^\mu \equiv \partial^\mu + iqA^\mu. \tag{4.3}$$

where A^μ is a vector field. The lagrangian (4.1) with the replacement $\partial^\mu \rightarrow D^\mu$ is now invariant under the following local transformations

$$\psi(x) \rightarrow \psi'(x) = e^{-i\alpha(x)}\psi(x), \tag{4.4}$$

$$A^\mu(x) \rightarrow A^{\mu\prime}(x) = A^\mu(x) + \frac{1}{q}\partial^\mu\alpha(x) \tag{4.5}$$

and is finally given by

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi - q\bar{\psi}\gamma_\mu A^\mu\psi. \tag{4.6}$$

The second term has been derived from the imposition of local invariance. It describes the interaction of the fermionic matter ψ already existent on the theory, with the gauge field A^μ. Therefore, a theory that had in principle only matter fields, needs vector fields to provide interactions amongst fermions.

One could thus generalize this principle for all interactions, i.e. generate interaction terms for the weak, electromagnetic, strong *and also gravitational* forces through specific symmetries imposed on the theory. This is basically the idea for constructing the SM initially proposed by Glashow [34] and independently by Salam and Ward [49], extended later by Weinberg [35] and Salam [36]. Of course the complexity is larger than previously explained, since it works perfectly for abelian theories, however the world is not only described by them. Before going ahead, it is worth reminding that the SM of particle physics was created as a puzzle, and each piece was put together at different moments of History.

Initially, the electroweak theory was developed, and the gauge group $SU(2)_L \otimes U(1)_Y$ was used to relate electric charge with the isospin and the leptonic hypercharge of a particle. Besides matter fields, four gauge bosons are included due to the gauge symmetry:

one triplet associated with the $SU(2)$ group and one singlet associated to the $U(1)$. The corresponding lagrangian, restricted to the leptonic fields, is given by

$$\mathcal{L} = \bar{L}\gamma^\mu D_\mu L + \bar{e}_R\gamma^\mu D'_\mu e_R - \frac{1}{4}W^{\mu\nu i}W^i_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{4.7}$$

where the covariant derivatives are defined by

$$\begin{aligned} D_\mu &= \partial_\mu + i\frac{g}{2}\sigma^i W^i_\mu + i\frac{g'}{2}B_\mu, \\ D'_\mu &= \partial_\mu + ig'B_\mu, \end{aligned} \tag{4.8}$$

and L is the isospin doublet that contains the left-handed neutrino and electron, e_R is the right-handed electron; '*Right-handed neutrinos are not included in the theory*', σ^i are the Pauli matrices, and g and $\acute{g}$ are the coupling constants. $W^i_{\mu\nu}$ and $B_{\mu\nu}$ are the field-strength tensors:

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{4.9}$$

The matter content of the world is not only formed by leptons but also by hadrons, and it has been discovered that hadrons are composed by quarks. The symmetry that relates the color charge in quarks is $SU(3)_C$. The SM is consequently defined to have $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as the gauge symmetry group. Each gauge boson field is associated to the generator of the algebra of each group. Therefore, eight colored spin-1 particles associated to the $SU(3)_C$ gauge group exist and are also known as gluons. In addition, there are still the four uncolored particles, $W^i_{\mu\nu}$ and $B_{\mu\nu}$ quoted previously, that will mix to form the massive $W^\pm$ and Z^0 gauge bosons and the massless photon.

In a nutshell, the final particle content of the SM is summarized in **Fig**.4.1 [50]. The masses and most significant quantum numbers of each particle are stated in the figure. One important piece of the SM, the scalar field, has not been mentioned. It has been stated that conservation laws are a consequence of imposed symmetries. The SM theory is therefore based on gauge symmetries, and Lagrangians are constructed from the assumption of massless fields. The mechanism to generate mass to the particles requires the existence of at least one scalar particle, the Brout-Englert-Higgs boson; '*From the present moment on we shall abbreviate Brout-Englert-Higgs boson by Higgs boson, which is the usual shortened form used in the literature*'. This is the so-called BEH mechanism [45, 46, 51], which will be explained in what follows.

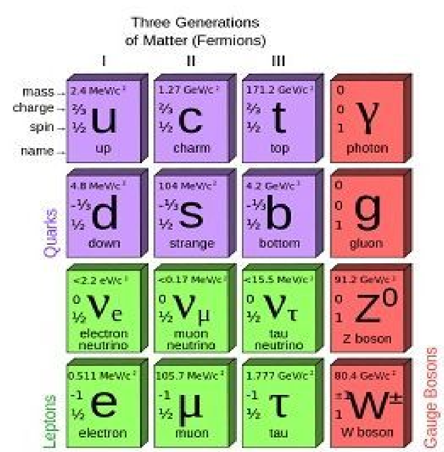


Fig.4.1 An illustration of the particle content of the Standard Model, excluding scalar fields. This is an interpretation of the periodic table of elements adapted to fundamental particle physics based on [50]

4.3 The Brout–Englert–Higgs Mechanism

It is known from Noether's theorem that symmetries imply conservation laws. It is a consequence of the invariance of both the lagrangian and the vacuum of a theory. However, there could be a situation where the lagrangian is invariant under a symmetry in which the vacuum is not. If such circumstance occurs, the symmetry is defined as to be broken, or hidden.

Consider for instance a scalar field ϕ whose lagrangian is given by [52]:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)\,, \text{ with } V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \tag{4.10}$$

where ϕ is real and $\lambda > 0$. The lagrangian is clearly invariant under the transformation $\phi \to -\phi$. When the vacuum expected value (vev) is calculated, two separated situations arise:

(a) If $\mu^2 > 0$, the vacuum is invariant under such symmetry,

$$\langle\phi\rangle_0 \equiv \langle 0 |\phi| 0\rangle = 0 \quad (\mu^2 > 0). \tag{4.11}$$

(b) However, if $\mu^2 < 0$:

$$\langle\phi\rangle_0 = \pm\sqrt{\frac{-\mu^2}{2\lambda}} \equiv \pm\frac{\upsilon}{\sqrt{2}} \quad (\mu^2 < 0). \tag{4.12}$$

The vacuum state in this case is degenerated, and it depends on the choice between $+\upsilon$ and $-\upsilon$. Both situations are illustrated in **Fig**.4.2. On the left side, the potential is shown as a function ϕ for $\mu^2 > 0$. As **Eq**.(4.11) revealed, only one vev is obtained. However, on the right, the plot of same potential is shown for the choice of $\mu^2 < 0$. Equation (4.12) exhibits a degenerate vacuum state, displayed by the two local minimums in **Fig**.4.2*b*.

(**a**) (**b**)

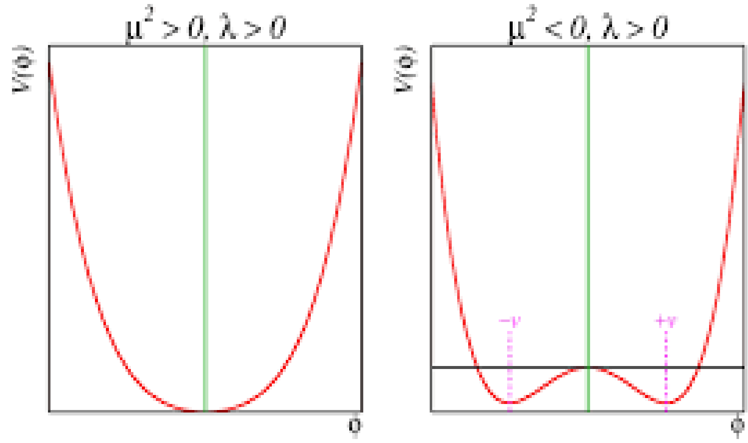


Fig.4.2 Potential $V(\phi)$ of **Eq**.(4.10) as a function of ϕ for the two situations presented: **a** with $\mu^2 > 0$ and **b** for $\mu^2 < 0$. For case (**a**), the minimum is at $\phi_0 = 0$ and for case (**b**) $\phi_0 = \pm\upsilon$, where the vacuum is degenerated.

We could therefore choose one state, for instance $\langle\phi\rangle_0 = +\upsilon$, and re-define the field in order for the vacuum to be at the origin:

$$\xi(x) \equiv \phi(x) - \langle\phi\rangle_0 = \phi(x) - \upsilon. \tag{4.13}$$

Now $\langle\xi\rangle_0 = 0$, and the lagrangian (4.10) becomes

$$\mathcal{L}_\xi = \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - \lambda\upsilon^2\xi^2 - \lambda\upsilon\xi^3 - \frac{1}{4}\lambda\xi^4. \tag{4.14}$$

This is the lagrangian of a free scalar field ξ with mass term $m_\xi = \sqrt{-2\mu^2}$; '*Notice the mass is positive, since we are analyzing the case where* $\mu^2 < 0$'. The symmetry was therefore hidden behind the primary definition of the field. By redefining the field ϕ it was possible to obtain the massive ξ. This concept is named *spontaneous symmetry breaking* and it illustrates the main mechanism used to generate the mass of fermionic and bosonic fields of the SM.

In the SM, it is necessary to define a doublet composed of two complex scalar fields. Suppose the doublet ϕ can be defined in a Hermitian basis as [53-55]

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i\phi_2 \\ \phi_3 - i\phi_4 \end{pmatrix} \tag{4.15}$$

The scalar lagrangian to be added to (4.7) will be similar to the one given in (4.10). However, in order to maintain the gauge invariance under $SU(2)_L \otimes U(1)_Y$, the covariant derivative (4.8) should be used:

$$\mathcal{L}_{BEH} = |D_\mu \phi|^2 - V(\phi). \tag{4.16}$$

In the new basis, the potential is

$$V(\phi) = \frac{1}{2}\mu^2 \left(\sum_{i=1}^{4} \phi_i^2 \right) + \frac{1}{4}\lambda \left(\sum_{i=1}^{4} \phi_i^2 \right)^2 . \tag{4.17}$$

We can choose the position of the minimum as $\phi_1 = \phi_2 = \phi_4 = 0$ and $\phi_3 = \upsilon$, being υ the Higgs vev. A new field h can be defined to be a radial excitation around the vev. The symmetry $SU(2)_L \otimes U(1)_Y$ is broken to $U(1)_{em}$ if we choose the specific value for the vev to be $\sqrt{-\mu^2/2\lambda}$. The field can thus be re-written as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \upsilon + h \end{pmatrix}, \tag{4.18}$$

The potential $V(\phi)$ of the lagrangian becomes

$$V_{BEH} = -\frac{1}{4}\lambda\upsilon^4 + \lambda\upsilon^2 h^2 + \lambda\upsilon h^3 + \frac{1}{4}\lambda h^4 \tag{4.19}$$

The second term indicates the mass of the Higgs field, $m_h^2 = 2\upsilon^2\lambda$. The third and the forth terms, i.e. the terms related to h^3 and h^4, indicate self-couplings of the Higgs boson.

The kinetic term of the lagrangian, $|D_\mu\phi|^2$ includes terms with the W_μ^i and B_μ fields

$$\left|\left(i\frac{g}{2}\sigma^i W_\mu^i + i\frac{g'}{2}B_\mu\right)\phi\right|^2 = \frac{(\upsilon+h)^2}{8}\left[g^2\left(W_\mu^1\right)^2 + g^2\left(W_\mu^2\right)^2 + \left(-gW_\mu^3 + g'B_\mu\right)^2\right]. \tag{4.20}$$

If four new vector fields are defined as:

$$\begin{aligned} Z_\mu^0 &= \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right), \qquad W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right), \\ A^\mu &= \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right), \end{aligned} \tag{4.21}$$

Equation (4.20) can be written in terms of the new fields. The boson masses can be identified by the following terms of the lagrangian

$$m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}\left(m_Z^2 Z_\mu Z^\mu + m_A^2 A_\mu A^\mu\right). \tag{4.22}$$

With the above expression, it is straightforward to find the masses related to the W_μ, Z_μ and A_μ fields [53]:

$$m_W = \frac{\upsilon g}{2}, \qquad m_Z = \frac{\upsilon}{2}\sqrt{g^2+g'^2} \quad \text{and} \quad m_A = 0. \tag{4.23}$$

Notice however that covariant derivatives can now be written in terms of the new bosonic fields. Consequently, it is possible to re-write **Eq.**(4.8) in a more useful format, in terms of the electromagnetic coupling, given by

$$g_e = \frac{gg'}{\sqrt{g^2+g'^2}} \tag{4.24}$$

or in terms of the *weak mixing angle*,

$$g_e = g \sin\theta_\omega, \qquad \cos\theta_\omega = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \text{and} \quad \sin\theta_\omega = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{4.25}$$

The mass of the $W^\pm$ and Z bosons are therefore related through the expression

$$m_W = m_Z \cos\theta_\omega. \tag{4.26}$$

As a result, all effects of $W^\pm$ and Z exchange processes (at tree level) can be exhibited as a function of the g_e, θ_ω and m_W parameters.

Finally, to find the value for the vev υ we can replace the expression found for the mass of the $W^\pm$ bosons in terms of the Fermi constant:

$$G_F = \sqrt{2}\frac{g^2}{8m_W} = \frac{\sqrt{2}}{2\upsilon^2}. \tag{4.27}$$

Because the Fermi constant is experimentally known with a very good accuracy, $G_F = 1.16637(1) \times 10^{-5} GeV^{-2}$ [56], the value for the Higgs vev can be deduced as $\upsilon \sim 246\ GeV$. This is the value where the $SU(2)_L \otimes U(1)_Y$ is broken. However, because the parameter λ is not known, the mass of the Higgs boson cannot be predicted.

A similar mechanism is used to obtain the mass of the fermions. The invariant lagrangian that should be added to (4.7) can be exhibited as

$$\mathcal{L}_{yuk} = -\lambda_e \bar{L}\phi e_R - \lambda_d \bar{Q}_L \phi d_R - \lambda_u \bar{Q}_L \tilde{\phi} u_R + h.c. \tag{4.28}$$

where Q_L is the isospin doublet that contains the left-handed up and down quarks, u_R and d_R are the right-handed up and down quarks, $\tilde{\phi} = i\sigma_2\phi^*$, and σ_2 is one of the Pauli matrices. After spontaneous symmetry breaking, the mass of the fermions '*except neutrinos*' are generated as $m_f = \lambda_f \upsilon/2$, and neutrinos continue to be massless.

The couplings λ_f are called Yukawa couplings [53], and are determined depending on the experimental values of fermionic masses.

Chapter 5

Noncommutative Standard Model (NCSM)

5.1 Introduction

The approach to noncommutative field theory based on star products and Seiberg-Witten (SW) maps allows the generalization of the Standard Model (SM) of particle physics to the case of noncommutative space-time, keeping the original gauge group and particle content [6, 9, 25-27, 30, 57, 58].

In this chapter we present the electroweak charged and neutral currents in the Noncommutative Standard Model (NCSM) [9] and also present the Higgs and Yukawa parts of the NCSM action. Among the features which are novel in comparison with the SM is the appearance of additional gauge boson interaction terms and of interaction terms without Higgs boson which include additional mass dependent contributions. All relevant expressions are given in terms of physical fields and selected Feynman rules are provided with the aim to make the model more accessible to phenomenological considerations.

In the star product formulation of noncommutative field theory, one retains the ordinary functions '*and fields*' on Minkowski space, but introduces a new noncommutative product which encodes the noncommutative structure of space-time. For a constant antisymmetric matrix $\Theta^{\mu\nu}$, the relevant product is the Moyal-Weyl star product

$$f \star g = \sum_{n=0}^{\infty} \left(\frac{i}{2}\right)^n \frac{1}{n!} \Theta^{\mu_1\nu_1} ... \Theta^{\mu_n\nu_n} \left(\partial_{\mu_1} ... \partial_{\mu_n} f\right) \left(\partial_{\nu_1} ... \partial_{\nu_n} g\right). \tag{5.1}$$

For coordinates: $x^\mu \star x^\nu - x^\nu \star x^\mu = i\Theta^{\mu\nu}$. More generally, a star product has the form

$$(f \star g)(x) = f(x)g(x) + \frac{i}{2}\Theta^{\mu\nu}(x)\partial_\mu f(x)\partial_\nu g(x) + O\left(\Theta^2\right), \tag{5.2}$$

where the Poisson tensor $\Theta^{\mu\nu}(x)$ may be x-dependent and satisfies the Jacobi identity. Higher-order terms in the star product are chosen in such a way that the overall star product is associative. In general, they involve derivatives of Θ.

Carefully studying noncommutative gauge transformations one finds that in general, noncommutative gauge fields are valued in the enveloping algebra of the gauge group [26, 30]. '*Only for $U(N)$ in the fundamental representation it is possible to stick to Lie-algebra valued gauge fields*'. A priori this would imply an infinite number of degrees of freedom if all coefficient functions of the monomials that form an infinite basis of the enveloping algebra were independent. That is the place where the second important ingredient of gauge theory on noncommutative spaces comes into play, Seiberg-Witten maps [6, 26] which relate noncommutative gauge fields and ordinary fields in commutative theory via a power series expansion in Θ. Since higher-order terms are now expressed in terms of the zeroth-order fields, we do have the same number of degrees of freedom as in the commutative case. Noncommutative fermion and gauge fields read

$$\widehat{\psi} = \widehat{\psi}\,[V] = \psi - \frac{1}{2}\Theta^{\alpha\beta}V_\alpha\partial_\beta\psi + \frac{i}{8}\Theta^{\alpha\beta}\,[V_\alpha, V_\beta]\,\psi + O\left(\Theta^2\right), \tag{5.3}$$

$$\widehat{V}_\mu = \widehat{V}_\mu\,[V] = V_\mu + \frac{1}{4}\Theta^{\alpha\beta}\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\} + O\left(\Theta^2\right), \tag{5.4}$$

where ψ and V_μ are ordinary fermion and gauge fields, respectively. Noncommutative fields throughout this work are denoted by a hat. The Seiberg-Witten maps are not unique. The free parameters are chosen such that the noncommutative gauge fields are hermitian and the action is real.

In [9], it was shown how to construct a model with noncommutative gauge invariance, which stays as close as possible to the regular Standard Model. The distinguishing feature of this minimal NCSM (mNCSM) is the absence of new triple neutral gauge boson interactions in the gauge sector. However, as shown here, triple Z coupling does appear from the Higgs action. Triple gauge boson interactions do quite naturally arise in the gauge sector of extended versions [9, 59-61] of the NCSM and have been discussed in [59, 60]. Another interesting novel feature of NCSM, introduced by Seiberg-Witten (SW) maps, is the appearance of mixing of the strong and electroweak interactions already at the tree level [9, 11, 59].

We consider the Θ-expanded NCSM up to first order in the noncommutativity parameter with an emphasis made on the electroweak interactions only. In this chapter, we give an introductory overview of the NCSM. We discuss different choices for representations of the gauge group which then yield minimal and non-minimal versions of the NCSM. We carefully discuss electroweak charged and neutral currents of the NCSM. Explicit expressions for the NCSM corrections in the Higgs and Yukawa sectors are also presented in this chapter. The Feynman rules for the selected three- and four-field electroweak vertices are given at the end.

5.2 Noncommutative Standard Model

The action of the NCSM formally resembles the action of the classical SM; the usual point-wise products in the Lagrangian are replaced by the Moyal-Weyl product and *matter* and *gauge* fields are replaced by the appropriate Seiberg-Witten expansions. The action of the NCSM is

$$S_{NCSM} = S_{fermions} + S_{gauge} + S_{Higgs} + S_{Yukawa}, \tag{5.5}$$

where

$$\begin{aligned} S_{fermions} &= \int d^4x \sum_{i=1}^{3} (\overline{\widehat{L}}_L^{(i)} \star \left(i\widehat{\not{D}}\widehat{L}_L^{(i)} \right) + \overline{\widehat{Q}}_L^{(i)} \star \left(i\widehat{\not{D}}\widehat{Q}_L^{(i)} \right) + \overline{\widehat{e}}_R^{(i)} \star \left(i\widehat{\not{D}}\widehat{e}_R^{(i)} \right) \\ &\quad + \overline{\widehat{u}}_R^{(i)} \star \left(i\widehat{\not{D}}\widehat{u}_R^{(i)} \right) + \overline{\widehat{d}}_R^{(i)} \star \left(i\widehat{\not{D}}\widehat{d}_R^{(i)} \right)), \end{aligned} \tag{5.6}$$

$$\begin{aligned} S_{Higgs} &= \int d^4x (h_0^\dagger \left(\widehat{D}_\mu \widehat{\Phi} \right) \star h_0 \left(\widehat{D}^\mu \widehat{\Phi} \right) - \mu^2 h_0^\dagger(\widehat{\Phi}) \star h_0(\widehat{\Phi}) \\ &\quad - \lambda h_0^\dagger(\widehat{\Phi}) \star h_0(\widehat{\Phi}) \star h_0^\dagger(\widehat{\Phi}) \star h_0(\widehat{\Phi})), \end{aligned} \tag{5.7}$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	T_3
$e_R^{(i)}$	1	1	-1	-1	0
$L_L^{(i)} = \begin{pmatrix} \nu_L^{(i)} \\ e_L^{(i)} \end{pmatrix}$	1	2	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$u_R^{(i)}$	3	1	$2/3$	$2/3$	0
$d_R^{(i)}$	3	1	$-1/3$	$-1/3$	0
$Q_L^{(i)} = \begin{pmatrix} u_L^{(i)} \\ d_L^{(i)} \end{pmatrix}$	3	2	$1/6$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	$1/2$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$W^+,\ W^-,\ Z$	1	3	0	$(\pm 1, 0)$	$(\pm 1, 0)$
A	1	1	0	0	0
G^b	8	1	0	0	0

Table.5.1: The Standard Model fields. Here $i \in \{1,2,3\}$ denotes the generation index. The electric charge is given by the Gell-Mann-Nishijima relation $Q = (T_3 + Y)$. The physical electroweak fields A, W^+, W^- and Z are expressed through the unphysical $U(1)_Y$ and $SU(2)_L$ fields A and B_a $(a \in \{1,2,3\})$ in **Eq**.(5.26). The gluons G^b $(b \in \{1,2,...,8\})$ are in the octet representation of $SU(3)_C$.

$$\begin{aligned} S_{Yukawa} &= -\int d^4x \sum_{i,j=1}^{3} \\ &\quad (G_e^{(ij)} \left(\overline{\widehat{L}}_L^{(i)} \star h_e(\widehat{\Phi}) \star \widehat{e}_R^{(j)}\right) + G_e^{\dagger(ij)} \left(\overline{\widehat{e}}_R^{(i)} \star h_e(\widehat{\Phi})^\dagger \star \widehat{L}_L^{(j)}\right) \\ &\quad + G_u^{(ij)} \left(\overline{\widehat{Q}}_L^{(i)} \star h_u(\widehat{\Phi}_c) \star \widehat{u}_R^{(j)}\right) + G_u^{\dagger(ij)} \left(\overline{\widehat{u}}_R^{(i)} \star h_u(\widehat{\Phi}_c)^\dagger \star \widehat{Q}_L^{(j)}\right) \\ &\quad + G_d^{(ij)} \left(\overline{\widehat{Q}}_L^{(i)} \star h_d(\widehat{\Phi}) \star \widehat{d}_R^{(j)}\right) + G_d^{\dagger(ij)} \left(\overline{\widehat{d}}_R^{(i)} \star h_d(\widehat{\Phi})^\dagger \star \widehat{Q}_L^{(j)}\right)). \end{aligned} \tag{5.8}$$

The gauge part S_{gauge} of the action is given in the next section. The particle spectrum of the SM, as well as that of the NCSM, is given in **Table**.5.1. Analogously to the usual SM definitions for fermion fields, we define $\overline{\widehat{\psi}} = \widehat{\psi}^\dagger \gamma^0$. *'The γ matrix can be pulled out of* *the SW expansion because it commutes with the matrices representing internal symmetries*'. The indices L and R denote the standard left and right components $\psi_L = 1/2(1-\gamma_5)\psi$ and $\psi_R = 1/2(1+\gamma_5)\psi$. For the conjugate Higgs field, we have $\Phi_c = i\tau_2\Phi^*$, 'τ_2 *is the usual Pauli matrix*'. In **Eqs.**(5.6) and (5.8) the generation index is denoted by $i, j \in \{1, 2, 3\}$. The matrices G_e, G_u and G_d are the Yukawa couplings.

