الجمهورية الجزائرية الديمقراطية الشعبية
People's Democratic Republic of Algeria
وزارة التعليم العالي والبحث العلمي
Ministry of Higher Education and Scientific Research
جامعة الإخوة منتوري - قسنطينة 1
Frères Mentouri Constantine 1 University
كلية علوم التكنولوجيا
Faculty of Technology Sciences
قسم الإلكترونيك
Department of Electronics

Ordre N^{O}: **39 / DS / 2021**
Series: **02 / ELE / 2021**

A Dissertation

Submitted to the Department of Electronics

in Fulfilment of the Requirements for the Degree of

DOCTOR of SCIENCE

in Electronics

Specialty: Control Systems

TITLE:

Contribution to Robust Fault-Tolerant Predictive Control with Constraints for Hybrid Systems

By

Abdelmalek ZAHAF

Defended on April 11, 2021

<u>Committee Members:</u>

Chairman :	***Abdelatah Charef***	***Prof.***	***Frères Mentouri University- Constantine 1***
Advisor :	***Sofiane Bououden***	***Prof.***	***Abbes Laghrour University, Khenchela***
Co-Advisor :	***Mohamed Chemachema***	***Prof.***	***Frères Mentouri University- Constantine 1***
Examiners :	***Mohamed Lashab***	***Prof.***	***Larbi Ben M'Hidi University, Oum el-Bouaghi***
	Salim Ziani	***M.C.A***	***Frères Mentouri University- Constantine 1***
	Fouâd Allouani	***M.C.A***	***Abbes Laghrour University, Khenchela***

Constantine 2021

No fear to boldly go

Where no man has gone before

Since all hard things must come to an end.

To whom I did not find words of gratitude and thankfulness, my parents

To my Soulmate

To my Sisters

To my Brothers and their sweet families

Abdelmalek

Acknowledgments

At First, I thank God for giving me strength and health to bit the obstacles and to go through the way He planned for me.

*I would first like to thank **Prof. Bououden Sofiane**, my advisor from Abbes Laghrour University of Khenchela. I express my gratefulness to him for his patience, motivation, enthusiasm, and immense knowledge. I worked with him since my Magister study in 2013. Now, almost seven years have passed that we were working together; I could not wish for a better or friendlier advisor, My deep gratefulness to you my friend and brother.*

*Special thanks go to **Prof. Chemachema Mohamed**, my co-advisor at Frères Mentouri Constantine 1 University. I would like to say thank you for your unconditional help, guidance and pieces of advices throughout my thesis. Whatever to say, thank you for attentive readings and your suggestions concerning my dissertation over these years. I had the pleasure to have a friendly advisor like you, My deep gratefulness to you my friend.*

*I offer my sincere gratitude to **Prof. Abdelfatah Charef**, from Frères Mentouri Constantine 1 University, who accepted to be the Chair of my thesis committee. I thank you for your suggestions concerning my dissertation.*

*I am also grateful to **Prof. Mohamed Lashab**, from Larbi Ben M'Hidi University of Oum el-Bouaghi, who accepted to be a member of my thesis committee. I thank you for your feedback concerning my dissertation.*

*I owe special thanks to **Dr. Salim Ziani**, Maître de Conference at Frères Mentouri Constantine 1 University, who accepted to examine my thesis. I thank you for your valuable suggestions concerning my dissertation.*

*I am also thankful to **Dr. Fouâd Allouani**, Maître de Conference at Abbes Laghrour University of Khenchela, who accepted to be a member of my thesis committee. I thank you for your valuable suggestions.*

I am grateful to Laboratory team of Thermodynamics and Surface Treatments of Materials (LTTSM) at Frères Mentouri Constantine 1 University, for their support I have needed to complete my thesis.

Finally, this dissertation is for you my parents, my sisters, my brothers and their sweet families; to you my beloved for your tremendous support, encouragement, tolerance, and …… It is to you that I dedicate this work.

Abstract

In this dissertation, the main objective is to focus on proposing a new reliable control scheme to handle the undesirable associated inputs (faults/time – delay) of HS. In this regard, we raised two aspects to be studied in this dissertation: the hybrid control (HC) and the control of hybrid systems in the presence of faults and time-delay with constraints (HFTPC). The hybrid control design (HFTPC) is based on the interaction of two components: a robust model predictive control (RMPC) to cope with time delay as continuous dynamic and robust fault tolerant predictive control as discrete dynamic.

The aim of this work is designing a robust optimal fault-tolerant predictive control (HFTPC) for a trajectory tracking, applied to a class of non-linear hybrid actuator systems subject to faults and time-delay. In fact, the introduction of time-delay and actuator faults into a hybrid system model results in a dynamic system converted to a strict feedback model. To improve the dynamic performances and decrease the conservatism, a dynamic mechanism of estimation is employed to estimate the actuators faults, in order to compute the optimal solution, while the performance of the hybrid system is preserved. The optimal solution of the HFTPC approach is computed online, by minimizing an upper bound of a specific cost function on infinite horizon, using *min-max* optimization method to derive necessary conditions in terms of LMIs; subject to the imposed constraints, faults and time-delays.