The noncommutative Higgs field $\widehat{\Phi}$ is given by the hybrid SW map

$$\begin{aligned}\widehat{\Phi} &\equiv \widehat{\Phi}\left[\Phi, V, V'\right] \\ &= \Phi + \frac{1}{2}\Theta^{\alpha\beta}V_\beta\left(\partial_\alpha\Phi - \frac{i}{2}\left(V_\alpha\Phi - \Phi V'_\alpha\right)\right) \\ &\quad + \frac{1}{2}\Theta^{\alpha\beta}\left(\partial_\alpha\Phi - \frac{i}{2}\left(V_\alpha\Phi - \Phi V'_\alpha\right)\right)V'_\beta + O\left(\Theta^2\right),\end{aligned} \tag{5.9}$$

which generalizes the Seiberg-Witten maps of both gauge bosons and fermions. $\widehat{\Phi}$ is a functional of two gauge fields V and V' and transforms covariantly under gauge transformations:

$$\delta\widehat{\Phi}\left[\Phi, V, V'\right] = i\widehat{\Lambda}\star\widehat{\Phi} - i\widehat{\Phi}\star\widehat{\Lambda}', \tag{5.10}$$

where $\widehat{\Lambda}$ and $\widehat{\Lambda}'$ are the corresponding gauge parameters. Hermitian conjugation yields $\widehat{\Phi}\left[\Phi, V, V'\right]^\dagger = \widehat{\Phi}\left[\Phi^\dagger, V', V\right]$. The covariant derivative for the noncommutative Higgs field $\widehat{\Phi}$ is given by

$$\widehat{D}_\mu\widehat{\Phi} = \partial_\mu\widehat{\Phi} - i\widehat{V}_\mu\star\widehat{\Phi} + i\widehat{\Phi}\star\widehat{V}'_\mu. \tag{5.11}$$

As explained in [9], the precise representations of the gauge fields V and V' in the Yukawa couplings are inherited from the fermions on the left $\left(\bar{\psi}\right)$ and on the right side (ψ) of the Higgs field found in (5.8), respectively. The following notation was introduced in **Eqs.**(5.7) and (5.8)

$$\begin{aligned} h_0(\widehat{\Phi}) &= \widehat{\Phi}\left[\Phi, \frac{1}{2}g'A + gB^a T_L^a, 0\right], \\ h_\psi(\widehat{\Phi}) &= \widehat{\Phi}\left[\Phi, R_{\psi_L}(V), R_{\psi_R}(V)\right], \\ h_\psi(\widehat{\Phi}_c) &= \widehat{\Phi}\left[\Phi_c, R_{\psi_L}(V), R_{\psi_R}(V)\right]. \end{aligned} \tag{5.12}$$

The representations R_ψ, determined by the multiplet ψ, are listed in **Table**.5.2. Note that $R_\psi(f(V_\mu)) = f(R_\psi(V_\mu))$ for any function f. Gauge invariance does not restrict the choice of representation for the Higgs field in S_{Higgs}. The simplest choice for h_0 which is adopted in the NCSM closely follows the SM representation for the Higgs field. For a better understanding of the gauge invariance, let us consider the hypercharges in two examples:

$$\begin{array}{ccccc} & \overline{\widehat{L}}_L[V] \star & \widehat{\Phi}\left[\Phi, V, V'\right] & \star\, \widehat{e}_R\left[V'\right] & \\ Y: & 1/2 & \underbrace{-1/2; 1}_{1/2} & -1, & \end{array}$$

$$\begin{array}{ccccc} & \overline{\widehat{Q}}_L[V] \star & \widehat{\Phi}\left[\Phi, V, V'\right] & \star\, \widehat{d}_R\left[V'\right] & \\ Y: & -1/6 & \underbrace{1/6; 1/3}_{1/2} & -1/3. & \end{array} \tag{5.13}$$

The choice of representation allows us to assign separate left and right hypercharges to the noncommutative Higgs field $\widehat{\Phi}$, which add up to Higgs usual hypercharge [9]. Because of the minus sign in (5.10), the right hypercharge attributed to the Higgs is effectively $-Y_{\psi_R}$.

ψ	$R_\psi(V_\mu)$
$e_R^{(i)}$	$-g'A_\mu$
$L_L^{(i)} = \begin{pmatrix} \nu_L^{(i)} \\ e_L^{(i)} \end{pmatrix}$	$-\frac{1}{2}g'A_\mu + gB_\mu^a T_L^a$
$u_R^{(i)}$	$\frac{2}{3}g'A_\mu + g_s G_\mu^b T_S^b$
$d_R^{(i)}$	$-\frac{1}{3}g'A_\mu + g_s G_\mu^b T_S^b$
$Q_L^{(i)} = \begin{pmatrix} u_L^{(i)} \\ d_L^{(i)} \end{pmatrix}$	$\frac{1}{6}g'A_\mu + gB_\mu^a T_L^a + g_s G_\mu^b T_S^b$

Table.5.2: The gauge fields in the covariant derivatives of the fermions and in the Seiberg-Witten maps of the fermions in the Noncommutative Standard Model. The matrices $T_L^a = \tau^a/2$ and $T_S^b = \lambda^b/2$ correspond to the Pauli and Gell-Mann matrices respectively, and the summation over the indices $a \in \{1, 2, 3\}$ and $b \in \{1, ..., 8\}$ is understood.

In Grand Unified Theories (GUT) it is more natural to first combine the left-handed and right-handed fermion fields and then contract the resulting expression with Higgs fields to obtain a gauge-invariant Yukawa term. Consequently, in NC GUTs we need to use the hybrid SW map for the left-handed fermion fields and then sandwich them between the NC Higgs on the left and the right-handed fermion fields on the right [62].

5.3 Gauge Sector of the NCSM Action

The general form of the gauge kinetic terms is [62]

$$S_{gauge} = -\frac{1}{2}\int d^4x \sum_R c_R \mathbf{Tr}\left(R(\widehat{F}_{\mu\nu}) \star R(\widehat{F}^{\mu\nu})\right), \tag{5.14}$$

where the noncommutative field strength $\widehat{F}_{\mu\nu}$

$$
\begin{aligned}
\widehat{F}_{\mu\nu} &= \partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu - i \left[\widehat{V}_\mu \stackrel{\star}{,} \widehat{V}_\nu \right] \\
&= F_{\mu\nu} + \frac{1}{2} \Theta^{\alpha\beta} \left\{ F_{\mu\alpha}, F_{\nu\beta} \right\} \\
&\quad - \frac{1}{4} \Theta^{\alpha\beta} \left\{ V_\alpha, \left(\partial_\beta + D_\beta \right) F_{\mu\nu} \right\} + O\left(\Theta^2\right),
\end{aligned}
\tag{5.15}
$$

was obtained from the SW map for the noncommutative vector potential (5.4). Ordinary field strength $F_{\mu\nu}$ is given by

$$
F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i \left[V_\mu, V_\nu \right], \tag{5.16}
$$

while its covariant derivative reads

$$
D_\beta F_{\mu\nu} = \partial_\beta F_{\mu\nu} - i \left[V_\beta, F_{\mu\nu} \right]. \tag{5.17}
$$

Here V_μ represents the whole of the gauge potential for the SM gauge group,

$$
V_\mu(x) = g' A_\mu(x) Y + g \sum_{a=1}^{3} B_\mu^a(x) T_L^a + g_s \sum_{b=1}^{8} G_\mu^b(x) T_S^b. \tag{5.18}
$$

The sum in (5.14) is over all unitary, irreducible and inequivalent representations R of a gauge group. The freedom in the kinetic terms is parametrized by real coefficients c_R that are subject to the constraints

$$
\frac{1}{g_I^2} = \sum_R c_R \mathbf{Tr} \left(R(T_I^a) R(T_I^a) \right), \tag{5.19}
$$

where g_I are the usual "commutative" coupling constants g', g, g_s and T_I^a are generators of $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, respectively. Equations (5.14) and (5.19) can also be written more compactly as

$$
S_{gauge} = -\frac{1}{2} \int d^4x \mathbf{Tr} \frac{1}{\mathbf{G}^2} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}, \qquad \frac{1}{g_I^2} = \mathbf{Tr} \frac{1}{\mathbf{G}^2} T_I^a T_I^a, \tag{5.20}
$$

where the trace **Tr** is again over all representations and **G** is an operator that commutes with all generators T_I^a and encodes the coupling constants [59]. The trace in the kinetic terms for gauge bosons is not unique, it depends on the choice of representation. This would not be of importance if the gauge fields were Lie algebra valued, but in the noncommutative case they live in the enveloping algebra. The possibility of new parameters in gauge theories on noncommutative space-time is a consequence of the fact that the gauge fields can take any value in the enveloping algebra of the gauge group.

It is instructive to provide the general form of S_{gauge}, (5.14), in terms of SM fields:

$$\begin{aligned} S_{gauge} &= -\frac{1}{2}\int d^4x \mathbf{Tr}\frac{1}{\mathbf{G}^2}F_{\mu\nu}F^{\mu\nu} \\ &+\Theta^{\rho\sigma}\int d^4x \mathbf{Tr}\frac{1}{\mathbf{G}^2}\left[\left(\frac{1}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right] \\ &+O\left(\Theta^2\right). \end{aligned} \tag{5.21}$$

5.3.1 Minimal NCSM

In the minimal Noncommutative Standard Model (mNCSM) which adopts the whole of the gauge potential (5.18) for the SM gauge group, the mNCSM gauge action is given by

$$S_{gauge}^{mNCSM} = -\frac{1}{2}\int d^4x \left(\frac{1}{g'^2}\mathrm{Tr}_1 + \frac{1}{g^2}\mathrm{Tr}_2 + \frac{1}{g_s^2}\mathrm{Tr}_3\right)\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu}. \tag{5.22}$$

Here the simplest choice was taken, i.e., a sum of three traces over the $U(1)$, $SU(2)$, $SU(3)$ sectors with

$$Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.23}$$

in the definition of Tr_1 and the fundamental representations for $SU(2)$ and $SU(3)$ generators in Tr_2 and Tr_3, respectively. In terms of physical fields, the action then reads

$$\begin{aligned} S_{gauge}^{mNCSM} &= -\frac{1}{2}\int d^4x \left(\frac{1}{2}A_{\mu\nu}A^{\mu\nu} + \mathrm{Tr}B_{\mu\nu}B^{\mu\nu} + \mathrm{Tr}G_{\mu\nu}G^{\mu\nu}\right) \\ &\quad +\frac{1}{4}g_s d^{abc}\Theta^{\rho\sigma}\int d^4x \left(\frac{1}{4}G^a_{\rho\sigma}G^b_{\mu\nu} - G^a_{\rho\mu}G^b_{\sigma\nu}\right)G^{\mu\nu,c} + O\left(\Theta^2\right), \end{aligned} \tag{5.24}$$

where $A_{\mu\nu}$, $B_{\mu\nu}(= B^a_{\mu\nu}T^a_L)$ and $G_{\mu\nu}(= G^a_{\mu\nu}T^a_S)$ denote the $U(1)$, $SU(2)_L$ and $SU(3)_c$ field strengths, respectively:

$$\begin{aligned} A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ B^a_{\mu\nu} &= \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\epsilon^{abc}B^b_\mu B^c_\nu, \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu. \end{aligned} \tag{5.25}$$

Note that in order to obtain the above result, one makes use of the following symmetry properties of the group generators $T^a_L = \tau^a/2$ and $T^a_S = \lambda^a/2$:

$$\mathrm{Tr}\left(T^aT^b\right) = \frac{1}{2}\delta^{ab}, \quad \mathrm{Tr}\left(\tau^a\tau^b\tau^c\right) = 2i\epsilon^{abc}, \quad \mathrm{Tr}\left(\lambda^a\lambda^b\lambda^c\right) = 2\left(d^{abc} + if^{abc}\right),$$

where ϵ^{abc} is the usual antisymmetric tensor, while f^{abc} and d^{abc} are totally antisymmetric and totally symmetric structure constants of the $SU(3)$ group.

There are no new electroweak gauge boson interactions in **Eq.**(5.24) nor the vertices already present in SM, like $W^+W^-\gamma$ and W^+W^-Z, do acquire any corrections. This is a consequence of the choice of the hypercharge (5.23) and of the antisymmetry in both the Lorentz and the group representation indices. However, new couplings, like ZZZ, and Θ corrections to SM vertices enter from the Higgs kinetic terms as elaborated in Section

5.5.

$$\begin{aligned} W_\mu^{\pm} &= \frac{B_\mu^1 \mp i B_\mu^2}{\sqrt{2}}, \\ Z_\mu &= \frac{-g' A_\mu + g B_\mu^3}{\sqrt{g^2 + g'^2}} = -\sin\theta_W A_\mu + \cos\theta_W B_\mu^3, \\ A^\mu &= \frac{g A_\mu + g' B_\mu^3}{\sqrt{g^2 + g'^2}} = \cos\theta_W A_\mu + \sin\theta_W B_\mu^3, \end{aligned} \tag{5.26}$$

where electric charge $e = g\sin\theta_W = g\prime\cos\theta_W$.

5.3.2 Non-Minimal NCSM

We can use the freedom in the choice of traces in kinetic terms for gauge fields to construct non-minimal versions of the mNCSM (nmNCSM). Since the fermion-gauge boson interactions remain the same regardless on the choice of traces in the gauge sector, the matter sector of the action is not affected, i.e. it is the same for both versions of the NCSM.

The expansion in Θ is at the same time an expansion in the momenta. The Θ-expanded action can thus be interpreted as a low-energy effective action. In such an effective low-energy description it is natural to expect that all representations that appear in commutative theory '*matter multiplets and adjoint representation*' are important. All representations of gauge fields that appear in the SM then have to be considered in the definition of the trace (5.20). In [59] the trace was chosen over all particles on which covariant derivatives act and which have different quantum numbers. In the SM, these are, five multiplets of fermions for each generation and one Higgs multiplet. The operator **G**, which determines the coupling constants of the theory, must commute with all generators (Y, T_L^a, T_S^b) of the gauge group, so that it does not spoil the trace property of **Tr**. This implies that **G** takes on constant values $g_1, ..., g_6$ on the six multiplets **Table**.5.1. The operator **G** is in general a function of Y and of the Casimir operators of $SU(2)$ and $SU(3)$. The action derived from (5.21) for such nmNCSM takes the following form:

$$\begin{aligned}
S_{gauge}^{nmNCSM} &= S_{gauge}^{mNCSM} \\
&+g'^3 k_1 \Theta^{\rho\sigma} \int d^4x \left(\frac{1}{4} A_{\rho\sigma} A_{\mu\nu} - A_{\mu\rho} A_{\nu\sigma}\right) A^{\mu\nu} \\
&+g' g^2 k_2 \Theta^{\rho\sigma} \int d^4x \left[\left(\frac{1}{4} A_{\rho\sigma} B^a_{\mu\nu} - A_{\mu\rho} B^a_{\nu\sigma}\right) B^{\mu\nu,a} + c.p.\right] \\
&+g' g_s^2 k_3 \Theta^{\rho\sigma} \int d^4x \left[\left(\frac{1}{4} A_{\rho\sigma} G^b_{\mu\nu} - A_{\mu\rho} G^b_{\nu\sigma}\right) G^{\mu\nu,b} + c.p.\right] \\
&+O\left(\Theta^2\right), \qquad (5.27)
\end{aligned}$$

where c.p. denotes cyclic permutations of field strength tensors with respect to Lorentz indices. The constants k_1, k_2 and k_3 represent parameters of the model given in [59, 60]. In the following we comment only the pure triple electroweak gauge-boson interactions.

New anomalous triple-gauge boson interactions that are usually forbidden by Lorentz invariance, angular moment conservation and Bose statistics '*Landau-Pomeranchuk-Yang theorem*' can arise within the framework of the nmNCSM [59, 60], but also in the alternative approach to the NCSM given in [8]. Neutral triple-gauge boson terms which are not present in the SM Lagrangian can be extracted from the action (5.27). In terms of physical fields (A, Z) they are

$$\begin{aligned}
\mathcal{L}_{\gamma\gamma\gamma} &= \frac{e}{4} \sin 2\theta_W K_{\gamma\gamma\gamma} \Theta^{\rho\sigma} A^{\mu\nu} \left(A_{\mu\nu} A_{\rho\sigma} - 4 A_{\mu\rho} A_{\nu\sigma}\right), \\
\mathcal{L}_{Z\gamma\gamma} &= \frac{e}{4} \sin 2\theta_W K_{Z\gamma\gamma} \Theta^{\rho\sigma} [2 Z^{\mu\nu} \left(2 A_{\mu\rho} A_{\nu\sigma} - A_{\mu\nu} A_{\rho\sigma}\right) \\
&\quad +8 Z_{\mu\rho} A^{\mu\nu} A_{\nu\sigma} - Z_{\rho\sigma} A_{\mu\nu} A^{\mu\nu}], \\
\mathcal{L}_{ZZ\gamma} &= \mathcal{L}_{Z\gamma\gamma} \left(A_\mu \leftrightarrow Z_\mu\right), \\
\mathcal{L}_{ZZZ} &= \mathcal{L}_{\gamma\gamma\gamma} \left(A_\mu \to Z_\mu\right), \qquad (5.28)
\end{aligned}$$

where

$$\begin{aligned}
K_{\gamma\gamma\gamma} &= \frac{1}{2} g g' \left(k_1 + 3k_2\right), \\
K_{Z\gamma\gamma} &= \frac{1}{2}\left[g'^2 k_1 + \left(g'^2 - 2g^2\right) k_2\right], \\
K_{ZZ\gamma} &= \frac{-1}{2gg'}\left[g'^4 k_1 + g^2\left(g^2 - 2g'^2\right) k_2\right], \\
K_{ZZZ} &= \frac{-1}{2g^2}\left[g'^4 k_1 + 3g^4 k_2\right],
\end{aligned} \tag{5.29}$$

and here we have introduced the shorthand notation $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ for $X \in \{A, Z\}$. Details of the derivations of neutral triple-gauge boson terms and the properties of the coupling constants in (5.27) are explained in [59, 60].

Additionally, in contrast to the mNCSM (5.24), electroweak triple-gauge boson terms already present in the SM acquire Θ corrections in the nmNCSM. Such contributions which originate from (5.27) read

$$\begin{aligned}
\mathcal{L}_{WW\gamma} &= \mathcal{L}^{SM}_{WW\gamma} + \mathcal{L}^{\Theta}_{WW\gamma} + O\left(\Theta^2\right), \\
\mathcal{L}_{WWZ} &= \mathcal{L}^{SM}_{WWZ} + \mathcal{L}^{\Theta}_{WWZ} + O\left(\Theta^2\right), \\
\mathcal{L}^{\Theta}_{WW\gamma} &= \frac{e}{4}\sin 2\theta_W K_{WW\gamma} \Theta^{\rho\sigma}\{A^{\mu\nu}[2(W^+_{\mu\rho}W^-_{\nu\sigma} + W^-_{\mu\rho}W^+_{\nu\sigma}) \\
&\quad -(W^+_{\mu\nu}W^-_{\rho\sigma} + W^-_{\mu\nu}W^+_{\rho\sigma})] + 4A_{\mu\rho}\left[W^{+\mu\nu}W^-_{\nu\sigma} + W^{-\mu\nu}W^+_{\nu\sigma}\right] \\
&\quad -A_{\rho\sigma}W^+_{\mu\nu}W^{-\mu\nu}], \\
\mathcal{L}^{\Theta}_{WWZ} &= \mathcal{L}^{\Theta}_{WW\gamma}\left(A_\mu \leftrightarrow Z_\mu\right),
\end{aligned} \tag{5.30}$$

with

$$K_{WW\gamma} = -\frac{g}{2g'}\left(g'^2+g^2\right)k_2,$$
$$K_{WWZ} = -\frac{g'}{g}K_{WW\gamma}. \tag{5.31}$$

It is important to stress that in both the mNCSM and the nmNCSM there are additional Θ corrections to these vertices coming from the Higgs part of the action. This will be elaborated in detail in **Section** 5.1.

The new parameters in the non-minimal NCSM can be restricted by considering GUTs on noncommutative space-time [62].

5.4 Electroweak Matter Currents

In this section we concentrate on the fermion electroweak sector of the NCSM. Some terms are derivative valued. Nevertheless, the hermiticity of the Seiberg-Witten maps for the gauge field guarantees the reality of the action. Using the SW maps of the noncommutative fermion field $\widehat{\psi}$ with corresponding function $R_\psi(V_\alpha)$

$$\widehat{\psi} = \psi - \frac{1}{2}\Theta^{\alpha\beta}R_\psi(V_\alpha)\partial_\beta\psi + \frac{i}{8}\Theta^{\alpha\beta}\left[R_\psi(V_\alpha), R_\psi(V_\beta)\right]\psi + O\left(\Theta^2\right), \tag{5.32}$$

and it's covariant derivative

$$\begin{aligned}
\widehat{D}_\mu\widehat{\psi} &= \partial_\mu\widehat{\psi} - iR_\psi(\widehat{V}_\mu)\star\widehat{\psi} \\
&= D_\mu\left[\psi - \frac{1}{2}\Theta^{\alpha\beta}R_\psi(V_\alpha)\partial_\beta\psi + \frac{i}{8}\Theta^{\alpha\beta}\left[R_\psi(V_\alpha), R_\psi(V_\beta)\right]\psi\right] \\
&\quad -iR_\psi\left(\frac{1}{4}\Theta^{\alpha\beta}\left\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\right\}\right)\psi + \frac{1}{2}\Theta^{\alpha\beta}(\partial_\alpha R_\psi(V_\mu))\partial_\beta\psi + O\left(\Theta^2\right),
\end{aligned} \tag{5.33}$$

it is straightforward to derive the general expression

$$\begin{aligned} S_\psi &= \int d^4x \overline{\widehat{\psi}} \star i\widehat{\not{D}}\widehat{\psi} \\ &= \int d^4x \left(i\overline{\psi}\not{D}\psi - \frac{i}{4}\overline{\psi}\Theta^{\mu\nu\rho}R_\psi(F_{\mu\nu})D_\rho\psi + O\left(\Theta^2\right) \right), \end{aligned} \tag{5.34}$$

where $\Theta^{\mu\nu\rho}$ is a totally antisymmetric quantity:

$$\Theta^{\mu\nu\rho} = \Theta^{\mu\nu}\gamma^\rho + \Theta^{\nu\rho}\gamma^\mu + \Theta^{\rho\mu}\gamma^\nu. \tag{5.35}$$

The terms of the form given in **Eq.**(5.34) appear in $S_{fermions}$ (5.6). One can easily show that $S^\dagger_{fermions} = S_{fermions}$, to order $O\left(\Theta^2\right)$. From **Eq.**(5.34) we have

$$S^\dagger_\psi = S_\psi - \frac{i}{4}\int d^4x \left(\overline{\psi}\Theta^{\mu\nu\rho}R_\psi(D_\rho F_{\mu\nu})\psi\right) + O\left(\Theta^2\right).$$

Since $R_\psi(\Theta^{\mu\nu\rho}D_\rho F_{\mu\nu}) = \Theta^{\mu\nu\rho}R_\psi(D_\rho F_{\mu\nu})$ for constant Θ, and

$$\Theta^{\mu\nu\rho}(D_\rho F_{\mu\nu}) = \Theta^{\mu\nu}\gamma^\rho\left(D_\rho F_{\mu\nu} + D_\nu F_{\rho\mu} + D_\mu F_{\nu\rho}\right),$$

the Θ-dependent term vanishes due to the Bianchi identity

$$D_\rho F_{\mu\nu} + D_\nu F_{\rho\mu} + D_\mu F_{\nu\rho} = 0,$$

thereby proving the reality of the action S_ψ and, hence, the reality of the action $S_{fermions}$ to $O\left(\Theta^2\right)$. However, note that the reality of the action is not essential, but is very desirable [63].

Next, we express the NCSM results for the electroweak currents in terms of physical fields starting with the left-handed electroweak sector. In the following Ψ_L represents

$\Psi_L \in \left\{L_L^{(i)}, Q_L^{(i)}\right\}$ and has the general form

$$\Psi_L = \begin{pmatrix} \psi_{up,L} \\ \\ \psi_{down,L} \end{pmatrix}. \tag{5.36}$$

In this case, according to the **Table**.5.2 the representation $R_{\Psi_L}(V_\mu)$ without $SU(3)$ fields takes the form

$$R_{\Psi_L}(V_\mu) = g' A_\mu Y_{\Psi_L} + g B_\mu^a T_L^a. \tag{5.37}$$

The hypercharge generator Y_{Ψ_L} (see **Table**.5.1) can be rewritten as

$$Y_{\Psi_L} = Q_{\psi_{up}} - T_{3,\psi_{up,L}} = Q_{\psi_{down}} - T_{3,\psi_{down,L}}, \tag{5.38}$$

and we make use of **Eqs**.(5.26). The left-handed electroweak part of the action S_ψ can be cast in the form

$$\begin{aligned} S_{\psi,eW,L} &= \int d^4x \left(\overline{\Psi}_L i\partial\!\!\!/ \Psi_L + \overline{\Psi}_L \mathbf{J}^{(L)} \Psi_L\right) \\ &= \int d^4x (\overline{\Psi}_L i\partial\!\!\!/ \Psi_L + \overline{\psi}_{up,L} J_{12}^{(L)} \psi_{down,L} + \overline{\psi}_{down,L} J_{21}^{(L)} \psi_{up,L} \\ &\quad + \overline{\psi}_{up,L} J_{11}^{(L)} \psi_{up,L} + \overline{\psi}_{down,L} J_{22}^{(L)} \psi_{down,L}), \end{aligned} \tag{5.39}$$

where $\mathbf{J}^{(L)}$ is a 2×2 matrix whose off-diagonal elements $(J_{12}^{(L)}, J_{21}^{(L)})$ denote the charged currents and diagonal elements $(J_{11}^{(L)}, J_{22}^{(L)})$ the neutral currents. After some algebra we obtain

$$J_{12}^{(L)} = \frac{g}{\sqrt{2}}\not{W}^{+} + J_{12}^{(L,\Theta)} + O\left(\Theta^2\right), \tag{5.40a}$$

$$J_{21}^{(L)} = \frac{g}{\sqrt{2}}\not{W}^{-} + J_{21}^{(L,\Theta)} + O\left(\Theta^2\right), \tag{5.40b}$$

$$\begin{aligned} J_{11}^{(L)} &= \left[2Q_{\psi_{up}}\not{A} + \frac{g}{\cos\theta_W}\left(T_{3,\psi_{up,L}} - Q_{\psi_{up}}\sin^2\theta_W\right)\not{Z}\right] \\ &\quad + J_{11}^{(L,\Theta)} + O\left(\Theta^2\right), \end{aligned} \tag{5.40c}$$

$$\begin{aligned} J_{22}^{(L)} &= \left[2Q_{\psi_{down}}\not{A} + \frac{g}{\cos\theta_W}\left(T_{3,\psi_{down,L}} - Q_{\psi_{down}}\sin^2\theta_W\right)\not{Z}\right] \\ &\quad + J_{22}^{(L,\Theta)} + O\left(\Theta^2\right), \end{aligned} \tag{5.40d}$$

where

$$\begin{aligned} J_{12}^{(L,\Theta)} &= \frac{g}{2\sqrt{2}}\Theta^{\mu\nu\rho}W_{\mu}^{+}\{-i\overleftarrow{\partial}_{\nu}\overrightarrow{\partial}_{\rho} \\ &\quad + e\left[Q_{\psi_{up}}A_{\nu}\overrightarrow{\partial}_{\rho} + Q_{\psi_{down}}A_{\nu}\overleftarrow{\partial}_{\rho} + (Q_{\psi_{up}} + Q_{\psi_{down}})(\partial_{\rho}A_{\nu})\right] \\ &\quad + \frac{g}{\cos\theta_W}[\left(T_{3,\psi_{up,L}} - Q_{\psi_{up}}\sin^2\theta_W\right)Z_{\nu}\overrightarrow{\partial}_{\rho} \\ &\quad + \left(T_{3,\psi_{down,L}} - Q_{\psi_{down}}\sin^2\theta_W\right)Z_{\nu}\overleftarrow{\partial}_{\rho} \\ &\quad + \left(\left(T_{3,\psi_{up,L}} + T_{3,\psi_{down,L}}\right) - \left(Q_{\psi_{up}} + Q_{\psi_{down}}\right)\sin^2\theta_W\right)(\partial_{\rho}Z_{\nu})] \\ &\quad - \frac{ieg}{\cos\theta_W}\left(Q_{\psi_{up}}T_{3,\psi_{down,L}} - Q_{\psi_{down}}T_{3,\psi_{up,L}}\right)A_{\nu}Z_{\rho}\} \end{aligned} \tag{5.41}$$

and

$$
\begin{aligned}
J_{11}^{(L,\Theta)} \quad = \quad & \frac{1}{2}\Theta^{\mu\nu\rho}\{ieQ_{\psi_{up}}(\partial_\nu A_\mu)\overrightarrow{\partial}_\rho \\
& +\frac{ig}{\cos\theta_W}\left(T_{3,\psi_{up,L}} - Q_{\psi_{up}}\sin^2\theta_W\right)(\partial_\nu Z_\mu)\overrightarrow{\partial}_\rho \\
& -e^2Q^2_{\psi_{up}}(\partial_\rho A_\mu)A_\nu \\
& -\frac{g^2}{\cos^2\theta_W}\left(T_{3,\psi_{up,L}} - Q_{\psi_{up}}\sin^2\theta_W\right)^2(\partial_\rho Z_\mu)Z_\nu \\
& -\frac{eg}{\cos\theta_W}Q_{\psi_{up}}\left(T_{3,\psi_{up,L}} - Q_{\psi_{up}}\sin^2\theta_W\right)[(\partial_\rho A_\mu)Z_\nu - A_\mu(\partial_\rho Z_\nu)] \\
& -\frac{g^2}{2}\left[W^+_\mu W^-_\nu\overrightarrow{\partial}_\rho + (\partial_\rho W^+_\mu)W^-_\nu\right] \\
& +\frac{ieg^2}{2}\left(2Q_{\psi_{up}} - Q_{\psi_{down}}\right)W^+_\mu W^-_\nu A_\rho \\
& +\frac{ig^3}{2\cos\theta_W}\left[\left(2T_{3,\psi_{up,L}} - T_{3,\psi_{down,L}}\right) - \left(2Q_{\psi_{up}} - Q_{\psi_{down}}\right)\sin^2\theta_W\right] \\
& \times W^+_\mu W^-_\nu Z_\rho\},
\end{aligned} \tag{5.42}
$$

while

$$
\left.\begin{matrix} J_{21}^{(L,\Theta)} \\ J_{22}^{(L,\Theta)} \end{matrix}\right\} \quad = \quad \left\{\begin{matrix} J_{12}^{(L,\Theta)} \\ J_{11}^{(L,\Theta)} \end{matrix}\right. \left(W^+ \leftrightarrow W^-, Q_{\psi_{up}} \leftrightarrow Q_{\psi_{down}}, T_{3,\psi_{up,L}} \leftrightarrow T_{3,\psi_{down,L}}\right). \tag{5.43}
$$

Here and in the following we use the notation in which $\overrightarrow{\partial}_\rho$ denotes the partial derivative which acts *only* on the fermion field on the right side, while $\overleftarrow{\partial}_\rho$ denotes the partial derivative which acts *only* on the fermion field on the left side, i.e. $\overrightarrow{\partial}_\rho \overleftarrow{\partial}_\rho$

$$
\partial_\rho\psi \equiv \overrightarrow{\partial}_\rho\psi \qquad \partial_\rho\overline{\psi} \equiv \overline{\psi}\overleftarrow{\partial}_\rho. \tag{5.44}
$$

We note that in contrast to the SM case, although

$$
\left(\int d^4x\overline{\psi}_{up,L}J_{12}\psi_{down,L}\right)^\dagger = \int d^4x\overline{\psi}_{down,L}J_{21}\psi_{up,L},
$$

we have

$$J_{21}^{(L)} \neq \gamma^0 \left(J_{12}^{(L)} \right)^\dagger \gamma_0.$$

The reason is the specific form of the interaction term (see **Eq.**(5.34)) which contains derivatives, whose presence produces

$$J_{21}^{(L)} = \gamma^0 \left(J_{12}^{(L)}(\overrightarrow{\partial} \leftrightarrow \overleftarrow{\partial}) \right)^\dagger \gamma_0.$$

Now, we turn to the results for the right-handed electroweak sector. Here ψ_R represents $\psi_R \in \{e_R^{(i)}, u_R^{(i)}, d_R^{(i)}\}$, and the representation $R_{\psi_R}(V_\mu)$ from **Table**.5.2 without $SU(3)$ fields is given by

$$R_{\psi_R}(V_\mu) = g' A_\mu Y_{\psi_R} = eQ_\psi A_\mu - \frac{g}{\cos\theta_W} Q_\psi \sin^2\theta_W Z_\mu. \tag{5.45}$$

For the right-handed fermions, $T_{3,\psi_R} = 0$, $Y_{\psi_R} = Q_\psi$. The right-handed electroweak part of the action S_ψ is of the form

$$S_{\psi,eW,R} = \int d^4x \left(\overline{\psi}_R i\partial\!\!\!/ \psi_R + \overline{\psi}_R J^{(R)} \psi_R \right), \tag{5.46}$$

$$J^{(R)} = \left[eQ_\psi A\!\!\!/ - \frac{g}{\cos\theta_W} Q_\psi \sin^2\theta_W Z\!\!\!/ \right] + J^{(R,\Theta)} + O\left(\Theta^2\right), \tag{5.47}$$

$$\begin{aligned} J^{(R,\Theta)} \quad = \quad & \frac{1}{2}\Theta^{\mu\nu\rho}\{ieQ_\psi(\partial_\nu A_\mu)\overrightarrow{\partial}_\rho - \frac{ig}{\cos\theta_W} Q_\psi \sin^2\theta_W(\partial_\nu Z_\mu)\overrightarrow{\partial}_\rho \\ & -e^2 Q_\psi^2(\partial_\rho A_\mu)A_\nu - \frac{g^2}{\cos^2\theta_W} Q_\psi^2 \sin^4\theta_W(\partial_\rho Z_\mu)Z_\nu \\ & +\frac{eg}{\cos\theta_W} Q_\psi^2 \sin^2\theta_W\left[(\partial_\rho A_\mu)Z_\nu - A_\mu(\partial_\rho Z_\nu)\right]\}. \end{aligned} \tag{5.48}$$

Let us now present the results in a form suitable for further calculations, derivation of Feynman rules and phenomenological applications, i.e. in terms of $\Psi \in \left\{L^{(i)}, Q^{(i)}\right\}$, and thus $\psi_{up} \in \left\{\nu^{(i)}, u^{(i)}\right\}$, and $\psi_{down} \in \left\{e^{(i)}, d^{(i)}\right\}$. The electroweak part of the action S_ψ then takes the form

$$\begin{aligned}
S_{\psi,eW} &= \int d^4x\{\overline{\Psi}i\partial\!\!\!/\Psi \\
&+\overline{\psi}_{up}J_{12}^{(L)}\frac{1}{2}(1-\gamma_5)\psi_{down}+\overline{\psi}_{down}J_{21}^{(L)}\frac{1}{2}(1-\gamma_5)\psi_{up} \\
&+\overline{\psi}_{up}\frac{1}{2}\left[(J_{11}^{(L)}+J^{(R)})-(J_{11}^{(L)}+J^{(R)})\gamma_5\right]\psi_{up} \\
&+\overline{\psi}_{down}\frac{1}{2}\left[(J_{22}^{(L)}+J^{(R)})-(J_{22}^{(L)}+J^{(R)})\gamma_5\right]\psi_{down}\},
\end{aligned} \tag{5.49}$$

and the currents $J_{ij}^{(L)}$ can be read from **Eqs**.(5.40)-(5.43), while $J^{(R)}$ is given by **Eqs**.(5.47)-(5.48), '*with* Q_ψ *substituted by the corresponding* $Q_{\psi_{up}}$ *or* $Q_{\psi_{down}}$'.

Finally, we note that the fermion fields appearing in this section are not mass but weak-interaction eigenstates. In order to present the results in terms of mass eigenstates, the Cabbibo-Kobayashi-Maskawa matrix '*denoted by* V_{ij} *in the following*' enters the quark currents leading to mixing between generations and to the modification of the quark currents by V_{ij} factors:

$$\overline{q}_{up}^{(i)}V_{ij}J_{12}^{(L)}\frac{1}{2}(1-\gamma_5)q_{down}^{(j)}, \qquad \overline{q}_{down}^{(j)}V_{ij}^{*}J_{21}^{(L)}\frac{1}{2}(1-\gamma_5)q_{up}^{(i)},$$

where $q_{up}^{(i)}$ and $q_{down}^{(i)}$ represent mass eigenstates. In the NCSM, as in the SM, the neutrino masses are not considered and consequently the leptonic mixing matrix is diagonal in contrast to the neutrino mass extended models. The corresponding noncommutative extensions which include neutrino masses can be made along the lines sketched here, '*see* ***Section*** 5.2 *for further details on this subject*'.

In this section, only electroweak interactions were considered.

5.5 Higgs Sector of the NCSM Action

In the preceding section we have expanded the fermionic part of the action and performed a detailed analysis of the electroweak interactions. We devote this section to the analysis of S_{Higgs} and S_{Yukawa} to first order in Θ.

5.5.1 Higgs Kinetic Terms

The expansion of the Higgs part of the action (5.7) to first order in Θ yields '*In order to make the presentation more transparent, in this section, we denote the* 2×2 *matrices appearing in the action by bold letters'*.

$$
\begin{aligned}
S_{Higgs} &= \int d^4x \left((\mathbf{D}_\mu \Phi)^\dagger (\mathbf{D}^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \right) \\
&+ \frac{1}{2}\Theta^{\alpha\beta} \int d^4x \Phi^\dagger \left(\mathbf{U}_{\alpha\beta} + \mathbf{U}^\dagger_{\alpha\beta} + \frac{1}{2}\mu^2 \mathbf{F}_{\alpha\beta} - 2i\lambda\Phi(\mathbf{D}_\alpha \Phi)^\dagger \mathbf{D}_\beta \right) \Phi,
\end{aligned} \tag{5.50}
$$

where

$$
\begin{aligned}
\mathbf{U}_{\alpha\beta} &= (\overleftarrow{\partial^\mu} + i\mathbf{V}^\mu)(-\partial_\mu \mathbf{V}_\alpha \partial_\beta - \mathbf{V}_\alpha \partial_\mu \partial_\beta + \partial_\alpha \mathbf{V}_\mu \partial_\beta \\
&+ i\mathbf{V}_\mu \mathbf{V}_\alpha \partial_\beta + \frac{i}{2}\mathbf{V}_\alpha \mathbf{V}_\beta \partial_\mu + \frac{i}{2}\partial_\mu (\mathbf{V}_\alpha \mathbf{V}_\beta) \\
&+ \frac{1}{2}\mathbf{V}_\mu \mathbf{V}_\alpha \mathbf{V}_\beta + \frac{i}{2}\{\mathbf{V}_\alpha, \partial_\beta \mathbf{V}_\mu + \mathbf{F}_{\beta\mu}\}).
\end{aligned} \tag{5.51}
$$

Equation (5.50) contains the usual covariant derivative of the Higgs boson $\mathbf{D}_\mu = \partial_\mu \mathbf{1} - i\mathbf{V}_\mu$ where $\mathbf{V}_\mu = g' A_\mu Y_\Phi \mathbf{1} + g B^a_\mu T^a_L$, and $\mathbf{1}$ is a unit matrix suppressed in the following. Also $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{V}_\nu - \partial_\nu \mathbf{V}_\mu - i\,[\mathbf{V}_\mu, \mathbf{V}_\nu]$.

Let us construct explicit expressions for the electroweak gauge matrices occurring in (5.50) and (5.51). The gauge field $\mathbf{V}_\mu$ can be expressed in a matrix form as

$$\mathbf{V}_\mu = \begin{pmatrix} g'A_\mu Y_\Phi + gT_{3,\phi_{up}} B^3_\mu & \frac{g}{\sqrt{2}} W^+_\mu \\ \frac{g}{\sqrt{2}} W^-_\mu & g'A_\mu Y_\Phi + gT_{3,\phi_{down}} B^3_\mu \end{pmatrix}, \tag{5.52}$$

where from **Table**.5.1 one can read '*Note* $Y_\Phi = Q_{\phi_{up}} - T_{3,\phi_{up}} = Q_{\phi_{down}} - T_{3,\phi_{down}}$':

$$Y_\Phi = 1/2, \qquad T_{3,\phi_{up}} = 1/2, \qquad T_{3,\phi_{down}} = -1/2.$$

The diagonal matrix elements can also be expressed in terms of physical fields using **Eqs**.(5.26). Hence, one obtains

$$V_{11,\mu} = eA_\mu + \frac{g}{2\cos\theta_W}(1 - 2\sin^2\theta_W)Z_\mu,$$
$$V_{22,\mu} = -\frac{g}{2\cos\theta_W} Z_\mu. \tag{5.53}$$

The product of two gauge fields is given by

$$\mathbf{V}_\mu \mathbf{V}_\alpha = \begin{pmatrix} V_{11,\mu}V_{11,\alpha} + \frac{g^2}{2} W^+_\mu W^-_\alpha & \frac{g}{\sqrt{2}}(W^+_\alpha V_{11,\mu} + W^+_\mu V_{22,\alpha}) \\ \frac{g}{\sqrt{2}}(W^-_\alpha V_{22,\mu} + W^-_\mu V_{11,\alpha}) & V_{22,\mu}V_{22,\alpha} + \frac{g^2}{2} W^-_\mu W^+_\alpha \end{pmatrix}, \tag{5.54}$$

while the product of three gauge fields can be expressed as

$$\mathbf{V}_\mu \mathbf{V}_\alpha \mathbf{V}_\beta = \mathbf{M}_{\mu\alpha\beta}, \tag{5.55a}$$

with matrix elements

$$
\begin{aligned}
\mathbf{M}_{\mu\alpha\beta,11} &= V_{11,\mu}V_{11,\alpha}V_{11,\beta} \\
&\quad +\frac{g^2}{2}\left(V_{11,\mu}W_\alpha^+W_\beta^- + W_\mu^+W_\alpha^-V_{11,\beta} + W_\mu^+V_{22,\alpha}W_\beta^-\right), \\
\mathbf{M}_{\mu\alpha\beta,12} &= \frac{g}{\sqrt{2}}(V_{11,\mu}W_\alpha^+V_{22,\beta} + V_{11,\mu}V_{11,\alpha}W_\beta^+ + W_\mu^+V_{22,\alpha}V_{22,\beta} \\
&\quad +\frac{g^2}{2}W_\mu^+W_\alpha^-W_\beta^+), \\
\mathbf{M}_{\mu\alpha\beta,21} &= \frac{g}{\sqrt{2}}(V_{22,\mu}W_\alpha^-V_{11,\beta} + V_{22,\mu}V_{22,\alpha}W_\beta^- + W_\mu^-V_{11,\alpha}V_{11,\beta} \\
&\quad +\frac{g^2}{2}W_\mu^-W_\alpha^+W_\beta^-), \\
\mathbf{M}_{\mu\alpha\beta,22} &= V_{22,\mu}V_{22,\alpha}V_{22,\beta} \\
&\quad +\frac{g^2}{2}\left(V_{22,\mu}W_\alpha^-W_\beta^+ + W_\mu^-W_\alpha^+V_{22,\beta} + W_\mu^-V_{11,\alpha}W_\beta^+\right).
\end{aligned}
\tag{5.55b}
$$

For the field strength one obtains

$$
\begin{aligned}
\mathbf{F}_{\mu\nu} &= \begin{pmatrix} eA_{\mu\nu} + \frac{g}{2\cos\theta_W}(1-2\sin^2\theta_W)Z_{\mu\nu} & \frac{g}{\sqrt{2}}W_{\mu\nu}^+ \\ \frac{g}{\sqrt{2}}W_{\mu\nu}^- & -\frac{g}{2\cos\theta_W}Z_{\mu\nu} \end{pmatrix} \\
&\quad -\frac{ig^2}{2}\begin{pmatrix} W_\mu^+W_\nu^- - W_\nu^+W_\mu^- & \sqrt{2}(B_\mu^3W_\nu^+ - W_\mu^+B_\nu^3) \\ -\sqrt{2}(B_\mu^3W_\nu^- - W_\mu^-B_\nu^3 & -W_\mu^+W_\nu^- + W_\nu^+W_\mu^- \end{pmatrix},
\end{aligned}
\tag{5.56}
$$

where $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ for $X \in \{A, Z, W^+, W^-\}$. By making use of **Eq**.(5.26) one can completely express the off-diagonal elements in terms of the physical fields A_μ and Z_μ. The other combinations of fields appearing in **Eqs**.(5.50) and (5.51) can also be easily obtained. We will not provide the explicit expressions here.