However, an inspiring analysis is provided to improve the dynamics of a hybrid manipulator arm, which can be extended for some classes of hybrid systems, to decrease the computation burden. The state-space model has been reformulated by introducing the output tracking errors, in order to increase the hybrid controller degrees of freedom. Then, an optimal control strategy is designed to operate in the industrial robot arm with the desired position, with a compensation of the loss of efficiency or failure of the actuator in the presence of time-delays. To achieve this optimality, we have used the Lyapunov-Krasovskii function combined with an optimized cost function and observer error, to establish necessary paradigm to obtain a stable and less conservative conditions that is dependent delay-range in terms of LMIs, in order to enhance the feasibility and the stability of the closed loop system. The obtained results are improved and outperformed those obtained using the QP method. In addition, they have been compared with several existing works mentioned in this dissertation.

Key words: *Hybrid Systems (HS), Model predictive control (MPC), Fault Tolerant Control (FTC), Optimal Control, Observers, Linear matrix inequality (LMI), Stability.*

Résumé

Dans cette thèse, la contribution principale est basée sur la conception d'une stratégie de commande fiable dont le but est de traiter et compenser le comportement résultant des entrées associées indésirables "défauts et retards" des systèmes hybride. Par ailleurs, nous avons soulevé deux aspects à étudier : la commande hybride et la commande des systèmes hybrides défectueux avec retard sous contraintes. La conception de la commande hybride (HFTPC) est basée et réalisée à partir d'une interaction de deux approches : un MPC robuste pour compenser le retard en tant que stratégie de commande de la dynamique continu ; et la commande prédictive robuste et tolérante aux défauts en tant qu'une stratégie de commande de la dynamique discrète.

Le but de ce travail est de concevoir une commande robuste prédictive optimale tolérante aux défauts pour le suivi d'une trajectoire (HFTPC), appliquée sur une classe de systèmes d'actionneurs hybrides non linéaires soumis à des défauts et à un retard de temps. En fait, l'introduction du retard et des défauts d'actionneurs dans un modèle de système hybride donne un système dynamique converti en un modèle à rétroaction stricte. Pour améliorer les performances dynamiques et diminuer les conditions de conservatisme, un mécanisme d'estimation dynamique est mise en œuvre pour estimer les défauts des actionneurs, afin de calculer une solution optimale, tandis que les performances du système hybride sont maintenues. La solution optimale de l'approche HFTPC est calculée en ligne, en minimisant une borne supérieure d'une fonction de coût bien définie sur un horizon infini, à l'aide de la méthode d'optimisation *min-max* nous obtenons des conditions nécessaires en terme des LMIs ; qui sont soumises aux contraintes imposées, aux défauts et aux retards.

Cependant, une analyse inspirante est fournie pour améliorer la dynamique d'un bras manipulateur hybride qui peut être étendu pour certaines classes de systèmes hybrides, afin de diminuer la charge de calcul. Le modèle d'espace d'état a été étendu en introduisant l'erreur de suivi des sorties afin d'augmenter encore les degrés de liberté du contrôleur hybride. Ensuite, une commande optimale est conçue afin de faire fonctionner le bras du robot industriel dans la position parfaite et de compenser la perte d'efficacité ou la défaillance de l'actionneur en présence de retards. En établissant une fonction de Lyapunov-Krasovskii, nous obtenons des conditions stables, moins conservatrices et dépendantes de l'intervalle de retard, combinées à une fonction de coût optimisée et à l'erreur de l'observateur, en termes LMIs, dans le but d'améliorer la faisabilité et la stabilité du système en boucle fermée. Les résultats obtenus sont améliorés et ont mieux par rapport à ceux obtenus quand utilise la méthode QP, en plus ils sont comparés avec plusieurs travaux existants qui sont mentionnés dans la thèse.

Mots Clés : *Systèmes Hybride (HS), Commande Prédictive à Modèle (MPC), Commande Tolérante aux Défauts (FTC), La Commande Optimale, Observateurs, Inégalités matricielle linéaire (LMI), Stabilité.*

ملخص

في هذه الأطروحة، ترتكز المساهمة الرئيسية على إقتراح مخطط تحكم جديد فعال وموثوق به للتعامل مع المدخلات المرتبطة غير المرغوب فيها كالأخطاء و التأخيرات الزمنية. لقد أثرنا موضوعين للدراسة و التحسين في هذه الأطروحة: التحكم الهجين والتحكم في الأنظمة الهجينة في ظل وجود أعطال وتأخير زمني مع قيود. يعتمد تصميم التحكم الهجين HFTPCعلى تفاعل مكونين: نموذج تحكم تنبئي قوي للتعامل مع التأخير الزمني باعتباره نمط ديناميكي مستمر ، بالإضافة إلى التحكم التنبئي القوي المتسامح مع الأخطاء على أساس أنه نمط ديناميكي منفصل.