It is not difficult to see that the value of the Higgs field that minimizes the '*non-*

commutative' Higgs potential is the same as in the commutative case because, we are looking for the minimum value of the potential attained for constant fields and hence can ignore all derivative terms and all star products. This leaves terms like $\Theta^{\alpha\beta}V_\alpha V_\beta \Phi$ in the hybrid SW map that could possibly lead to corrections of the vacuum expectation value of the Higgs. Taking into account also the potential of the gauge fields it is, however, clear that we should consider only $V_\alpha = 0$, i.e. $\widehat{\Phi} = \Phi$ when fixing the vacuum expectation value.

The Higgs field is chosen to be in the unitary gauge

$$\Phi(x) \equiv \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) + \upsilon \end{pmatrix}, \tag{5.57}$$

where $\upsilon = \sqrt{-\mu^2/\lambda}$ represents the Higgs vacuum expectation value, while $h(x)$ is the physical Higgs field.

There are several points that need to be mentioned in connection with the NCSM version of the S_{Higgs} part of the action (5.50). From (5.57) one trivially obtains

$$\int d^4x \phi^\dagger H \phi = \int d^4x (h(x) + \upsilon) H_{22} (h(x) + \upsilon),$$

where H stands here for any 2×2 matrix. Taking into account this along with (5.52) and (5.54)-(5.56), it is easy to see that terms containing one or more Higgs fields $h(x)$ as well as terms containing solely gauge bosons reside in (5.50).

First, let us examine the contributions of the last two Θ-dependent terms in **Eq.**(5.50). By making use of (5.50)-(5.56) for the Higgs field in unitary gauge we find

$$\begin{aligned} & \frac{1}{2}\Theta^{\alpha\beta} \int d^4x \phi^\dagger \left(\frac{1}{2}\mu^2 \mathbf{F}_{\alpha\beta} - 2i\lambda\phi(\mathbf{D}_\alpha\phi)^\dagger \mathbf{D}_\beta \right) \phi \\ = \; & \frac{1}{8}\Theta^{\alpha\beta} \{ ig^2 \int d^4x (h+\upsilon)^2 \left[\mu^2 + \lambda(h+\upsilon)^2 \right] W_\alpha^+ W_\beta^- \\ & + \frac{g}{\cos\theta_W} \int d^4x (h+\upsilon)^2 \left[-\mu^2(\partial_\alpha Z_\beta) + 2\lambda(h+\upsilon)(\partial_\alpha h) Z_\beta \right] \}. \end{aligned} \tag{5.58}$$

Owing to the Stokes theorem the term containing only one Z field vanishes. Similarly, by performing partial integration and taking into account $v^2 = -\mu^2/\lambda$, the spuriously looking two-field terms vanish and (5.58) simplifies to

$$\frac{1}{8}\Theta^{\alpha\beta}\lambda \int d^4x h(h+v)(h+2v)\left\{ig^2(h+v)W^+_\alpha W^-_\beta + 2\frac{g}{\cos\theta_W}(\partial_\alpha h)Z_\beta\right\}. \tag{5.59}$$

Second, let us note that, in contrast to the SM case, in the NCSM action S_{Higgs} (5.50) there are terms proportional to v^2 that cannot be identified as the mass terms of the Higgs and weak gauge bosons fields but represent interaction terms. Hence, after the identification of the mass terms $(-1/2m_H^2h^2)$, $M_W^2 W^+_\mu W^{-\mu}$ and $1/2M_Z^2 Z_\mu Z^\mu$ with Higgs, W and Z boson masses

$$m_H^2 = 2\mu^2 = -2v^2\lambda,$$

$$M_W^2 = \frac{1}{4}v^2g^2, \qquad M_Z^2 = \frac{1}{4}v^2(g^2+g'^2) = \frac{M_W^2}{\cos^2\theta_W}, \tag{5.60}$$

respectively, additional terms remain which describe interactions of Higgs and gauge bosons and interactions of solely gauge bosons. The latter behavior is novel in comparison with the Standard Model and is introduced by the Seiberg-Witten mapping. The analysis of **Eq.**(5.50) reveals that, in addition to the interaction terms contained in S_{gauge} (5.21), the last three terms of the second bracket in $\mathbf{U}_{\alpha\beta}$ (5.51) give rise to order Θ contributions to the three- and four-gauge-boson couplings. Specifically, the three-gauge-boson interaction terms from S_{Higgs} read $(-1/4)v^2\Theta^{\alpha\beta}[I_{\alpha\beta}+I^\dagger_{\alpha\beta}]_{22}$, where $I_{\alpha\beta} = \mathbf{V}^\mu[(\partial_\mu\mathbf{V}_\alpha)\mathbf{V}_\beta + \mathbf{V}_\alpha(\partial_\beta\mathbf{V}_\mu) + (\partial_\beta\mathbf{V}_\mu)\mathbf{V}_\alpha]$. By making use of (5.55) one arrives at explicit expressions for the $W^+W^-\gamma$,

W^+W^-Z and ZZZ interaction terms:

$$\begin{aligned}
& \frac{-1}{4}\upsilon^2\Theta^{\alpha\beta}[I_{\alpha\beta}+I^\dagger_{\alpha\beta}]_{22} \\
= \; & \frac{e}{2}M_W^2\Theta^{\alpha\beta}\left[\left(W^{+\mu}W^-_\alpha + W^{-\mu}W^+_\alpha\right)A_{\mu\beta} + (\partial_\beta A_\alpha)W^{+\mu}W^-_\mu\right] \\
& - \frac{g}{4\cos\theta_W}M_W^2\Theta^{\alpha\beta}\{Z^\mu\left[W^+_\mu(\partial_\beta W^-_\alpha) + W^-_\mu(\partial_\beta W^+_\alpha)\right] \\
& + \left(Z^\mu W^+_\alpha + Z_\alpha W^{+\mu}\right)W^-_{\mu\beta} + \left(Z^\mu W^-_\alpha + Z_\alpha W^{-\mu}\right)W^+_{\mu\beta} \\
& - \cos 2\theta_W\left[\left(W^{+\mu}W^-_\alpha + W^{-\mu}W^+_\alpha\right)Z_{\mu\beta} + (\partial_\beta Z_\alpha)W^{+\mu}W^-_\mu\right] \\
& + \frac{g}{4\cos\theta_W}M_Z^2\Theta^{\alpha\beta}Z^\mu Z_\alpha(2\partial_\beta Z_\mu - \partial_\mu Z_\beta).
\end{aligned} \tag{5.61}$$

The four-gauge-boson interaction terms can be analyzed analogously.

5.5.2 Yukawa Terms

Next, we proceed to the Θ-expansion of the S_{Yukawa} action (5.8). Similarly to the analysis of the electroweak currents presented in **Section** 5.4, let us first analyse the general form for the Yukawa action,

$$\begin{aligned}
S_{\psi,Yukawa} \; = \; & -\int d^4x \sum_{i,j=1}^{3}[(G^{(ij)}_{down}(\overline{\widehat{\Psi}}^{(i)}_L \star h_{\psi_{down}}(\widehat{\Phi}) \star \widehat{\psi}^{(j)}_{down,R}) + h.c.) \\
& + (G^{(ij)}_{up}(\overline{\widehat{\Psi}}^{(i)}_L \star h_{\psi_{up}}(\widehat{\Phi}_c) \star \widehat{\psi}^{(j)}_{up,R}) + h.c.)].
\end{aligned} \tag{5.62}$$

Here G_{down} and G_{up} are general 3×3 matrices which comprise Yukawa couplings while $\psi^{(j)}_{up,R}$ and $\psi^{(j)}_{down,R}$ denote up and down fermion fields of the generation j. As we analyse a simple noncommutative extension of the SM, G^{ij}_{up} vanishes for leptons. Furthermore, as in the SM one can find a biunitary transformation that diagonalizes the G matrices

$$G_{down} = \frac{\sqrt{2}}{v} S_{down} M_{down} T^{\dagger}_{down}, \qquad G_{up} = \frac{\sqrt{2}}{v} S_{up} M_{up} T^{\dagger}_{up},$$

and obtain the diagonal 3×3 mass matrices M_{down} and M_{up}. Next, one redefines the fermion fields to mass eigenstates

$$\begin{array}{rclrcl}
\overline{\widehat{\psi}}^{(i)}_{down,L} S^{(ij)}_{down} & \rightarrow & \overline{\widehat{\psi}}^{(j)}_{down,L} & T^{\dagger(ij)}_{down} \widehat{\psi}^{(j)}_{down,R} & \rightarrow & \widehat{\psi}^{(i)}_{down,R} \\
\overline{\widehat{\psi}}^{(i)}_{up,L} S^{(ij)}_{up} & \rightarrow & \overline{\widehat{\psi}}^{(j)}_{up,L} & T^{\dagger(ij)}_{up} \widehat{\psi}^{(j)}_{up,R} & \rightarrow & \widehat{\psi}^{(i)}_{up,R}.
\end{array}$$

This redefinition of the fields introduces the fermion mixing matrix $V = S^{\dagger}_{up} S_{down}$ in the electroweak currents (5.49), and, owing to the hybrid SW mapping of the Higgs field, in the Yukawa part of the NCSM action as well. We introduce the matrix V_f, which like in the SM, corresponds to

$$V_f = \begin{cases} \mathbf{1} & \text{for } f = l \\ V \equiv V_{CKM} & \text{for } f = d \end{cases}, \tag{5.63}$$

where l and q denote leptons and quarks, respectively. Hence, the quark mixing is described by the CKM matrix, while the mixing in the lepton sector is absent but can be additionally introduced following the commonly accepted modifications of the SM which comprise neutrino masses. Furthermore, as the Higgs part of the NCSM action introduces mass dependent gauge boson couplings '*see* **Eq.**(5.61)', the Yukawa part of the NCSM action introduces fermion mass dependent interactions. In contrast to the NCSM, in the SM fermion mass dependent interactions always include an interaction with the Higgs field.

Using **Eq.**(5.12) we find

$$
\begin{aligned}
&\int d^4x \overline{\widehat{\Psi}}^{(i)}_L \star h_{\psi_{down}}(\widehat{\Phi}) \star \widehat{\psi}^{(j)}_{down,R} \\
= \;&\int d^4x (\overline{\Psi}^{(i)}_L \Phi \psi^{(j)}_{down,R}) + \frac{1}{2}\int d^4x \Theta^{\mu\nu} \overline{\Psi}^{(i)}_L [-i \overleftarrow{\partial}_\mu \Phi \overrightarrow{\partial}_\nu \\
&- \overleftarrow{\partial}_\nu R_{\Psi_L}(V_\mu)\Phi - \Phi R_{\psi_{down,R}}(V_\mu)\overrightarrow{\partial}_\nu \\
&- R_{\Psi_L}(V_\mu)(\partial_\mu \Phi) - (\partial_\mu \Phi) R_{\psi_{down,R}}(V_\mu) \\
&+ i R_{\Psi_L}(V_\mu) R_{\Psi_L}(V_\nu)\Phi + i\Phi R_{\psi_{down,R}}(V_\mu) R_{\psi_{down,R}}(V_\nu) \\
&- i R_{\Psi_L}(V_\mu)\Phi R_{\psi_{down,R}}(V_\nu)]\psi^{(j)}_{down,R}. \qquad (5.64)
\end{aligned}
$$

The representations $R_{\Psi_L}(V_\mu)$ and $R_{\psi_{down,R}}(V_\mu)$ can be read from **Table**.5.2. Expressions valid for both leptons and quarks, with strong interactions omitted, are given in **Eqs**.(5.37) and (5.45). For the Higgs field (5.57) is used.

Finally, using (5.64), after some algebra we obtain the following result for (5.62) expressed in terms of physical fields '*and with gluons omitted*':

$$
\begin{aligned}
S_{\psi,Yukawa} \;=\; &\int d^4x \sum_{i,j=1}^{3} [\overline{\psi}^{(i)}_{down}(N^{V(ij)}_{dd} + \gamma N^{A(ij)}_{dd})\psi^{(j)}_{down} \\
&+ \overline{\psi}^{(i)}_{up}(N^{V(ij)}_{uu} + \gamma_5 N^{A(ij)}_{uu})\psi^{(j)}_{up} \\
&+ \overline{\psi}^{(i)}_{up}(C^{V(ij)}_{ud} + \gamma_5 C^{A(ij)}_{ud})\psi^{(j)}_{down} \\
&+ \overline{\psi}^{(i)}_{down}(C^{V(ij)}_{du} + \gamma_5 C^{A(ij)}_{du})\psi^{(j)}_{up}]. \qquad (5.65)
\end{aligned}
$$

The neutral currents read

$$\begin{aligned}
N_{dd}^{V(ij)} &= -M_{down}^{(ij)}\left(1+\frac{h}{v}\right)+N_{dd}^{V,\Theta(ij)}+O\left(\Theta^2\right), \\
N_{dd}^{A(ij)} &= N_{dd}^{A,\Theta(ij)}+O\left(\Theta^2\right), \\
N_{uu}^{V(ij)} &= -M_{up}^{(ij)}\left(1+\frac{h}{v}\right)+N_{uu}^{V,\Theta(ij)}+O\left(\Theta^2\right), \\
N_{uu}^{A(ij)} &= N_{uu}^{A,\Theta(ij)}+O\left(\Theta^2\right),
\end{aligned} \tag{5.66}$$

where

$$\begin{aligned}
N_{dd}^{V,\Theta(ij)} &= -\frac{1}{2}\Theta^{\mu\nu}M_{down}^{(ij)}\{i\frac{(\partial_\mu h)}{v}\overrightarrow{\partial}_\nu \\
&-[eQ_{\psi_{down}}A_\mu+\frac{g}{2\cos\theta_W}(T_{3,\psi_{down,L}}-2Q_{\psi_{down}}\sin^2\theta_W)Z_\mu]\frac{(\partial_\mu h)}{v} \\
&+[eQ_{\psi_{down}}(\partial_\nu A_\mu)+\frac{g}{2\cos\theta_W}(T_{3,\psi_{down,L}}-2Q_{\psi_{down}}\sin^2\theta_W)(\partial_\nu Z_\mu) \\
&-i\frac{g^2}{2}W_\mu^+W_\nu^-]\left(1+\frac{h}{v}\right)\},
\end{aligned} \tag{5.67}$$

$$\begin{aligned}
N_{dd}^{A,\Theta(ij)} &= \frac{g}{4\cos\theta_W}T_{3,\psi_{down,L}}\Theta^{\mu\nu}M_{down}^{(ij)}\left(1+\frac{h}{v}\right)Z_\mu \\
&\times\left[\left(\overleftarrow{\partial}_\nu-\overrightarrow{\partial}_\nu\right)+2ieQ_{\psi_{down}}A_\nu\right],
\end{aligned} \tag{5.68}$$

and

$$\left.\begin{array}{l} N_{uu}^{V,\Theta(ij)} \\ \\ N_{uu}^{A,\Theta(ij)} \end{array}\right\} = \left\{\begin{array}{l} N_{dd}^{V,\Theta(ij)} \\ \\ N_{dd}^{A,\Theta(ij)} \end{array}\right. \quad (W^+\leftrightarrow W^-, down\rightarrow up). \tag{5.69}$$

The charged currents are given by

$$\begin{aligned}
C_{ud}^{V(ij)} &= C_{ud}^{V,\Theta(ij)}+O\left(\Theta^2\right), \\
C_{ud}^{A(ij)} &= C_{ud}^{A,\Theta(ij)}+O\left(\Theta^2\right),
\end{aligned} \tag{5.70}$$

where

$$
\begin{aligned}
& C_{ud}^{V,\Theta(ij)} \\
= \; & -\frac{g}{4\sqrt{2}}\Theta^{\mu\nu}\left(1+\frac{h}{\upsilon}\right)\{[((V_f M_{down})^{(ij)}+(M_{up}V_f)^{(ij)})(\partial_\nu W_\mu^+) \\
& +\left((V_f M_{down})^{(ij)}\overrightarrow{\partial}_\nu+(M_{up}V_f)^{(ij)}\overleftarrow{\partial}_\nu\right)W_\mu^+] \\
& +ie((V_f M_{down})^{(ij)}Q_{\psi_{up}}-(M_{up}V_f)^{(ij)}Q_{\psi_{down}})A_\mu W_\nu^+ \\
& +i\frac{g}{\cos\theta_W}[(V_f M_{down})^{(ij)}(2T_{3,\psi_{up,L}}-Q_{\psi_{up}}\sin^2\theta_W) \\
& -(M_{up}V_f)^{(ij)}(2T_{3,\psi_{down,L}}-Q_{\psi_{down}}\sin^2\theta_W)]Z_\mu W_\nu^+\},
\end{aligned}
\tag{5.71}
$$

and

$$
C_{ud}^{A,\Theta(ij)}=C_{ud}^{V,\Theta(ij)}(M_{up}\to -M_{up}),
\tag{5.72}
$$

while

$$
\begin{aligned}
C_{du}^{V(ij)} &= \left(C_{ud}^{V(ij)}(\overrightarrow{\partial}\leftrightarrow\overleftarrow{\partial})\right)^\dagger, \\
C_{du}^{A(ij)} &= -\left(C_{ud}^{A(ij)}(\overrightarrow{\partial}\leftrightarrow\overleftarrow{\partial})\right)^\dagger.
\end{aligned}
\tag{5.73}
$$

Note that $\overrightarrow{\partial}$ and $\overleftarrow{\partial}$ are defined in (5.44).

At the end, observe that the simplified introduction of the fermion mass and the use of the relation

$$
\begin{aligned}
S_{\psi,m} &= \int d^4x\overline{\widehat{\psi}}\star(i\widehat{\not{D}}-m)\widehat{\psi} \\
&= \int d^4x[\overline{\psi}(i\not{D}-m)\psi-\frac{1}{4}\overline{\psi}R_\psi(F_{\mu\nu})(i\Theta^{\mu\nu\rho}D_\rho-m\Theta^{\mu\nu})\psi \\
&\quad +O\left(\Theta^2\right)].
\end{aligned}
\tag{5.74}
$$

is valid only in the case of pure QED and pure QCD.

5.6 Feynman Rules

On the basis of the results presented in **Sections** 5.4 and 5.5, it is now straightforward to derive the Feynman rules needed for phenomenological applications of the NCSM, i.e. for the calculation of physical processes. In this section, we list a number of selected Feynman rules for the NCSM pure electroweak interactions up to order Θ. We omit interactions with the Higgs particle, boson interactions with four and more gauge fields, and fermion interactions with more than two gauge bosons.

The following notation for vertices has been adopted: *all gauge boson momenta are taken to be incoming; following the flow of the fermion line, the momenta of the incoming and outgoing fermions are given by p_{in} and p_{out}, respectively.* In the following we denote fermions by f, and the generation indices by i and j. Furthermore, $f_u^{(i)} \in \{\nu^{(i)}, u^{(i)}\}$ and $f_d^{(i)} \in \{e^{(i)}, d^{(i)}\}$.

For the Feynman rules we use the following definitions:

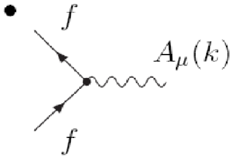
$$\begin{aligned} c_{V,f} &= T_{3,f_L} - 2Q_f \sin^2\theta_W, \\ c_{A,f} &= T_{3,f_L}. \end{aligned} \tag{5.75}$$

The charge Q and the weak isospin T_3 can be read from **Table**.5.1. The notation V_f is introduced in (5.63), while $\Theta^{\mu\nu\rho}$ is defined in (5.35). We also make use of $(\Theta k)^\mu \equiv \Theta^{\mu\nu} k_\nu = -k_\nu \Theta^{\nu\mu} \equiv -(k\Theta)^\mu$ and $(k\Theta p) \equiv k_\mu \Theta^{\mu\nu} p_\nu$.

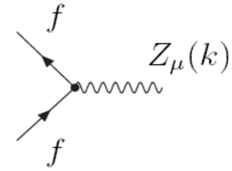
5.6.1 Minimal NCSM

In this subsection we present selected Feynman rules for the mNCSM containing SM contributions and Θ corrections. The Θ corrections to vertices containing fermions are obtained using **Eq.**(5.49) and the Yukawa part of the action (5.65) has to be taken into account as well, because it generates additional mass dependent terms which modify some interaction vertices. In comparison with the SM, this is a novel feature. Similarly, the gauge boson couplings present in (5.24) receive additional Θ dependent corrections from the Higgs part of the action (5.50) and even new three- and four-gauge boson couplings appear, see (5.61).

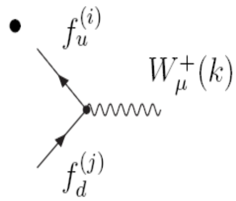
First, we list three-vertices that appear in the SM as well.

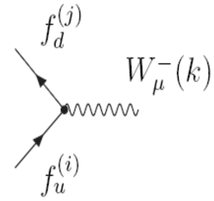
- 

$$
\begin{aligned}
& ieQ_f\left[\gamma_\mu - \frac{i}{2}k^\nu\left(\Theta_{\mu\nu\rho}p_{in}^\rho - \Theta_{\mu\nu}m_f\right)\right] \\
= \;& ieQ_f\gamma_\mu \qquad (5.76) \\
& +\frac{1}{2}eQ_f[(p_{out}\Theta p_{in})\gamma_\mu(p_{out}\Theta)_\mu(\not{p}_{in} - m_f) - (\not{p}_{out} - m_f)(\Theta p_{in})_\mu],
\end{aligned}
$$

- 

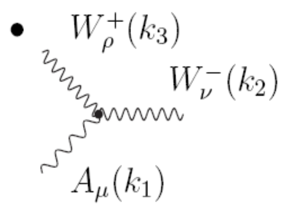
$$
\begin{aligned}
& \frac{ie}{\sin 2\theta_W}\{(\gamma_\mu - \frac{i}{2}k^\nu\Theta_{\mu\nu\rho}p_{in}^\rho)\left(c_{V,f} - c_{A,f}\gamma_5\right) \\
& -\frac{i}{2}\Theta_{\mu\nu}m_f[p_{in}^\nu\left(c_{V,f} - c_{A,f}\gamma_5\right) - p_{out}^\nu\left(c_{V,f} + c_{A,f}\gamma_5\right)]\}, \qquad (5.77)
\end{aligned}
$$





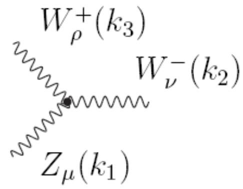
$$\frac{ie}{2\sqrt{2}\sin\theta_W}\begin{pmatrix} V_f^{(ij)} \\ \\ V_f^{*(ij)} \end{pmatrix}\{[\gamma_\mu - \frac{i}{2}\Theta_{\mu\nu\rho}k^\nu p_{in}^\rho](1-\gamma_5)$$

$$-\frac{i}{2}\Theta_{\mu\nu}[\begin{pmatrix} m_{f_u^{(i)}} \\ \\ m_{f_d^{(j)}} \end{pmatrix} p_{in}^\nu(1-\gamma_5) - \begin{pmatrix} m_{f_d^{(j)}} \\ \\ m_{f_u^{(i)}} \end{pmatrix} p_{out}^\nu(1+\gamma_5)]\}, \qquad (5.78)$$



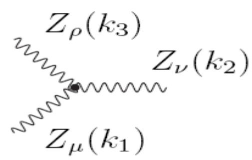
$$ie\{g^{\mu\nu}(k_1-k_2)^\rho + g^{\nu\rho}(k_2-k_3)^\mu + g^{\rho\mu}(k_3-k_1)^\nu$$

$$+\frac{i}{2}M_W^2[\Theta^{\mu\nu}k_1^\rho + \Theta^{\mu\rho}k_1^\nu + g^{\mu\nu}(\Theta k_1)^\rho - g^{\nu\rho}(\Theta k_1)^\mu + g^{\rho\mu}(\Theta k_1)^\nu]\}, \qquad (5.79)$$

- 

$$
\begin{aligned}
& ie\cot\theta_W\{g^{\mu\nu}(k_1-k_2)^\rho+g^{\nu\rho}(k_2-k_3)^\mu+g^{\rho\mu}(k_3-k_1)^\nu \\
& +\frac{i}{2}M_W^2[\Theta^{\mu\nu}k_1^\rho+\Theta^{\mu\rho}k_1^\nu+g^{\mu\nu}(\Theta k_1)^\rho-g^{\nu\rho}(\Theta k_1)^\mu+g^{\rho\mu}(\Theta k_1)^\nu] \\
& -\frac{i}{4}M_Z^2[\Theta^{\mu\nu}(k_1-k_2)^\rho+\Theta^{\nu\rho}(k_2-k_3)^\mu+\Theta^{\rho\mu}(k_3-k_1)^\nu \\
& -2g^{\mu\nu}(\Theta k_3)^\rho-2g^{\nu\rho}(\Theta k_1)^\mu-2g^{\rho\mu}(\Theta k_2)^\nu]\}. \qquad (5.80)
\end{aligned}
$$

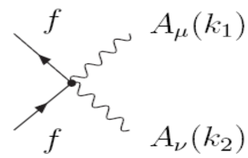
Here we give the new three-gauge-boson coupling which follows from the Higgs action (5.50), i.e., **Eq.**(5.61)

- 

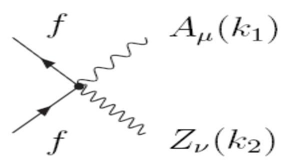
$$
\begin{aligned}
& \frac{eM_Z^2}{2\sin 2\theta_W}[\Theta^{\mu\nu}(k_1-k_2)^\rho+\Theta^{\nu\rho}(k_2-k_3)^\mu+\Theta^{\rho\mu}(k_3-k_1)^\nu \\
& -2g^{\mu\nu}(\Theta k_3)^\rho-2g^{\nu\rho}(\Theta k_1)^\mu-2g^{\rho\mu}(\Theta k_2)^\nu]. \qquad (5.81)
\end{aligned}
$$

Additionally, from the Higgs action (5.50) one can derive the Θ corrections to the electroweak four-gauge-boson vertices already present in SM '*see* (5.24)', as well as, new four-gauge-boson vertices.

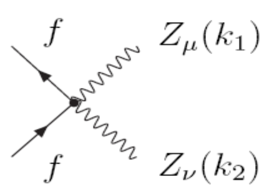
Equation (5.49) also describes the interaction vertices involving fermions and two or three gauge bosons. These do not appear in the SM. In the following we provide all contributions to such vertices with four legs and corresponding mass-dependent contributions from (5.65).

- 

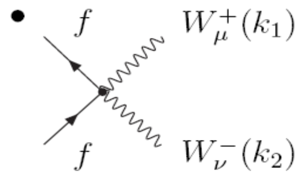
$$\frac{-e^2 Q_f^2}{2}\Theta_{\mu\nu\rho}(k_1^\rho - k_2^\rho), \tag{5.82}$$

- 

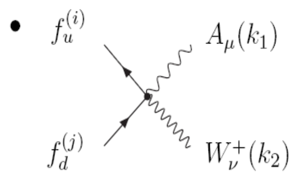
$$\frac{-e^2 Q_f}{2\sin 2\theta} \tag{5.83}$$
$$\times\left[\Theta_{\mu\nu\rho}(k_1^\rho - k_2^\rho)\left(c_{V,f} + c_{A,f}\gamma_5\right) - 2\Theta_{\mu\nu} m_f c_{A,f}\gamma_5\right],$$

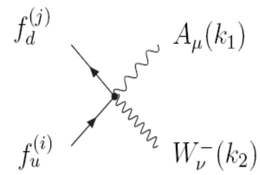
- 

$$\frac{-e^2}{2\sin^2 2\theta}\Theta_{\mu\nu\rho}(k_1^\rho - k_2^\rho)\left(c_{V,f} + c_{A,f}\gamma_5\right)^2, \tag{5.84}$$

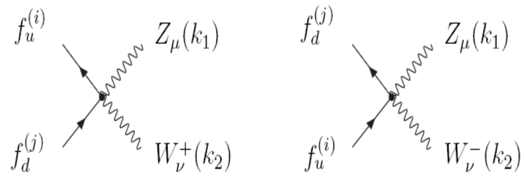
- 

$$\frac{-e^2}{8\sin^2\theta}\left[\Theta_{\mu\nu\rho}(p^\rho_{in}+k^\rho_1)(1-\gamma_5)+2\Theta_{\mu\nu}m_f\right], \tag{5.85}$$

- 



$$\begin{aligned}&\frac{-e^2}{4\sqrt{2}\sin\theta_W}\{\Theta_{\mu\nu\rho}[\begin{pmatrix} Q_{f_u^{(i)}} \\ \\ Q_{f_d^{(j)}}\end{pmatrix}(p^\rho_{in}+k^\rho_1)-\begin{pmatrix} Q_{f_d^{(j)}} \\ \\ Q_{f_u^{(i)}}\end{pmatrix}(p^\rho_{in}+k^\rho_2)](1-\gamma_5)\\ &+\Theta_{\mu\nu}[\begin{pmatrix} m_{f_u^{(i)}}Q_{f_d^{(j)}} \\ \\ m_{f_d^{(j)}}Q_{f_u^{(i)}}\end{pmatrix}(1-\gamma_5)-\begin{pmatrix} m_{f_d^{(j)}}Q_{f_u^{(i)}} \\ \\ m_{f_u^{(i)}}Q_{f_d^{(j)}}\end{pmatrix}(1+\gamma_5)]\}\begin{pmatrix} V_f^{(ij)} \\ \\ V_f^{*(ij)}\end{pmatrix},\end{aligned} \tag{5.86}$$

- 

$$
\begin{aligned}
&\frac{-e^2}{4\sqrt{2}\sin\theta_W \sin 2\theta_W}\begin{pmatrix} V_f^{(ij)} \\ V_f^{*(ij)} \end{pmatrix} \\
&\{\Theta_{\mu\nu\rho}[\begin{pmatrix} c_{V,f_u^{(i)}} + c_{A,f_u^{(i)}} \\ c_{V,f_d^{(j)}} + c_{A,f_d^{(j)}} \end{pmatrix}(p_{in}^\rho + k_1^\rho) - \begin{pmatrix} c_{V,f_d^{(j)}} + c_{A,f_d^{(j)}} \\ c_{V,f_u^{(i)}} + c_{A,f_u^{(i)}} \end{pmatrix}(p_{in}^\rho + k_2^\rho)](1-\gamma_5) \\
&+\Theta_{\mu\nu}[\begin{pmatrix} m_{f_u^{(i)}}\left[c_{V,f_d^{(j)}} + 3c_{A,f_d^{(j)}}\right] \\ m_{f_d^{(j)}}\left[c_{V,f_u^{(i)}} + 3c_{A,f_u^{(i)}}\right] \end{pmatrix}(1-\gamma_5) \\
&-\begin{pmatrix} m_{f_d^{(j)}}\left[c_{V,f_u^{(i)}} + 3c_{A,f_u^{(i)}}\right] \\ m_{f_u^{(i)}}\left[c_{V,f_d^{(j)}} + 3c_{A,f_d^{(j)}}\right] \end{pmatrix}(1+\gamma_5)]\}. \qquad (5.87)
\end{aligned}
$$

Similarly, $ffWWZ$, $ffWW\gamma$ and $ff\gamma WZ$ can be extracted from **Eq.**(5.49) as well. They have no mass-dependent corrections.

Chapter 6

New Limit for the Noncommutativity Parameter of the Noncommutative Standard Model

In this chapter, we discuss the limits on the scale of the noncommutativity parameter Λ_{NC} via studying the top-quark pair production through electron-positron collision in the framework of the minimal noncommutative standard model (mNCSM), using the Seiberg-Witten (SW) maps up to the first order of the noncommutativity parameter $\Theta^{\mu\nu}$.

6.1 Cross-Section of $t\bar{t}$ Pair Production and Numerical Analysis

The action of NCSM can be written as '*See* ***Chapter*** 5 *for more details*'

$$S_{NCSM} = S_{fermions} + S_{gauge} + S_{Higgs} + S_{Yukawa},$$

We just consider the matter sector of the action, i.e. the action of the fermions '*leptons and quarks*'; which there are terms taking part in this process $e^-e^+ \to \gamma, Z \to t\bar{t}$:

$$S_{fermions} = \int d^4x \sum_{i=1}^{3} \left(\bar{\hat{\psi}}_L^{(i)} \star \left(i\hat{\ } \not{D} \hat{\psi}_L^{(i)} \right) \right) + \int d^4x \sum_{i=1}^{3} \left(\bar{\hat{\psi}}_R^{(i)} \star \left(i\hat{\ } \not{D} \hat{\psi}_R^{(i)} \right) \right).$$

We study the process $e^-(p_1)e^+(p_2) \to t(p_3)\bar{t}(p_4)$ in the NCSM, which proceeds via the s-channel exchange of γ and Z bosons. We are interested in the first order of NC corrections on the cross-section of the $t\bar{t}$ production. The Feynman rules are given in [9].

The relative Feynman rule for

- $e(p_{in})$-$e(p_{out})$-$\gamma(k)$ vertex to the first order in Θ is

$$= ieQ_f \left[\gamma_\mu - \frac{i}{2} k^\nu \left(\Theta_{\mu\nu\rho} p_{in}^\rho - \Theta_{\mu\nu} m_f \right) \right]$$

- $e(p_{in})$-$e(p_{out})$-$Z(k)$ vertex to the 1^{st} order in Θ is

$$\begin{aligned} = \quad & \frac{ie}{\sin 2\theta_W} \{ (\gamma_\mu - \frac{i}{2} k^\nu \Theta_{\mu\nu\rho} p_{in}^\rho) \left(C_{V,f} - C_{A,f}\gamma_5 \right) \\ & - \frac{i}{2} \Theta_{\mu\nu} m_f [p_{in}^\nu \left(C_{V,f} - C_{A,f}\gamma_5 \right) - p_{out}^\nu \left(C_{V,f} + C_{A,f}\gamma_5 \right)] \} \end{aligned}$$

with $\Theta_{\mu\nu\rho} = \Theta_{\mu\nu}\gamma_\rho + \Theta_{\nu\rho}\gamma_\mu + \Theta_{\rho\mu}\gamma_\nu$

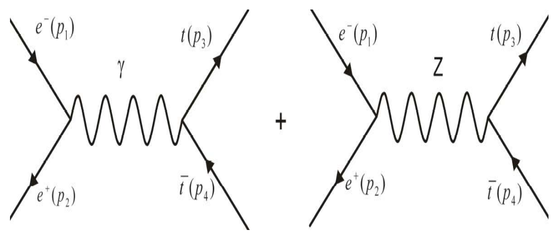


Fig.6.1 Feynman diagrams for the process $e^-e^+ \to \gamma, Z \to t\bar{t}$ in the NCSM.

The corresponding Feynman diagrams are shown in **Fig**.6.1. Differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \left|\bar{A}\right|^2 .$$

We can obtain the cross-section $\sigma = \sigma\left(\sqrt{s}, \Lambda_{NC}, \theta, \phi\right)$ as

$$\sigma = \int_{-1}^{1} d(\cos\theta) \int_{0}^{2\pi} d\phi \frac{d\sigma}{d\Omega}$$

where θ and ϕ are polar and azimuthal angles, respectively.

Most analysis studies have been assumed $c_{\mu\nu} = (\xi_i, \epsilon_{ijk}\chi^k)$ where $\xi_i = (\vec{E})_i$ and $\chi_k = (\vec{B})_k$. The vectors $\vec{E}$ and $\vec{B}$ are given in [15, 64], i.e. $\vec{E} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$,

$\vec{B} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$,where $c^{\mu\nu}$ has the following form:

$$c^{\mu\nu} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{pmatrix}.$$

In the present work, $c^{\mu\nu}$ is represented by the ansatz

$$c^{\mu\nu} = \frac{1}{2}\left(\sigma^{\mu\nu} + (\sigma^{\mu\nu})^{+}\right),$$

knowing that

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}\right),$$

where γ^{μ} are Dirac matrices.

All calculations of the cross-section scattering have been done using the packages FeynArts and FormCalc [20-23], for which we have written a complete FeynArts model file and have extended the corresponding FormCalc Fortran drivers for the NCSM. The implementation is described for the FeynArts model file in the **Appendix** A.1. and for the FormCalc Fortran drivers in A.2. In our numerical analysis we also have used LoopTools [66]. We analyze the total cross-section in the presence of space-time noncommutativity. In **Fig**.6.2, we have plotted the total cross-section for the process $e^{-}e^{+} \to \gamma, Z \to t\bar{t}$ [pb] as a function of center-of-mass energy $E_{com}(=\sqrt{s})$ [GeV]. The ordinary SM is presented by solid curve '*the black one*' and the NCSM with different curves short-dashed '*red*', dotted '*blue*' and long-dashed '*green*', with the corresponding $\Lambda_{NC} = 400, 300$ and 200 GeV, respectively.

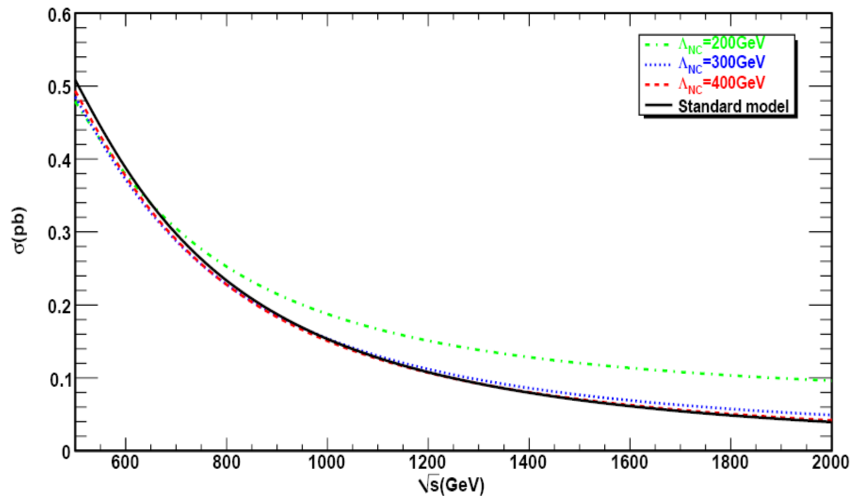


Fig.6.2. The total cross-section for the process $e^-e^+ \longrightarrow \gamma, Z \longrightarrow t\bar{t}$ [pb] as a function of center-of-mass energy $E_{com}(=\sqrt{s})$ [GeV].

We can see that the effect of the noncommutativity appears at around 700 GeV. Moreover, notice that the deviations become more important with the upper energy and with smaller values of Λ_{NC}. We find that for lower values of the noncommutativity scale Λ_{NC}, there is a significant deviation from the SM result and with the increase in NC, all the curves approach the SM value, i.e. the deviations from the standard model are significant for small values of the noncommutative characteristic scale. The cross-section for $e^+e^- \to \mu^+\mu^-$ up to the Θ^2 order was studied in [64]. An interesting result is that all the contribution from Θ, Θ^2 and Θ^3 terms to the cross-section canceled out. We confirm this result that this process is not sensible to the noncommutative corrections at the first order in Θ, i.e. there is no noncommutative effect at $O(\Theta)$.

Chapter 7

Conclusions

We have presented the Standard Model on noncommutative spacetime and its main ingredients, the Moyal-Weyl $\star$-product of functions on ordinary space-time which reproduces thus the noncommutativity inherent to the noncommutative operators $\hat{x}^\mu$ on an algebra of functions on the ordinary spacetime, and the Seiberg-Witten maps. The latter map the ordinary fields to noncommutative fields in such a way that ordinary gauge transformations induce noncommutative transformations. This requirement was described mathematically by the so called gauge equivalence conditions for the gauge and matter field, and the consistency equation for the gauge parameter. These differential equations can be solved order by order in the noncommutative parameter $\Theta^{\mu\nu}$ and their solutions are the Seiberg-Witten maps, determined nonuniquely, since they differ by homogeneous solutions of the differential equations. The result is an effective theory as expansion in powers of Θ, which preserves noncommutative gauge invariance, is anomaly free, does not modify the SM particle content.

The extension of the standard model of elementary particle physics to noncommutative space-time opens a window on a rich variety of new physical phenomena. The presence of the noncommutativity parameter $\Theta^{\mu\nu}$ which breaks Lorentz invariance at a scale Λ_{NC}, results in deviation of the production cross-section from the one of SM prediction. In this work, we have studied the top-quark pair production process $e^-e^+ \to \gamma, Z \to t\bar{t}$ in the framework of the NCSM in order to derive bounds on the NC scale Λ_{NC}. Our analysis has been made up to the first order in $O(\Theta)$. We have defined the noncommutative parameter with the help of the gamma matrices. The noncommutative structure is determined by some spinor background on which the gamma-dependent $\Theta^{\mu\nu}$ acts.

The noncommutative effects seem to be completely hidden under the shadow of the SM results for most part of the parameter space and it is only in a very small range, for very low values of the noncommutativity scale that there are any significant deviations from the SM values. The NC effects are found to be significant only for low values of the NC characteristic scale Λ_{NC}, which is in the range 0.1-0.2 TeV and we have noticed also that the noncommutativity effects become more pronounced for $E_{com} \geq 700$ GeV. We got the same results that has been obtained by [65]. Another lowest bound on the NC scale which is surprisingly low, has been given in [61] which are in the order of 0.1- 0.2 TeV.

To conclude, regarding the top-quark pair production through electron-positron collision and the possibility that the space-time noncommutativity is observed in such a scattering, our theoretical predicted signature is relatively small. However, there is much more work that can be done in this direction. For further phenomenological consequences, one could go beyond calculating the cross-sections of this to the second order $O(\Theta^2)$.

In this spirit, it is much better if we can expand the model to higher orders in theta, because it takes more corrections into account. The contributions of the corrections to higher orders will become less and less important, but still they might play a role for certain processes.

Appendix A

Implementation of the NCSM in FeynArts and FormCalc

Serious perturbative calculations in physics can generally no longer done by hand: required accuracy, Models with many particles, etc. In order to calculate some physical processes; hybrid programming techniques are necessary: Computer algebra is an indispensable tool because many manipulations must be done symbolically and fast number crunching can only be achieved in compiled language. Using FeynArts, FormCalc and LoopTools [20-23, 66] which are Mathematica packages is quite convenient for studying particular processes. These programs are available [67] and up to now the FeynArts package does not contain model files for the NCSM. In the following sections we describe how we have implemented the new model file for calculations in the NCSM in FeynArts, and how we have extended the corresponding FormCalc fortran drivers. The reason why we give this summary is to pass on the knowledge we have, such that it can be used by others who want to perform calculations in the NCSM using FeynArts and FormCalc.

We have performed our changes using: the versions FeynArts 3.9., FormCalc 9.2. and LoopTools-2.13..

Operating system: Linux

Programming language used: Mathematica

A.1 FeynArts Model File for the NCSM

In FeynArts, the fields, the propagators and the couplings are defined in special files, i.e. the information about the physical model is provided in two fields: The Generic model file defines the representation of the kinematical quantities like spinors or vectors fields '*the generic analytical propagators*' and couplings. The Classes model file defines the particles contents and specifies the actual coupling. In the NCSM, no new particles are introduced so we work with the same fields and with the same parameters as in the SM, there are no NC corrections to the propagators but the couplings are modified and we should introduce the scale of the noncommutativity Λ_{NC}, so obviously we need a new model files. We have collected the information about the couplings of the NCSM into Generic model file and Classes model file. As we have mentioned in **Chapter** 6. We have assumed that $c^{\mu\nu}$ has the form:

$$c^{\mu\nu} = \frac{i}{4}\left[(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) - (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)^{+}\right] \tag{A.1}$$

The diagrams and the amplitudes are generated with the new FeynArts model files NCSM.mod and NCSM.gen. In the following part we are going to present the new model files that have been created in FeynArts for calculations in the NCSM.