الهدف من هذا العمل هو تصميم تحكم تنبؤي أمثل متسامح مع الخطأ (HFTPC) لتتبع مسار معين ، مطبق على فئة من الأنظمة الهجينة ذات المشغلات الهجينة غير الخطية المعرضة لأعطال المحرك وتأخير الوقت. في الواقع ، يؤدي إدخال أخطاء التأخير والمشغل في نموذج نظام هجين إلى تحويل نظام ديناميكي إلى نموذج ذو ردود فعل صارم. فلتحسين الأداء الديناميكي و الحصول على شروط أقل تحفظا ، يتم تنفيذ آلية تقدير ديناميكي لتقدير أعطال المشغلات ، من أجل تعيين الحل الأمثل لمتغير الحالة ، مع الحفاظ على أداء المطلوب للنظام الهجين. حيث يتم حساب الحل الأمثل لنهج HFTPC أنيا، عن طريق تقليل الحد الأعلى لوظيفة التكلفة المحددة جيدا على أفق غير محدود باستخدام طريقة *min-max* للتحسين، أين نحصل على شروط كافية وضرورية على شكل LMIs ؛ التي تخضع للقيود المفروضة والأخطاء والتأخير الزمني.

لأجل ذلك ، قمنا بإقتراح تحليل ملهم لتحسين ديناميكيات ذراع روبوت ذو الطبيعة الهجينة ، والتي يمكن تمديدها لبعض فئات الأنظمة الهجينة ، وذلك لتقليل عبء الحساب. حيث تمت إعادة صياغة نموذج النظام المدروس من خلال إدخال أخطاء تتبع مخرجات النظام ، من أجل زيادة درجات حرية التحكم الهجين. بعد ذلك ، تم تصميم استراتيجية لحساب التحكم الأمثل لتشغيل ذراع الروبوت الصناعي إلى الموضع المطلوب ، مع تعويض فقدان الكفاءة أو فشل المشغل ، مع وجود تأخير زمني. لتحقيق هذا المبتغى ، نستخدم وظيفة Lyapunov-Krasovskii جنبًا إلى جنب مع وظيفة التكلفة المحددة وكذا الخطأ الموصوف من المراقب ، لإنشاء نموذج ضروري للحصول على شروط ثابتة وأقل تحفظًا تعتمد على مدى التأخير على شكل LMIs ، من أجل تعزيز جدوى واستقرار نظام الحلقة المغلقة. تم تحسين النتائج التي تم الحصول عليها وتفوقت في الأداء مقارنة بتلك التي تم الحصول عليها باستخدام طريقة QP ، بالإضافة إلى مقارنتها بالعديد من الأعمال الذات الصلة المذكورة في هذه الأطروحة.

كلمات مفتاحية : *الأنظمة الهجينة، النموذج التحكم التنبؤي (MPC) التحكم المتسامح مع الأعطال، الحل الأمثل، مصفوفة عدم المساواة الخطية LMI، المراقبين و إستقرار الأنظمة.*

PUBLISHED WORKS

Below is the published work of my Ph.D. studies. Most of these papers contributed to make up this dissertation. The contributions are listed in ascending order of publication date.

Journal Papers:

- A. Zahaf, S. Bououden, M. Chadli and M. Chemachema, Robust Fault Tolerant Optimal Predictive Control of Hybrid Actuators with Time-Varying Delay for Industrial Robot Arm. Asian J Control. 2020. https://doi.org/10.1002/asjc.2444.

Chapters Book:

- A. Zahaf, M. Chemachema, S. Bououden, and I. Boulkaibet, Fault Tolerant Predictive Control for Constraints Hybrid Systems with Sensors Failures. Proceedings of the 4th International Electrical Engineering and Control Applications. ICEECA'2019. Lecture Notes in Electrical Engineering, vol. 682, pp. 973 - 984, Springer, Singapore, 2021. https://doi.org/10.1007/978-981-15-6403-1_67.
- A. Zahaf, A. Beunemeur, S. Bououden, and I. Boulkaibet, Fault Diagnosis of Uncertain Hybrid Actuators Based Model Predictive Control. Proceedings of the 4th International Electrical Engineering and Control Applications. ICEECA'2019. Lecture Notes in Electrical Engineering, vol. 682, pp. 961 – 971, Springer, Singapore, 2021. https://doi.org/10.1007/978-981-15-6403-1_66.
- A. Beunemeur, M. Chemachema, A. Zahaf and S. Bououden, Adaptive Fuzzy Fault-Tolerant Control Using Nussbaun Gain for a class of SISO Nonlinear Systems with Unknown Directions. Proceedings of the 4th International Electrical Engineering and Control Applications. ICEECA'2019. Lecture

Notes in Electrical Engineering, vol. 682, pp. 493 – 510, Springer, Singapore, 2021. https://doi.org/10.1007/978-981-15-6403-1_34.