- **Classes model file:** NCSM.mod

```
(*
    NCSM.mod

        last modified 10 March 11 by Linda GHEGAL
This file contains the definition of a Classes model for FeynArts.
It needs the Generic model file Lorentz.gen.
When you change things, remember:
– All particles are arranged in classes. For single particle
model definitions each particle lives in its own class.
– For each class the common SelfConjugate behavior and the
IndexRange MUST be present in the definitions.
– IMPORTANT: The coupling matrices MUST be declared in the
SAME order as the Generic coupling.
Reference:
    Ansgar Denner, "Techniques for the calculation of electroweak
    radiative corrections at one-loop level and results for
    W-physics at LEP200", Fortschr. d. Physik, 41 (1993) 4
Oct 95: one-loop counter terms added by Stefan Bauberger:
    Some corrections and addition of all one-loop counter terms
    according to A. Denner. The gauge-fixing terms are assumed not
```

to be renormalized. The Denner conventions are extended to

include field renormalization of the Goldstone bosons.

The counter terms associated with quark mixing are not well

tested yet.

Apr 99: Christian Schappacher added colour indices for the quarks

Apr 99: Terms for ghost sector updated by Ayres Freitas.

The gauge-fixing terms are still assumed not to be renormalized

but the renormalized gauge parameters follow the R_xi-gauge.

In addition, renormalization for the ghost fields is included.

The 2-loop counter terms for vector-boson selfenergies and for

the W-nu-l vertex have been added.

Old versions of the changes of sbau are removed!

Apr 01: Thomas Hahn added the definitions of the renormalization

constants a la A. Denner.

This file introduces the following symbols:

coupling constants and masses:

EL: electron charge (Thomson limit)

CW, SW: cosine and sine of Weinberg angle

MW, MZ, MH: W, Z, Higgs masses

MLE: lepton class mass

ME, MM, ML: lepton masses (e, mu, tau)

MQU: u-type quark class mass

MU, MC, MT: u-type quark masses (up, charm, top)

MQD: d-type quark class mass

MD, MS, MB: d-type quark masses (down, strange, bottom)

CKM: quark mixing matrix

(set CKM = IndexDelta for no quark-mixing)

GaugeXi[A, W, Z]: photon, W, Z gauge parameters

one-loop renormalization constants (RCs):

---

dZe1: electromagnetic charge RC

dSW1, dCW1: Weinberg angle sine/cosine RC

dZH1, dMHsq1: Higgs field and mass RC

dZW1, dMWsq1: W field and mass RC

dMZsq1: Z mass RC

dZZZ1, dZZA1,

dZAZ1, dZAA1: Z and photon field RCs

dMf1: fermion mass RCs

dZfL1, dZfR1: fermion field RCs

dCKM1: quark mixing matrix RCs

dZG01, dZGp1: field RC for unphysical scalars

dUZZ1, dUZA1,

dUAZ1, dUAA1: field RCs for photon and Z ghosts dUW1: field RC for +/- ghosts

two-loop renormalization constants:

---

dZe2: electromagnetic charge RC

dSW2: weak mixing angle sine/cosine RC

dZW2, dMWsq2: W field and mass RC

dMZsq2: Z mass RC

dZZZ2, dZZA2,

dZAZ2, dZAA2: Z and photon field RCs

dZfL2: fermion field RCs

*)

IndexRange[Index[Generation]] = Range[3]

IndexRange[Index[Colour]] = NoUnfold[Range[3]]

IndexStyle[Index[Generation, i_Integer]] := Alph[i + 8]

MaxGenerationIndex = 3

ViolatesQ[q__] := Plus[q] =!= 0

mdZfLR1[type_, j1_, j2_] :=

Mass[F[type, {j1}]]/2 dZfL1[type, j1, j2] +

Mass[F[type, {j2}]]/2 Conjugate[dZfR1[type, j2, j1]]

mdZfRL1[type_, j1_, j2_] :=

Mass[F[type, {j1}]]/2 dZfR1[type, j1, j2] +

Mass[F[type, {j2}]]/2 Conjugate[dZfL1[type, j2, j1]]

```
(* the leptonic field RCs are diagonal: *)
dZfL1[ type:1 | 2, j1_, j2_ ] :=
IndexDelta[j1, j2] dZfL1[type, j1, j1] /; j1 =!= j2
dZfR1[ type:1 | 2, j1_, j2_ ] :=
IndexDelta[j1, j2] dZfR1[type, j1, j1] /; j1 =!= j2
(* some short-hands for fermionic couplings: *)
FermionCharge[1] = 0;
FermionCharge[2] = -1;
FermionCharge[3] = 2/3;
FermionCharge[4] = -1/3
gR[ type_ ] :=
-SW/CW FermionCharge[type];
gL[ type_ ] :=
(If[ OddQ[type], 1/2, -1/2 ] - SW^2 FermionCharge[type])/(SW CW);
dgR[ type_ ] :=
gR[type] (dZe1 + 1/(CW^2 SW) dSW1);
dgL[ type_ ] :=
If[ OddQ[type], 1/2, -1/2 ]/(SW CW) *
(dZe1 + (SW^2 - CW^2)/(CW^2 SW) dSW1) + dgR[type]
M$ClassesDescription = {
    (* Leptons (neutrino): I_3 = +1/2, Q = 0 *)
F[1] == {
```

```
    SelfConjugate -> False,
    Indices -> {Index[Generation]},
    Mass -> 0,
    QuantumNumbers -> LeptonNumber,
    PropagatorLabel -> ComposedChar["\\nu", Index[Generation]],
    PropagatorType -> Straight,
    PropagatorArrow -> Forward },
    (* Leptons (electron): I_3 = -1/2, Q = -1 *)
F[2] == {
    SelfConjugate -> False,
    Indices -> {Index[Generation]},
    Mass -> MLE,
    QuantumNumbers -> {-Charge, LeptonNumber},
    PropagatorLabel -> ComposedChar["e", Index[Generation]],
    PropagatorType -> Straight,
    PropagatorArrow -> Forward },
    (* Quarks (u): I_3 = +1/2, Q = +2/3 *)
F[3] == {
    SelfConjugate -> False,
    Indices -> {Index[Generation], Index[Colour]},
    Mass -> MQU,
    QuantumNumbers -> 2/3 Charge,
```

```
    PropagatorLabel -> ComposedChar["u", Index[Generation]],
    PropagatorType -> Straight,
    PropagatorArrow -> Forward },
    (* Quarks (d): I_3 = -1/2, Q = -1/3 *)
F[4] == {
    SelfConjugate -> False,
    Indices -> {Index[Generation], Index[Colour]},
    Mass -> MQD,
    QuantumNumbers -> -1/3 Charge,
    PropagatorLabel -> ComposedChar["d", Index[Generation]],
    PropagatorType -> Straight,
    PropagatorArrow -> Forward },
    (* Gauge bosons: Q = 0 *)
V[1] == {
    SelfConjugate -> True,
    Indices -> {},
    Mass -> 0,
    PropagatorLabel -> "\\gamma",
    PropagatorType -> Sine,
    PropagatorArrow -> None },
V[2] == {
    SelfConjugate -> True,
```

```
    Indices -> {},
    Mass -> MZ,
    PropagatorLabel -> "Z",
    PropagatorType -> Sine,
    PropagatorArrow -> None },
    (* Gauge bosons: Q = -1 *)
V[3] == {
    SelfConjugate -> False,
    Indices -> {},
    Mass -> MW,
    QuantumNumbers -> -Charge,
    PropagatorLabel -> "W",
    PropagatorType -> Sine,
    PropagatorArrow -> Forward },
(*
V[4] == {
    SelfConjugate -> True,
    Indices -> {},
    Mass -> MAZ,
    MixingPartners -> {V[1], V[2]},
    PropagatorLabel -> {"\\gamma", "Z"},
    PropagatorType -> Sine,
```

```
    PropagatorArrow -> None },
*)
    (* mixing Higgs gauge bosons: Q = 0 *)
SV[2] == {
    SelfConjugate -> True,
    Indices -> {},
    Mass -> MZ,
    MixingPartners -> {S[2], V[2]},
    PropagatorLabel -> {ComposedChar["G", Null, "0"], "Z"},
    PropagatorType -> {ScalarDash, Sine},
    PropagatorArrow -> None },
    (* mixing Higgs gauge bosons: charged *)
SV[3] == {
    SelfConjugate -> False,
    Indices -> {},
    Mass -> MW,
    QuantumNumbers -> -Charge,
    MixingPartners -> {S[3], V[3]},
    PropagatorLabel -> {"G", "W"},
    PropagatorType -> {ScalarDash, Sine},
    PropagatorArrow -> Forward },
    (* physical Higgs: Q = 0 *)
```

```
S[1] == {
    SelfConjugate -> True,
    Indices -> {},
    Mass -> MH,
    PropagatorLabel -> "H",
    PropagatorType -> ScalarDash,
    PropagatorArrow -> None },
    (* unphysical Higgs: neutral *)
S[2] == {
    SelfConjugate -> True,
    Indices -> {},
    Mass -> MZ,
    PropagatorLabel -> ComposedChar["G", Null, "0"],
    PropagatorType -> ScalarDash,
    PropagatorArrow -> None },
    (* unphysical Higgs: Q = -1 *)
S[3] == {
    SelfConjugate -> False,
    Indices -> {},
    Mass -> MW,
    QuantumNumbers -> -Charge,
    PropagatorLabel -> "G",
```

```
    PropagatorType -> ScalarDash,
    PropagatorArrow -> Forward },
    (* Ghosts: neutral *)
U[1] == {
    SelfConjugate -> False,
    Indices -> {},
    Mass -> 0,
    QuantumNumbers -> GhostNumber,
    PropagatorLabel -> ComposedChar["u", "\\gamma"],
    PropagatorType -> GhostDash,
    PropagatorArrow -> Forward },
U[2] == {
    SelfConjugate -> False,
    Indices -> {},
    Mass -> MZ,
    QuantumNumbers -> GhostNumber,
    PropagatorLabel -> ComposedChar["u", "Z"],
    PropagatorType -> GhostDash,
    PropagatorArrow -> Forward },
    (* Ghosts: charged *)
U[3] == {
    SelfConjugate -> False,
```

```
    Indices -> {},
    Mass -> MW,
    QuantumNumbers -> {-Charge, GhostNumber},
    PropagatorLabel -> ComposedChar["u", "-"],
    PropagatorType -> GhostDash,
    PropagatorArrow -> Forward },
U[4] == {
    SelfConjugate -> False,
    Indices -> {},
    Mass -> MW,
    QuantumNumbers -> {Charge, GhostNumber},
    PropagatorLabel -> ComposedChar["u", "+"],
    PropagatorType -> GhostDash,
    PropagatorArrow -> Forward }
}
MLE[1] = ME;
MLE[2] = MM;
MLE[3] = ML;
MQU[1] = MU;
MQU[2] = MC;
MQU[3] = MT;
MQD[1] = MD;
```

```
MQD[2] = MS;

MQD[3] = MB;

MQU[gen_, _] := MQU[gen];

MQD[gen_, _] := MQD[gen]

TheLabel[ F[1, {1}] ] = ComposedChar["\\nu", "e"];

TheLabel[ F[1, {2}] ] = ComposedChar["\\nu", "\\mu"];

TheLabel[ F[1, {3}] ] = ComposedChar["\\nu", "\\tau"];

TheLabel[ F[2, {1}] ] = "e";

TheLabel[ F[2, {2}] ] = "\\mu";

TheLabel[ F[2, {3}] ] = "\\tau";

TheLabel[ F[3, {1, ___}] ] = "u";

TheLabel[ F[3, {2, ___}] ] = "c";

TheLabel[ F[3, {3, ___}] ] = "t";

TheLabel[ F[4, {1, ___}] ] = "d";

TheLabel[ F[4, {2, ___}] ] = "s";

TheLabel[ F[4, {3, ___}] ] = "b"

GaugeXi[ V[1] ] = GaugeXi[A];

GaugeXi[ V[2] ] = GaugeXi[Z];

GaugeXi[ V[3] ] = GaugeXi[W];

GaugeXi[ S[1] ] = 1;

GaugeXi[ S[2] ] = GaugeXi[Z];

GaugeXi[ S[3] ] = GaugeXi[W];
```

```
GaugeXi[ U[1] ] = GaugeXi[A];

GaugeXi[ U[2] ] = GaugeXi[Z];

GaugeXi[ U[3] ] = GaugeXi[W];

GaugeXi[ U[4] ] = GaugeXi[W]

M$CouplingMatrices = {

    (* F-F-V: *)

C[ -F[1, {j1}], F[1, {j2}], V[1] ] == I EL FermionCharge[1] IndexDelta[j1, j2] *

{ {1},

{1},

{(I/4)*(1/LambdaNC^2)},

{(I/4)*(1/LambdaNC^2)},

{0},

{0},

{0},

{0},

{(-I/4)*(1/LambdaNC^2) Mass[F[1, {j1}]]},

{(-I/4)*(1/LambdaNC^2) Mass[F[1, {j1}]]} },

C[ -F[2, {j1}], F[2, {j2}], V[1] ] == I EL FermionCharge[2] IndexDelta[j1, j2] *

{ {1},

{1},

{(I/4)*(1/LambdaNC^2)},

{(I/4)*(1/LambdaNC^2)},
```

```
{0},
{0},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]]},
{(-I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]]} },
C[ -F[3, {j1, o1}], F[3, {j2, o2}], V[1] ] == I EL FermionCharge[3] IndexDelta[j1, j2] IndexDelta[o1, o2] *
{ {1},
{1},
{(I/4)*(1/LambdaNC^2)},
{(I/4)*(1/LambdaNC^2)},
{0},
{0},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]]},
{(-I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]]} },
C[ -F[4, {j1, o1}], F[4, {j2, o2}], V[1] ] == I EL FermionCharge[4] IndexDelta[j1, j2] IndexDelta[o1, o2] *
{ {1},
{1},
```

```
{(I/4)*(1/LambdaNC^2)},
{(I/4)*(1/LambdaNC^2)},
{0},
{0},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]]},
{(-I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]]} },
C[ -F[1, {j1}], F[1, {j2}], V[2] ] == I EL IndexDelta[j1, j2] *
{ {gL[1]},
{0},
{(I/4)*(1/LambdaNC^2) gL[1]},
{0},
{(I/4)*(1/LambdaNC^2) Mass[F[1, {j1}]] gL[1]},
{0},
{0},
{-(I/4)*(1/LambdaNC^2) Mass[F[1, {j1}]] gL[1]},
{0},
{0} },
C[ -F[2, {j1}], F[2, {j2}], V[2] ] == I EL IndexDelta[j1, j2] *
{ {gL[2]},
{gR[2]},
```

{(I/4)*(1/LambdaNC^2) gL[2]},

{(I/4)*(1/LambdaNC^2) gR[2]},

{(I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]] gL[2]},

{(I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]] gR[2]},

{(-I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]] gR[2]},

{(-I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]] gL[2]},

{0},

{0} },

C[-F[3, {j1, o1}], F[3, {j2, o2}], V[2]] == I EL IndexDelta[j1, j2] IndexDelta[o1, o2] *

{ {gL[3]},

{gR[3]},

{(I/4)*(1/LambdaNC^2) gL[3]},

{(I/4)*(1/LambdaNC^2) gR[3]},

{(I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]] gL[3]},

{(I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]] gR[3]},

{(-I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]] gR[2]},

{(-I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]] gL[3]},

{0},

{0} },

C[-F[4, {j1, o1}], F[4, {j2, o2}], V[2]] == I EL IndexDelta[j1, j2] IndexDelta[o1, o2] *

```
{ {gL[4]},
{gR[4]},
{(I/4)*(1/LambdaNC^2) gL[4]},
{(I/4)*(1/LambdaNC^2) gR[4]},
{(I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]] gL[4]},
{(I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]] gR[4]},
{(-I/4)*(1/LambdaNC^2) Mass[F[4 {j1}]] gR[4]},
{(-I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]] gL[4]},
{0},
{0} },
C[ -F[1, {j1}], F[2, {j2}], -V[3] ] ==
I EL/(Sqrt[2] SW) IndexDelta[j1, j2] *
{ {1},
{0},
{(I/4)*(1/LambdaNC^2)},
{0},
{(I/4)*(1/LambdaNC^2) Mass[F[1, {j1}]]},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]]},
{0},
{0} },
```

```
C[ -F[2, {j1}], F[1, {j2}], V[3] ] ==
I EL/(Sqrt[2] SW) IndexDelta[j1, j2] *
{ {1},
{0},
{(I/4)*(1/LambdaNC^2)},
{0},
{(I/4)*(1/LambdaNC^2) Mass[F[2, {j1}]]},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[1, {j1}]]},
{0},
{0} },
C[ -F[3, {j1, o1}], F[4, {j2, o2}], -V[3] ] ==
I EL/(Sqrt[2] SW) CKM[j1, j2] IndexDelta[o1, o2] *
{ {1},
{0},
{(I/4)*(1/LambdaNC^2)},
{0},
{(I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]]},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]]},
```

```
{0},
{0} },
C[ -F[4, {j2, o2}], F[3, {j1, o1}], V[3] ] ==
I EL/(Sqrt[2] SW) Conjugate[CKM[j1, j2]] IndexDelta[o1, o2] *
{ {1},
{0},
{(I/4)*(1/LambdaNC^2)},
{0},
{(I/4)*(1/LambdaNC^2) Mass[F[4, {j1}]]},
{0},
{0},
{(-I/4)*(1/LambdaNC^2) Mass[F[3, {j1}]]},
{0},
{0} },
(* F-F-V-V: *)
C[ -F[1, {j1}], F[1, {j2}], V[1], V[1] ] == -(1/2) EL^2 *(1/LambdaNC^2)
{ {0},
{0},
{0},
{0},
{0},
{0} },
```

C[-F[2, {j1}], F[2, {j2}], V[1], V[1]] == -(1/2) EL^2 *(1/LambdaNC^2) FermionCharge[2]^2 IndexDelta[j1, j2]

{ {1},

{1},

{-1},

{-1},

{0},

{0} },

C[-F[3, {j1, o1}], F[3, {j2, o2}], V[1], V[1]] == -(1/2)* EL^2 *(1/LambdaNC^2) FermionCharge[3]^2 IndexDelta[j1, j2] IndexDelta[o1, o2] *

{ {1},

{1},

{-1},

{-1},

{0},

{0} },

C[-F[4, {j1, o1}], F[4, {j2, o2}], V[1], V[1]] == -(1/2)* EL^2 *(1/LambdaNC^2) FermionCharge[4]^2 IndexDelta[j1, j2] IndexDelta[o1, o2] *

{ {1},

{1},

{-1},

{-1},

{0},

{0} },

C[-F[1, {j1}], F[1, {j2}], V[1], V[2]] == -(1/2)* EL^2 *(1/LambdaNC^2) FermionCharge[1] IndexDelta[j1, j2] *

{ {gL[1]},

{gR[1]},

{-gL[1]},

{-gR[1]},

{Mass[F[1, {j1}]] (gR[1]-gL[1])},

{Mass[F[1, {j1}]] (gL[1]-gR[1])} },

C[-F[2, {j1}], F[2, {j2}], V[1], V[2]] == -(1/2)* EL^2 *(1/LambdaNC^2) FermionCharge[2] IndexDelta[j1, j2] *

{ {gL[2]},

{gR[2]},

{-gL[2]},

{-gR[2]},

{Mass[F[2, {j1}]] (gR[2]-gL[2])},

{Mass[F[2, {j1}]] (gL[2]-gR[2])} },

C[-F[3, {j1, o1}], F[3, {j2, o2}], V[1], V[2]] == -(1/2)* EL^2 *(1/LambdaNC^2) FermionCharge[3] IndexDelta[j1, j2] IndexDelta[o1, o2] *

{ {gL[3]},

{gR[3]},

{-gL[3]},

{-gR[3]},

{Mass[F[3, {j1}]] (gR[3]-gL[3])},

{Mass[F[3, {j1}]] (gL[3]-gR[3])} },

C[-F[4, {j1, o1}], F[4, {j2, o2}], V[1], V[2]] == -(1/2)* EL^2 *(1/LambdaNC^2) FermionCharge[4] IndexDelta[j1, j2] IndexDelta[o1, o2] *

{ {gL[4]},

{gR[4]},

{-gL[4]},

{-gR[4]},

{Mass[F[4, {j1}]] (gR[4]-gL[4])},

{Mass[F[4, {j1}]] (gL[4]-gR[4])} },

C[-F[1, {j1}], F[1, {j2}], V[2], V[2]] == -(1/2)* EL^2 *(1/LambdaNC^2) IndexDelta[j1, j2] *

{ {gL[1]^2},

{gR[1]^2},

{-gL[1]^2},

{-gR[1]^2},

{0},

{0} },

C[-F[2, {j1}], F[2, {j2}], V[2], V[2]] == -(1/2)* EL^2 *(1/LambdaNC^2)

IndexDelta[j1, j2] *

```
{ {gL[2]^2},
{gR[2]^2},
{-gL[2]^2},
{-gR[2]^2},
{0},
{0} },
C[ -F[3, {j1, o1}], F[3, {j2, o2}], V[2], V[2] ] == -(1/2)* EL^2 *(1/LambdaNC^2)
IndexDelta[j1, j2] IndexDelta[o1, o2] *
{ {gL[3]^2},
{gR[3]^2},
{-gL[3]^2},
{-gR[3]^2},
{0},
{0} },
C[ -F[4, {j1, o1}], F[4, {j2, o2}], V[2], V[2] ] == -(1/2)* EL^2 *(1/LambdaNC^2)
IndexDelta[j1, j2] IndexDelta[o1, o2] *
{ {gL[4]^2},
{gR[4]^2},
{-gL[4]^2},
{-gR[4]^2},
{0},
{0} },
```

```
C[ -F[1, {j1}], F[1, {j2}], -V[3], V[3] ] == EL^2 *(1/LambdaNC^2) *
-1/(4 SW^2) IndexDelta[j1, j2] *
{ {1},
{0},
{0},
{0},
{Mass[F[1, {j1}]]},
{Mass[F[1, {j1}]]} },
C[ -F[2, {j1}], F[2, {j2}], -V[3], V[3] ] == EL^2 *(1/LambdaNC^2) *
-1/(4 SW^2) IndexDelta[j1, j2] *
{ {1},
{0},
{0},
{0},
{Mass[F[2, {j1}]]},
{Mass[F[2, {j1}]]} },
C[ -F[3, {j1, o1}], F[3, {j2, o2}], -V[3], V[3] ] == EL^2 *(1/LambdaNC^2)
*IndexDelta[o1, o2] *
-1/(4 SW^2) IndexDelta[j1, j2] *
{ {1},
{0},
{0},
```

```
{0},
{Mass[F[3, {j1}]]},
{Mass[F[3, {j1}]]} },
C[ -F[4, {j1, o1}], F[4, {j2, o2}], -V[3], V[3] ] == EL^2 *(1/LambdaNC^2)
IndexDelta[o1, o2] *
-1/(4 SW^2) IndexDelta[j1, j2] *
{ {1},
{0},
{0},
{0},
{Mass[F[4, {j1}]]},
{Mass[F[4, {j1}]]} },
C[ -F[1, {j1}], F[2, {j2}],V[1], -V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) IndexDelta[j1, j2] *
{ {FermionCharge[1]},
{0},
{-FermionCharge[2]},
{0},
{FermionCharge[2] Mass[F[1, {j1}]]},
{-FermionCharge[1] Mass[F[2, {j1}]]} },
C[ -F[2, {j1}], F[1, {j2}],V[1], V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) IndexDelta[j1, j2] *
```

```
{ {FermionCharge[2]},
{0},
{-FermionCharge[1]},
{0},
{FermionCharge[1] Mass[F[2, {j1}]]},
{-FermionCharge[2] Mass[F[1, {j1}]]} },
C[ -F[3, {j1, o1}], F[4, {j2, o2}], V[1], -V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) IndexDelta[j1, j2] *
{ {FermionCharge[3]},
{0},
{-FermionCharge[4]},
{0},
{FermionCharge[4] Mass[F[3, {j1}]] CKM[j1, j2]},
{-FermionCharge[3] Mass[F[4, {j1}]] CKM[j1, j2]} },
C[ -F[4, {j2, o2}], F[3, {j1, o1}], V[1], V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) IndexDelta[j1, j2] *
{ {FermionCharge[4]},
{0},
{-FermionCharge[3]},
{0},
{FermionCharge[3] Mass[F[4, {j1}]] Conjugate[CKM[j1, j2]]},
{-FermionCharge[4] Mass[F[3, {j1}]] Conjugate[CKM[j1, j2]]} },
```

```
C[ -F[1, {j1}], F[2, {j2}], V[2], -V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) IndexDelta[j1, j2] *
{ {gL[1]},
{0},
{-gL[2]},
{0},
{Mass[F[1, {j1}]] (2gL[2]-gR[2])},
{Mass[F[2, {j1}]] (2gL[1]-gR[1])} },
C[ -F[2, {j1}], F[1, {j2}],V[2], V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) IndexDelta[j1, j2] *
{ {gL[2]},
{0},
{-gL[1]},
{0},
{Mass[F[2, {j1}]] (2gL[1]-gR[1])},
{Mass[F[1, {j1}]] (2gL[2]-gR[2])} },
C[ -F[3, {j1, o1}], F[4, {j2, o2}], V[2], -V[3] ] == EL^2 *(1/LambdaNC^2)
-1/(2 Sqrt[2] SW) CKM[j1, j2] IndexDelta[j1, j2] *
{ {gL[3]},
{0},
{-gL[4]},
{0},
```

{Mass[F[3, {j1}]] (2gL[4]-gR[4])},

{Mass[F[4, {j1}]] (2gL[3]-gR[3])} },

C[-F[4, {j2, o2}], F[3, {j1, o1}], V[2], V[3]] == EL^2 *(1/LambdaNC^2)

-1/(2 Sqrt[2] SW) Conjugate[CKM[j1, j2]] IndexDelta[j1, j2] *

{ {gL[4]},

{0},

{-gL[3]},

{0},

{Mass[F[4, {j1}]] (2gL[3]-gR[3])},

{Mass[F[3, {j1}]] (2gL[4]-gR[4])} },

(* V-V-V: *)

C[V[1], -V[3], V[3]] == I EL *

{ {1},

{(I/4) MW^2*(1/LambdaNC^2)},

{0},

{0} },

C[V[2], -V[3], V[3]] == I EL CW/SW *

{ {1},

{(-I/4) MW^2*(1/LambdaNC^2)},

{(I/8) MZ^2*(1/LambdaNC^2)},

{(-I/4) MZ^2*(1/LambdaNC^2)} },

C[V[2], V[2], V[2]] == (EL MZ^2)/(4 CW SW) * (1/LambdaNC^2)

```
{ {0},
{0},
{(-1/2)*(1/LambdaNC^2)},
{(1/LambdaNC^2)} },
(* S-S-V: *)
C[ S[1], S[1], V[2] ] == EL /(4 CW) *
{ {(1/LambdaNC^2) MH^2} }
}
M$LastModelRules = {}
(* some short-hands for excluding classes of particles *)
QEDOnly = ExcludeParticles -> {F[1], V[2], V[3], S, SV, U[2], U[3], U[4]}
NoGeneration1 = ExcludeParticles -> F[_, {1, ___}]
NoGeneration2 = ExcludeParticles -> F[_, {2, ___}]
NoGeneration3 = ExcludeParticles -> F[_, {3, ___}]
NoElectronHCoupling =
ExcludeFieldPoints -> {
FieldPoint[_][-F[2, {1}], F[2, {1}], S],
FieldPoint[_][-F[2, {1}], F[1, {1}], S] }
NoLightFHCoupling =
ExcludeFieldPoints -> {
FieldPoint[_][-F[2], F[2], S],
FieldPoint[_][-F[2], F[1], S],
```

```
FieldPoint[_][-F[3, {1, ___}], F[3, {1, ___}], S],

FieldPoint[_][-F[3, {2, ___}], F[3, {2, ___}], S],

FieldPoint[_][-F[4], F[4], S],

FieldPoint[_][-F[4], F[3, {1, ___}], S],

FieldPoint[_][-F[4], F[3, {2, ___}], S] }

NoQuarkMixing =

ExcludeFieldPoints -> {

FieldPoint[_][-F[4, {1, ___}], F[3, {2, ___}], S[3]],

FieldPoint[_][-F[4, {1, ___}], F[3, {2, ___}], V[3]],

FieldPoint[_][-F[4, {1, ___}], F[3, {3, ___}], S[3]],

FieldPoint[_][-F[4, {1, ___}], F[3, {3, ___}], V[3]],

FieldPoint[_][-F[4, {2, ___}], F[3, {1, ___}], S[3]],

FieldPoint[_][-F[4, {2, ___}], F[3, {1, ___}], V[3]],

FieldPoint[_][-F[4, {2, ___}], F[3, {3, ___}], S[3]],

FieldPoint[_][-F[4, {2, ___}], F[3, {3, ___}], V[3]],

FieldPoint[_][-F[4, {3, ___}], F[3, {1, ___}], S[3]],

FieldPoint[_][-F[4, {3, ___}], F[3, {1, ___}], V[3]],

FieldPoint[_][-F[4, {3, ___}], F[3, {2, ___}], S[3]],

FieldPoint[_][-F[4, {3, ___}], F[3, {2, ___}], V[3]] }

(* The following definitions of renormalization constants

are for the on-shell renormalization of the Standard Model in

the scheme of A. Denner, Fortschr. d. Physik, 41 (1993) 4.
```

The renormalization constants are not directly used by

FeynArts, and hence do not restrict the generation of diagrams

and amplitudes in any way. *)

```
Clear[RenConst]

RenConst[ dMf1[type_, j1_] ] := MassRC[F[type, {j1}]]

RenConst[ dZfL1[type_, j1_, j2_] ] :=

FieldRC[F[type, {j1}], F[type, {j2}]][[1]]

RenConst[ dZfR1[type_, j1_, j2_] ] :=

FieldRC[F[type, {j1}], F[type, {j2}]][[2]]

RenConst[ dCKM1[j1_, j2_] ] := 1/4 IndexSum[

(dZfL1[3, j1, gn] - Conjugate[dZfL1[3, gn, j1]]) CKM[gn, j2] -

CKM[j1, gn] (dZfL1[4, gn, j2] - Conjugate[dZfL1[4, j2, gn]]),

{gn, MaxGenerationIndex} ]

RenConst[ dMZsq1 ] := MassRC[V[2]]

RenConst[ dMWsq1 ] := MassRC[V[3]]

RenConst[ dMHsq1 ] := MassRC[S[1]]

RenConst[ dZAA1 ] := FieldRC[V[1]]

RenConst[ dZAZ1 ] := FieldRC[V[1], V[2]]

RenConst[ dZZA1 ] := FieldRC[V[2], V[1]]

RenConst[ dZZZ1 ] := FieldRC[V[2]]

RenConst[ dZG01 ] := FieldRC[S[2]]

RenConst[ dZW1 ] := FieldRC[V[3]]
```

```
RenConst[ dZGp1 ] := FieldRC[S[3]]

RenConst[ dZH1 ] := FieldRC[S[1]]

RenConst[ dTH1 ] := TadpoleRC[S[1]]

RenConst[ dSW1 ] := CW^2/SW/2 (dMZsq1/MZ^2 - dMWsq1/MW^2)

RenConst[ dZe1 ] := -1/2 (dZAA1 + SW/CW dZZA1)
```

- **Generic Model file:** NCSM.gen

```
(* NCSM.gen

        last modified 10 March 11 by Linda GHEGAL

*)

    (* Kinematic indices are ‘transported’ along a propagator line.

    KinematicIndices[X] = {Name} means that the generic field X

    will carry an index Index[Name, i] along the line:

    X[ n, {m..}, p, {Index[Name, i]} -> {Index[Name, i + 1]} ] *)

KinematicIndices[ F ] = {};

KinematicIndices[ V ] = {Lorentz};

KinematicIndices[ S ] = {};

KinematicIndices[ SV ] = {Lorentz};

KinematicIndices[ U ] = {}

$FermionLines = True

P$NonCommuting = F | U

Attributes[ MetricTensor ] = Attributes[ ScalarProduct ] = {Orderless}

ThSlash[mu_, nu_, ro_] :=
```

```
NonCommutative[DiracMatrix[mu], DiracMatrix[nu],DiracMatrix[ro]]
-NonCommutative[DiracMatrix[nu], DiracMatrix[mu], DiracMatrix[ro]]
+NonCommutative[DiracMatrix[nu], DiracMatrix[ro], DiracMatrix[mu]]
-NonCommutative[DiracMatrix[ro], DiracMatrix[nu], DiracMatrix[mu]]
+NonCommutative[DiracMatrix[ro], DiracMatrix[mu], DiracMatrix[nu]]
-NonCommutative[DiracMatrix[mu], DiracMatrix[ro], DiracMatrix[nu]]
FourVector/: -FourVector[ mom_, mu___ ] := FourVector[Expand[-mom], mu]
FourVector[ 0, ___ ] = 0
SpinorType[j_Integer, ___] := MajoranaSpinor /; SelfConjugate[F[j]]
SpinorType[_Integer, __] = DiracSpinor
M$GenericPropagators = {
    (* general fermion propagator: *)
AnalyticalPropagator[External][ s1 F[j1, mom] ] ==
NonCommutative[ SpinorType[j1][-mom, Mass[F[j1]]] ],
    (* Remarks:
    Fermionic propagators have (like all others, too) their
    momentum flowing from left to right. The fermion flow (for
    Dirac fermions: fermion number flow) is from right to left.
    If the fermion inside the propagator has no sign (i.e. fermion
    number flow is opposite to fermion flow or fermion is self
    conjugate) we just use the internal propagator S(-p).
    If the fermion has a sign, we have to use the Feynman rule S(p)
```

```
    according to the Majorana paper. However, this rule is given
    for a momentum flowing against the fermion flow so, again, we
    end up with S(-p). *)
AnalyticalPropagator[Internal][ s1 F[j1, mom] ] ==
NonCommutative[ DiracSlash[-mom] + Mass[F[j1]] ] *
I PropagatorDenominator[mom, Mass[F[j1]]],
    (* general vector boson propagator: *)
AnalyticalPropagator[External][ s1 V[j1, mom, {li2}] ] ==
PolarizationVector[V[j1], mom, li2],
AnalyticalPropagator[Internal][ s1 V[j1, mom, {li1} -> {li2}] ] ==
-I PropagatorDenominator[mom, Mass[V[j1]]] *
(MetricTensor[li1, li2] - (1 - GaugeXi[V[j1]]) *
FourVector[mom, li1] FourVector[mom, li2] *
PropagatorDenominator[mom, Sqrt[GaugeXi[V[j1]]] Mass[V[j1]]]),
    (* general mixing scalar-vector propagator: *)
AnalyticalPropagator[Internal][ s1 SV[j1, mom, {li1} -> {li2}] ] ==
I Mass[SV[j1]] PropagatorDenominator[mom, Mass[SV[j1]]] *
FourVector[mom, If[s1 == 1 || s1 == -2, li1, li2]],
    (* general scalar propagator: *)
AnalyticalPropagator[External][ s1 S[j1, mom] ] == 1,
AnalyticalPropagator[Internal][ s1 S[j1, mom] ] ==
I PropagatorDenominator[mom, Sqrt[GaugeXi[S[j1]]] Mass[S[j1]]],
```

(* general Fadeev-Popov ghost propagator: *)

AnalyticalPropagator[External][s1 U[j1, mom]] == 1,

AnalyticalPropagator[Internal][s1 U[j1, mom]] ==

I PropagatorDenominator[mom, Sqrt[GaugeXi[U[j1]]] Mass[U[j1]]]

}

(* Definition of the generic couplings.

The couplings must be defined as a Dot product of the (generic)

coupling vector G[+/-][field1, field2, ..] and the

kinematical vector Gamma = {Gamma1, Gamma2, ...}.

The kinematical vector must have the following properties:

a) the entries of Gamma must close under permutation of the

fields, i.e. under permutation of the momenta and

kinematical indices. One exception is allowed: if the

elements of Gamma only change their signs under certain

permutations (e.g. Gamma1 = mom1 - mom2), a coupling vector

G[-] can be used.

This leads to the following behaviour during the

construction of the classes couplings: if a permuted

coupling was found and the corresponding permutation doesn't

resolve the coupling vector entry, then the program tries

the negative expression of the corresponding Gamma and

multiplies the coupling with (-1).

b) the entries of the kinematical vector have to be closed
under application of the M$FlippingRules, i.e. fermionic
couplings have to be written such that the flipped couplings
are present in the generic coupling. Again, it is possible
to define flippings that change the sign of Gamma and to
take care for those signs by using a G[-]. *)

```
M$GenericCouplings = {
    (* F-F-V: *)
AnalyticalCoupling[ s1 F[j1, mom1], s2 F[j2, mom2],
s3 V[j3, mom3, {li3}] ] ==
G[-1][s1 F[j1], s2 F[j2], s3 V[j3]] .
{ NonCommutative[DiracMatrix[li3], ChiralityProjector[-1]],
NonCommutative[DiracMatrix[li3], ChiralityProjector[+1]],
NonCommutative[ThSlash[li3, mom3, mom2], ChiralityProjector[-1]],
NonCommutative[ThSlash[li3, mom3, mom2], ChiralityProjector[+1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[mom2], ChiralityProjector[-1]]
-NonCommutative[DiracMatrix[mom2], DiracMatrix[li3], ChiralityProjector[-1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[mom2], ChiralityProjector[+1]]
-NonCommutative[DiracMatrix[mom2], DiracMatrix[li3], ChiralityProjector[+1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[mom1], ChiralityProjector[-1]]
-NonCommutative[DiracMatrix[mom1], DiracMatrix[li3], ChiralityProjector[-1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[mom1], ChiralityProjector[+1]]
```

```
-NonCommutative[DiracMatrix[mom1], DiracMatrix[li3], ChiralityProjector[+1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[mom3], ChiralityProjector[-1]]
-NonCommutative[DiracMatrix[mom3], DiracMatrix[li3], ChiralityProjector[-1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[mom3], ChiralityProjector[+1]]
-NonCommutative[DiracMatrix[mom3], DiracMatrix[li3], ChiralityProjector[+1]]
},
(* F-F-V-V: *)
AnalyticalCoupling[ s1 F[j1, mom1], s2 F[j2, mom2],
s3 V[j3, mom3, {li3}], s4 V[j4, mom4, {li4}] ] ==
G[-1][s1 F[j1], s2 F[j2], s3 V[j3], s4 V[j4]] .
{ NonCommutative[ThSlash[li3, li4, mom2 + mom3], ChiralityProjector[-1]],
NonCommutative[ThSlash[li3, li4, mom2 + mom3], ChiralityProjector[+1]],
NonCommutative[ThSlash[li3, li4, mom2 + mom4], ChiralityProjector[-1]],
NonCommutative[ThSlash[li3, li4, mom2 + mom4], ChiralityProjector[+1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[li4], ChiralityProjector[-1]]
-NonCommutative[DiracMatrix[li4], DiracMatrix[li3], ChiralityProjector[-1]],
NonCommutative[DiracMatrix[li3], DiracMatrix[li4], ChiralityProjector[+1]]
-NonCommutative[DiracMatrix[li4], DiracMatrix[li3], ChiralityProjector[+1]] },
(* V-V-V: *)
AnalyticalCoupling[ s1 V[j1, mom1, {li1}], s2 V[j2, mom2, {li2}],
s3 V[j3, mom3, {li3}] ] ==
G[-1][s1 V[j1], s2 V[j2], s3 V[j3]] .
```

{ MetricTensor[li1, li2] FourVector[mom1 - mom2, li3] +

MetricTensor[li2, li3] FourVector[mom2 - mom3, li1] +

MetricTensor[li3, li1] FourVector[mom3 - mom1, li2],

NonCommutative[DiracMatrix[li1], DiracMatrix[li2], FourVector[mom1, li3]]

-NonCommutative[DiracMatrix[li2], DiracMatrix[li1], FourVector[mom1, li3]]

+NonCommutative[DiracMatrix[li1], DiracMatrix[li3], FourVector[mom1, li2]]

-NonCommutative[DiracMatrix[li3], DiracMatrix[li1], FourVector[mom1, li2]]

+MetricTensor[li1, li2] NonCommutative[DiracMatrix[li3], DiracMatrix[mom1]]

-MetricTensor[li1, li2] NonCommutative[DiracMatrix[mom1], DiracMatrix[li3]]

-MetricTensor[li2, li3] NonCommutative[DiracMatrix[li1], DiracMatrix[mom1]]

+MetricTensor[li2, li3] NonCommutative[DiracMatrix[mom1], DiracMatrix[li1]]

+MetricTensor[li3, li1] NonCommutative[DiracMatrix[li2], DiracMatrix[mom1]]

-MetricTensor[li3, li1] NonCommutative[DiracMatrix[mom1], DiracMatrix[li2]],

NonCommutative[DiracMatrix[li1], DiracMatrix[li2], FourVector[mom1 - mom2, li3]]

-NonCommutative[DiracMatrix[li2], DiracMatrix[li1], FourVector[mom1 - mom2, li3]] +NonCommutative[DiracMatrix[li2], DiracMatrix[li3], FourVector[mom2 - mom3, li1]]

-NonCommutative[DiracMatrix[li3], DiracMatrix[li2], FourVector[mom2 - mom3, li1]] +NonCommutative[DiracMatrix[li3], DiracMatrix[li1], FourVector[mom3 - mom1, li2]]

-NonCommutative[DiracMatrix[li1], DiracMatrix[li3], FourVector[mom3 - mom1, li2]],

MetricTensor[li1, li2] NonCommutative[DiracMatrix[li3], DiracMatrix[mom3]]

```
- MetricTensor[li1, li2] NonCommutative[DiracMatrix[mom3], DiracMatrix[li3]]+
MetricTensor[li2, li3] NonCommutative[DiracMatrix[li1], DiracMatrix[mom1]]
- MetricTensor[li2, li3] NonCommutative[DiracMatrix[mom1], DiracMatrix[li1]]+
MetricTensor[li3, li1] NonCommutative[DiracMatrix[li2], DiracMatrix[mom2]]
- MetricTensor[li3, li1] NonCommutative[DiracMatrix[mom2], DiracMatrix[li2]]},
(* S-S-V: *)
AnalyticalCoupling[ s1 S[j1, mom1], s2 S[j2, mom2],
s3 V[j3, mom3, {li3}] ] ==
G[-1][s1 S[j1], s2 S[j2], s3 V[j3]] .
{ DiracMatrix[mom1 - mom2] DiracMatrix[li3]
-DiracMatrix[li3] DiracMatrix[mom1 - mom2]}}
    (* FlippingRules: the flipping rules determines how Dirac
    objects change when the order of fermion fields in the
    coupling is reversed. In other words, it defines how the
    coupling C[F, -F, ...] is derived from C[-F, F, ...].
    Of the elements of the Dirac algebra we need to consider
    only gamma_mu omega_pm since the others are either
    unchanged or not used (sigma_{mu,nu}).
    See Denner, Eck, Hahn, Kueblbeck, NPB 387 (1992) 467. *)
M$FlippingRules =
NonCommutative[dm:_DiracMatrix | _DiracSlash, ChiralityProjector[pm_]] ->
-NonCommutative[dm, ChiralityProjector[-pm]]
```

```
    (* TruncationRules: rule for omitting the wave functions of
    external Propagators defined in this file. *)
M$TruncationRules = {
_PolarizationVector -> 1,
_DiracSpinor -> 1,
_MajoranaSpinor -> 1
}
    (* LastGenericRules: the very last rules that are applied to an
    amplitude before it is returned by CreateFeynAmp. *)
M$LastGenericRules = {
PolarizationVector[p_, _. mom:FourMomentum[Outgoing, _], li_] :>
Conjugate[PolarizationVector][p, mom, li]
}
    (* cosmetics: *)
    (* left spinor in chain + mom incoming -> \bar v
    left spinor in chain + mom outgoing -> \bar u
    right spinor in chain + mom incoming -> u
    right spinor in chain + mom outgoing -> v *)
Format[ ThSlash ] = "Th"
Format[
FermionChain[
NonCommutative[_[s1_. mom1_, mass1_]],
```

```
r___,
NonCommutative[_[s2_. mom2_, mass2_]]] ] :=
Overscript[If[FreeQ[mom1, Incoming], "u", "v"], "_"][mom1, mass1] .
r . If[FreeQ[mom2, Outgoing], "u", "v"][mom2, mass2]
Format[ DiracSlash ] = "gs"
Format[ DiracMatrix ] = "ga"
Format[ ChiralityProjector[1] ] = SequenceForm["om", Subscript["+"]]
Format[ ChiralityProjector[-1] ] = SequenceForm["om", Subscript["-"]]
Format[ GaugeXi[a_] ] := SequenceForm["xi", Subscript[a]]
Format[ PolarizationVector ] = "ep"
Unprotect[Conjugate];
Format[ Conjugate[a_] ] = SequenceForm[a, Superscript["*"]];
Protect[Conjugate]
Format[ MetricTensor ] = "g"
Format[ ScalarProduct[a__] ] := Dot[a]
Format[ FourVector[a_, b_] ] := a[b]
Format[ FourVector[a_] ] := a
```

A.2 FormCalc Drivers for the NCSM

After the diagram generation with the new FeynArts model file NCSM.mod, FormCalc calculates the squared matrix elements with the help of Form [23] and the resulting expressions are translated into Fortran for the further numerical evaluation. For consistency, the Fortran drivers necessary for the initialization of the NCSM parameters. The scale of the noncommutativity Λ_{NC} of the NCSM is initiated into two specific files NCSM.F and NCSM.h as real parameter. In the following part we are going to present the corresponding FormCalc Fortran drivers.

- **NCSM.F**

```
* xsection.F
* routines to compute the cross-section
* this file is part of FormCalc
* last modified 3 Mar 11 th
#include "decl.h"
#include "process.h"
#include MODEL
#ifdef BREMSSTRAHLUNG
#include "softphoton.F"
#endif
************************************************************************
** ProcessIni translates the polarization string into bit-encoded
** helicities, determines the averaging factor, and initializes the
** model defaults.
    subroutine ProcessIni(fail, pol,
& sqrtSfrom, sqrtSto, sqrtSstep)
    implicit none
```

```
      integer fail
      character*(*) pol
      double precision sqrtSfrom, sqrtSto, sqrtSstep
#include "xsection.h"
#if U77EXT
      integer lnblnk
      external lnblnk
#endif
      integer i, c, bits, df
#define SCALAR Z'001'
#define FERMION Z'10A'
#define PHOTON Z'20A'
#define GLUON PHOTON
#define VECTOR Z'20E'
#define GRAVITINO Z'31B'
#define GRAVITON Z'41B'
#define TENSOR Z'41F'
      integer type(LEGS)
      data type /TYPES/
      if( lnblnk(pol) .ne. LEGS ) then
      fail = 1
      return
```

```
endif
df = 2**(LEGS)
do i = 1, LEGS
bits = iand(type(i), 255)
c = ior(ichar(pol(i:i)), 32)
if( c .eq. ichar('t') ) then
c = ior(c, ibits(bits, 3, 1))
bits = iand(bits, 16+8+2+1)
else if( c .eq. ichar('p') ) then
bits = iand(bits, 16)
else if( c .eq. ichar('+') ) then
bits = iand(bits, 8)
else if( c .eq. ichar('l') ) then
bits = iand(bits, 4)
else if( c .eq. ichar('-') ) then
bits = iand(bits, 2)
else if( c .eq. ichar('m') ) then
bits = iand(bits, 1)
else if( c .ne. ichar('u') ) then
bits = 0
endif
if( bits .eq. 0 ) then
```

```
      Error("Invalid polarization for leg "/ /Digit(i))
      return
      endif
      if( i .le. LEGS_IN ) df = df*(ibits(bits, 4, 1) +
& ibits(bits, 3, 1) + ibits(bits, 2, 1) +
& ibits(bits, 1, 1) + ibits(bits, 0, 1))
#ifdef DIRACFERMIONS
      if( type(i) .eq. FERMION ) then
      df = df/2
      bits = 1
      endif
#endif
      pol(i:i) = char(c)
      helicities = helicities*32 + bits
      enddo
      Lower(SQRTS) = sqrtSfrom
      Upper(SQRTS) = sqrtSto
      Step(SQRTS) = sqrtSstep
      Var(FIXED) = 0
      Step(FIXED) = 1
      Var(TRIVIAL) = 0
      Step(TRIVIAL) = 0
```

```
      avgfac = 2**(LEGS)*DBLE(COLOURFACTOR)
*DBLE(IDENTICALFACTOR)/df
      sqrtS = -1
      threshold = -1
      scale = -1
      sqrtSinvalid = 1
      call ltini
#ifdef SAMURAI
* args are:
* 1. imeth = "diag" (numerators) or "tree" (products of tree amps)
* 2. isca = 1 (QCDloop) or 2 (OneLOop)
* 3. verbosity = 0, 1, 2, 3
* 4. itest = 0 (none), 1 (powertest), 2 (nntest), 3 (lnntest)
      call initsamurai("diag", 2, 1, 1)
#endif
      call ModelDefaults
      call LumiDefaults
      fail = 0
      end
************************************************************************
** ProcessExi wraps up the calculation, e.g. prints a summary of
** messages, deallocates arrays etc.
```

```
      subroutine ProcessExi
      implicit none
      call ltexi
      end
************************************************************************
** SetEnergy sets the energy for the partonic scattering process.
** All scale-dependent quantities are initialized at SCALE.
      subroutine SetEnergy(fail, newsqrtS)
      implicit none
      integer fail
      double precision newsqrtS
#include "xsection.h"
      double precision oldscale, oldmass_in, oldmass_out
      logical reset
      integer i
      fail = 0
      reset = abs(newsqrtS - sqrtS) .gt. 1D-9
      sqrtS = newsqrtS
      oldscale = max(DBLE(SCALE), 1D0)
      if( sqrtSinvalid .eq. 1 .or.
& btest(flags, BIT_RESET) .or.
& abs(oldscale - scale) .gt. 1D-9 ) then
```

```
      oldmass_in = MASS_IN
      oldmass_out = MASS_OUT
      do i = 1, 10
      call ModelVarIni(fail, oldscale)
      scale = max(DBLE(SCALE), 1D0)
      mass_in = MASS_IN
      mass_out = MASS_OUT
      if( abs(scale - oldscale) +
& abs(mass_in - oldmass_in) +
& abs(mass_out - oldmass_out) .lt. 1D-9 ) goto 1
      oldscale = scale
      oldmass_in = mass_in
      oldmass_out = mass_out
      enddo
1       threshold = max(mass_in, mass_out)
      sqrtSinvalid = fail
      reset = .TRUE.
      endif
      if( reset ) flags = ibset(flags, BIT_RESET)
      if( fail .ne. 0 .or. sqrtS .lt. threshold ) then
      fail = 1
      else
```

```
      call InitialState
      endif
      end
************************************************************************
** ParameterScan performs the actual calculation.
** It scans over the parameter loops LOOP* declared in run.F.
      subroutine ParameterScan(dir,
& serialfrom, serialto, serialstep)
      implicit none
      character*(*) dir
      integer serialfrom, serialto, serialstep
#include "xsection.h"
      integer openlog
      external openlog
      integer serial, next
      next = serialfrom
      serial = 0
#ifdef LOOP1
      LOOP1
#endif
#ifdef LOOP2
      LOOP2
```

```
#endif
#ifdef LOOP3
    LOOP3
#endif
#ifdef LOOP4
    LOOP4
#endif
#ifdef LOOP5
    LOOP5
#endif
#ifdef LOOP6
    LOOP6
#endif
#ifdef LOOP7
    LOOP7
#endif
#ifdef LOOP8
    LOOP8
#endif
#ifdef LOOP9
    LOOP9
#endif
```

```
#ifdef LOOP10
    LOOP10
#endif
#ifdef LOOP11
    LOOP11
#endif
#ifdef LOOP12
    LOOP12
#endif
#ifdef LOOP13
    LOOP13
#endif
#ifdef LOOP14
    LOOP14
#endif
#ifdef LOOP15
    LOOP15
#endif
#ifdef LOOP16
    LOOP16
#endif
#ifdef LOOP17
```

```
      LOOP17
#endif
#ifdef LOOP18
      LOOP18
#endif
#ifdef LOOP19
      LOOP19
#endif
#ifdef LOOP20
      LOOP20
#endif
      serial = serial + 1
      if( serial .lt. next ) goto 1
      call flush(6)
      if( openlog(dir, serial) .eq. 0 ) then
      call IntegratedCS
      call flush(6)
      call closelog
      endif
      next = next + serialstep
      if( next .gt. serialto ) return
1       continue
```

```
      end
************************************************************************
** IntegratedCS computes the integrated cross-section at one point in
** parameter space. The cross-section is differential in all variables
** with non-zero step, i.e. integration variables have step = 0.
      subroutine IntegratedCS
      implicit none
#include "xsection.h"
      integer nvars
      parameter (nvars = MAXVAR - (MINVAR) + 1)
      external DifferentialCS
      double precision result(NCOMP), error(NCOMP), show(nvars)
      integer fail, f, v, nfix, ndim, fix(nvars)
      call ModelConstIni(fail)
      if( fail .ne. 0 ) goto 999
      flags = 2**BIT_RESET + 2**BIT_LOOP
#ifdef MUDIM
      call setmudim(DBLE(MUDIM))
#endif
#ifdef DELTA
      call setdelta(DBLE(DELTA))
#endif
```

```
#ifdef LAMBDA
	call setlambda(DBLE(LAMBDA))
#endif
	Divergence = getdelta()
	mudim = getmudim()
	lambda = getlambda()
	epscoeff = -dim(0, int(lambda))
	call KinIni(fail)
	if( fail .ne. 0 ) goto 999
	nfix = 0
	do v = MINVAR, MAXVAR
	if( Step(v) .ne. 0 ) then
	nfix = nfix + 1
	fix(nfix) = v
	Var(v) = Lower(v)
	endif
	enddo
	ndim = nvars - nfix
	call LumiIni(fail)
	if( fail .ne. 0 ) goto 999
	call ModelDigest
#define SHOW print 100,
```

```
100     format("|# ", A, "=", F10.4, SP, F10.4, " I")
#ifdef MMA
    call MmaSetPara
#endif
#ifdef PRINT1
    PRINT1
#endif
#ifdef PRINT2
    PRINT2
#endif
#ifdef PRINT3
    PRINT3
#endif
#ifdef PRINT4
    PRINT4
#endif
#ifdef PRINT5
    PRINT5
#endif
#ifdef PRINT6
    PRINT6
#endif
```

```
#ifdef PRINT7
    PRINT7
#endif
#ifdef PRINT8
    PRINT8
#endif
#ifdef PRINT9
    PRINT9
#endif
#ifdef PRINT10
    PRINT10
#endif
#ifdef PRINT11
    PRINT11
#endif
#ifdef PRINT12
    PRINT12
#endif
#ifdef PRINT13
    PRINT13
#endif
#ifdef PRINT14
```

```
    PRINT14
#endif
#ifdef PRINT15
    PRINT15
#endif
#ifdef PRINT16
    PRINT16
#endif
#ifdef PRINT17
    PRINT17
#endif
#ifdef PRINT18
    PRINT18
#endif
#ifdef PRINT19
    PRINT19
#endif
#ifdef PRINT20
    PRINT20
#endif
#ifdef MMA
    call MmaEndSet
```

```
      call MmaSetData
#endif
1      call Cuba(ndim, DifferentialCS, result, error)
      do f = 1, nfix
      show(f) = Show(fix(f))
      enddo
#ifdef MMA
      call MmaData(show, nfix, result, error, NCOMP)
#else
* Note: "real" data lines are tagged with "|" in the output.
101      format("| ", 10(4G19.10, :, /"|+"))
      print 101, (show(f), f = 1, nfix)
102      format("|+ ", NCOMP G24.15)
      print 102, LambdaNC, result(1)
      print 102, error(1)
      call flush(6)
#endif
      do f = nfix, 1, -1
      v = fix(f)
      Var(v) = Var(v) + Step(v)
      if( (Var(v) - Upper(v))/Step(v) .lt. 1D-10 ) goto 1
      Var(v) = Lower(v)
```

```
      enddo
#ifdef MMA
      call MmaEndSet
#else
103   format("|"/"|"/)
      print 103
#endif
999   continue
      end
*************************************************************************
** DifferentialCS computes the differential cross-section at x.
** For all integration variables (those with zero step) it factors in
** the Jacobian, too.
      subroutine DifferentialCS(ndim, x, ncomp, result)
      implicit none
      integer ndim, ncomp
      double precision x(ndim), result(ncomp)
#include "xsection.h"
#ifdef BREMSSTRAHLUNG
      double precision SoftPhotonFactor
      external SoftPhotonFactor
#endif
```

```
double precision fac, range, flux
integer v, d, c
fac = avgfac
d = 0
do v = MINVAR, MAXVAR
if( Step(v) .eq. 0 ) then
range = Upper(v) - Lower(v)
d = d + 1
Var(v) = Lower(v) + x(d)*range
fac = fac*range
endif
Show(v) = Var(v)
enddo
do c = 1, ncomp
result(c) = 0
enddo
call Luminosity(fac)
if( fac .eq. 0 ) return
call FinalState(fac)
if( fac .eq. 0 ) return
if( btest(flags, BIT_RESET) ) then
call clearcache
```

```
#ifndef NO_RENCONST
      call CalcRenConst
#endif
      endif
      call SquaredME(result, helicities, flags)
      flags = ibclr(flags, BIT_RESET)
#if LEGS_IN == 1
      flux = 2*sqrtS
#else
      flux = 4/hbar_c2*sqrtS*momspec(SPEC_K, 1)
#endif
      fac = fac/((2*pi)**(3*LEGS_OUT - 4)*2*sqrtS*flux)
      do c = 1, ncomp
      if( .not. abs(result(c)) .lt. 1D16 ) then
      Warning("Got strange values from SquaredME:")
      INFO result
      INFO "(Did you compute the colour matrix elements?)"
      stop
      endif
      result(c) = result(c)*fac
      enddo
#ifdef BREMSSTRAHLUNG
```

```
      result(2) = result(2) + SoftPhotonFactor()*result(1)
#endif
#ifdef WF_RENORMALIZATION
      result(2) = result(2) + (WF_RENORMALIZATION)*result(1)
#endif
      end
************************************************************************
** Cuba is a chooser for the Cuba routines, with special cases
** for ndim = 0 (integrand evaluation) and ndim = 1 (Patterson
** integration).
      subroutine Cuba(ndim, integrand, result, error)
      implicit none
      integer ndim
      external integrand
      double precision result(NCOMP), error(NCOMP)
      integer nregions, neval, fail, c
      double precision prob(NCOMP)
#include "xsection.h"
#define GAUSS 1
#define PATTERSON 2
#define VEGAS 3
#define SUAVE 4
```

```
#define DIVONNE 5
#define CUHRE 6
#define FLAGS_LAST 4
#define FLAGS_PSEUDO 8
      if( ndim .eq. 0 ) then
      call integrand(0, 0D0, NCOMP, result)
      do c = 1, NCOMP
      error(c) = 0
      enddo
      return
      else if( ndim .eq. 1 ) then
#if METHOD == GAUSS
      neval = 32
      call Gauss(NCOMP, 0D0, 1D0, integrand,
& neval, result)
      do c = 1, NCOMP
      error(c) = -1
      prob(c) = -1
      enddo
      nregions = 1
      fail = 0
      INFO "Gauss integration results:"
```

```
#else
      call Patterson(NCOMP, 0D0, 1D0, integrand,
& DBLE(RELACCURACY), DBLE(ABSACCURACY),
& neval, fail, result, error)
      do c = 1, NCOMP
      prob(c) = -1
      enddo
      nregions = 1
      INFO "Patterson integration results:"
#endif
      else
#if METHOD == VEGAS
      call vegas(ndim, NCOMP, integrand, USERDATA,
& DBLE(RELACCURACY), DBLE(ABSACCURACY),
& VERBOSE, SEED, MINEVAL, MAXEVAL,
& NSTART, NINCREASE, NBATCH,
& GRIDNO, STATEFILE,
& neval, fail, result, error, prob)
      nregions = 1
      INFO "Vegas integration results:"
#elif METHOD == SUAVE
      call suave(ndim, NCOMP, integrand, USERDATA,
```

```
& DBLE(RELACCURACY), DBLE(ABSACCURACY),
& VERBOSE + FLAGS_LAST, SEED, MINEVAL, MAXEVAL,
& NNEW, DBLE(FLATNESS),
& nregions, neval, fail, result, error, prob)
    INFO "Suave integration results:"
#elif METHOD == DIVONNE
    call divonne(ndim, NCOMP, integrand, USERDATA,
& DBLE(RELACCURACY), DBLE(ABSACCURACY),
& VERBOSE, SEED, MINEVAL, MAXEVAL,
& KEY1, KEY2, KEY3, MAXPASS,
& DBLE(BORDER), DBLE(MAXCHISQ), DBLE(MINDEVIATION),
& 0, NDIM, 0, 0, 0,
& nregions, neval, fail, result, error, prob)
    INFO "Divonne integration results:"
#else
    call cuhre(ndim, NCOMP, integrand, USERDATA,
& DBLE(RELACCURACY), DBLE(ABSACCURACY),
& VERBOSE + FLAGS_LAST, MINEVAL, MAXEVAL,
& KEY,
& nregions, neval, fail, result, error, prob)
    INFO "Cuhre integration results:"
#endif
```

```
      endif
      INFO "nregions =", nregions
      INFO "neval =", neval
      INFO "fail =", fail
      if( fail .gt. 0 ) then
      Warning("Failed to reach the desired accuracy")
      else if( fail .lt. 0 ) then
      Error("Integration error")
      endif
100   format(I2, G24.15, " +- ", G24.15, " p = ", F6.3)
      print 100, (c, result(c), error(c), prob(c), c = 1, NCOMP)
      end
```

- **NCSM.h**

```
* process.h
* defines all process-dependent parameters
* this file is part of FormCalc
* last modified 12 May 09 th
* Definition of the external particles.
* Each TYPEn is one of SCALAR, FERMION, PHOTON (= GLUON), or VEC-
TOR.
* (PHOTON/GLUON is equivalent to VECTOR, except that longitudinal
* modes are not allowed)
```

```
* Note: The initial definitions for particles 2...5 are of course
* sample entries for demonstration purposes.
#define TYPE1 FERMION
#define MASS1 ME
#define CHARGE1 1
#define TYPE2 FERMION
#define MASS2 ME
#define CHARGE2 -1
#define TYPE3 FERMION
#define MASS3 MT
#define CHARGE3 2/3D0
#define TYPE4 FERMION
#define MASS4 MT
#define CHARGE4 -2/3D0
* When using Dirac fermions (FermionChains -> Chiral|VA) and
* the trace technique (HelicityME), the following flag should be
* defined to compute unpolarized cross-sections efficiently,
* i.e. without actually summing up the different helicities.
* This has no effect on the result, only on the speed of the
* calculation.
* Note: DIRACFERMIONS must NOT be defined when using Weyl fermions,
* i.e. FermionChains -> Weyl in CalcFeynAmp.
```

```
c#define DIRACFERMIONS

* The combinatorial factor for identical particles in the final state:

* 1/n! for n identical particles, 1 otherwise

#define IDENTICALFACTOR 1

* Possibly a colour factor, e.g.

* - an additional averaging factor if any of the incoming particles

* carry colour,

* - the overall colour factor resulting from the external particles

* if that cannot computed by FormCalc (e.g. if the model has no

* colour indices, as SMew.mod).

#define COLOURFACTOR 1

* The scale at which the interaction takes place

* (= the factorization scale for an hadronic process).

#define SCALE sqrtS

* Whether to include soft-photon bremsstrahlung.

* ESOFTMAX is the maximum energy a soft photon may have and may be

* defined in terms of sqrtS, the CMS energy.

c#define BREMSSTRAHLUNG

#define ESOFTMAX .1D0*sqrtS

* Possibly some wave-function renormalization

* (e.g. if calculating in the background-field method)

c#define WF_RENORMALIZATION (nW*dWFW1 + nZ*dWFZ1)
```

```
* NCOMP is the number of components of the result vector. Currently
* the components are 1 = tree-level result, 2 = one-loop result.
#define NCOMP 2
* Choose the appropriate luminosity for the collider:
* - lumi_parton.F for a "parton collider" (e.g. e+ e- -> X),
* - lumi_hadron.F for a hadron collider (e.g. p pbar -> X),
* - lumi_photon.F for a photon collider (gamma gamma -> X)
#define LUMI "lumi_parton.F"
* for lumi_parton.F: whether to force the decaying particle to
* be on-shell, independent of the command-line choices for sqrtS;
* the value specifies the maximum value of |sqrtS - sum_masses_in|
c#define FORCE_ONSHELL 1D-9
* for lumi_hadron.F: PARTON1 and PARTON2 identify the
* incoming partons by their PDG code, where
* 0 = gluon
* 1 = down 3 = strange 5 = bottom
* 2 = up 4 = charm 6 = top
#define PARTON1 1
#define PARTON2 1
#define PDFSET "cteq5l.LHgrid"
#define PDFMEM 0
* Include the kinematics-dependent part.
```

```
#include "2to2.F"
```

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دراسة بعض النماذج الفيزيائية في إطار الهندسة اللاتبديلية

ملخص:

ننافش الحدود على سلم طاقة اللاتبديلية Λ_{NC} و ذلك بدراسة تكوين كوارك العلوي كوارك علوي مضاد الناتج من اصطدام الكترون ببوزيترون و هذا في اطار النموذج المعياري الأصغري في زمكان لا تبديلي ، و هذا باستعمال خريطة سيبارق-ويتن (SW) في الدرجة الأولى بالنسبة لمعامل اللاتبديلية $\Theta_{\mu\nu}$.

في هذا البحث استعملنا اختيار مناسب لمعامل اللاتبديلية $\Theta_{\mu\nu}$ و وجدنا حد جديد لسلم طاقة اللاتبديلية Λ_{NC} في المجال من 0.1 إلى 0.2 **TeV**.

تتوافق نتائجنا مع تلك المتحصل عليها من تشكل زوج من الميونات.

الكلمات المفتاحية:

زمكان لا تبديلي، التموذج المعياري اللاتبديلي، المقطع الفعّال.

Etude de Quelques Modèles Physiques dans le Cadre de la Géométrie Non Commutative

Résumé:

On discute les limites sur l'échelle du paramètre de la non commutativité Λ_{NC} en étudiant le processus de production d'une paire quark top-antiquark top à partir de la collision d'un électron avec un positron dans le cadre du modèle standard minimal non commutatif (mNCSM), et cela en utilisant la carte de Seiberg-Witten au premier ordre du paramètre de la non commutativité $\Theta_{\mu\nu}$.

Dans ce travail on suppose un ansatz du paramètre $\Theta_{\mu\nu}$ et on trouve une nouvelle limite de l'échelle Λ_{NC} dans le domaine 0.1-0.2 TeV.

Les résultats trouvés coïncident avec ceux obtenus à partir de la production d'une paire de muons.

Mots Clé : Espace-temps non commutatif ; le modèle standard non commutatif ; la section efficace de diffusion.

Abstract:

We discuss the limits on the scale of the noncommutative (NC) parameter Λ_{NC} via studying the top-quark pair production through electron-positron collision in the framework of the minimal noncommutative standard model (mNCSM), using the Seiberg-Witten(SW) maps to the first order of the NC parameter $\Theta_{\mu\nu}$.

In this work we assume an ansatz for the NC parameter $\Theta_{\mu\nu}$ and we find new limit on the NC scale Λ_{NC}, which is in the range 0.1-0.2 TeV. We confirm the results obtained in muon pair production.

Keywords: Noncommutative space-time; noncommutative standard model; scattering cross-section.