- A. Zahaf, S. Bououden, M. Chadli, I. Zelinka and I. Boulkaibet, Observer Based Model Predictive Control of Hybrid Systems. Advanced Control Engineering Methods in Electrical Engineering Systems. ICEECA'2017. Lecture Notes in Electrical Engineering, vol. 522, pp. 198 – 207, Springer, Cham, 2019. https://doi.org/10.1007/978-3-319-97816-1_15.
- A. Zahaf, B. Boutamina, S. Bououden and S. Filali, New Approach of Model Based T-S Fuzzy Predictive Control Using LMI Approach. 15th International Conference on Sciences and Techniques of Automatic Control & Computer Engineering – STA'2014. vol. 00, pp. 38-43, 2014. https://ieeexplore.ieee.org/document/7086708
- A. Zahaf, S. Bououden, M. Chadli, Constrained Fuzzy Predictive Control Design Based on the PDC Approach. Recent Advances in Electrical Engineering and Control Applications. ICEECA 2014. Lecture Notes in Electrical Engineering, vol. 411, pp. 140-154, Springer, Cham, 2017. https://doi.org/10.1007/978-3-319-48929-2_11

Contents

List of Notations

Mathematical Symbols

R	Field of Real Numbers
R^{n}	n Dimensional Euclidean Space
Z	Field of Complex Numbers
$\mathcal{D}$	The Finite Set of Discrete Dynamics
$\mathcal{C}$	The Set of Continuous Dynamics
$\mathcal{U}_{\mathcal{D}}$	Describes the Set of Discrete Inputs
$\mathcal{U}_{\mathcal{C}}$	Defines the Set of Continuous Inputs
I_n (I)	Identity matrix with n Dimension
P	Symmetric Positive Definite matrix
$(.)^T$	Transpose Matrix
$\widehat{(.)}$	Estimated Value

Abbreviations

HS	Hybrid System
HC	Hybrid Control
SS	Switched System
CSS	Continuous Switched System
EHS	Embedded Hybrid Systems
MPC	Model Predictive Control
RMPC	Robust Model Predictive Control
FHS	Faulty Hybrid System
FTC	Fault Tolerant Control
FD	Fault Diagnosis
AFTC	Active Fault Tolerant Control

PFTC	Passive Fault Tolerant Control
HFTPC	Hybrid Fault Tolerant Predictive Control
LMI	Linear Matrix Inequalities
QP	Quadratic Programming

Functions, Function Spaces and Operators

J	Total Cost Function
V	Lyapunov Function
h_σ	Transition Function
$\sigma(k)$	Switched Function
Co	Convex Hull of the Set

List of Figures

List of Tables

INTRODUCTION

1. Motivation

Motivated by the performances study of complex systems in real applications and innovated technologies, this dissertation explores new control approaches to complex dynamic systems that fundamentally have hybrid nature. Typically, the notion “Hybrid” in control engineering and control applications can be associated to two aspects: the hybrid control, which refers to the hierarchical structure of combined controller designs for complex systems, and hybrid systems as a second aspect. Usually, the concept of hybrid systems refers to the description of complex systems with different characteristics or a combination of two different natures, for example, hybrid actuators in industry (with Pneumatic and Electrical Actuators), hybrid Vehicles, hybrid network systems (that include heterogeneous technologies, services and products), as well as chemical and biological systems …. etc.

More specifically, the hybrid systems concept is defined as an interaction ”can be extended to hybrid control” between continuous and discrete dynamics where each dynamic behavior influences other’s dynamic; moreover, the continuous dynamics concern the included systems process defined as modeled framework described by state variables, inputs, outputs and unknown inputs. Besides, the discrete dynamics describe the rules and logics concerning the continuous dynamics; for example, using several operating modes of the dynamic model is considered one among the privileged classes of hybrid systems, entitled switched systems, where each mode is controlled by a specific control law. Therefore, designing a stable dynamic controller is in the heart of diverse issues that encounter researchers with this class of systems, starting from the modeling aspect to the design of a reliable control strategies to meet the required performances of hybrid systems.

To the best of our knowledge, Robust Fault-Tolerant Predictive Control with constraints for the delayed hybrid systems is not well investigated in the literature. Therefore, this dissertation aims to improve the hybrid systems control reliability; where an effective dynamic performance control law is required, to guarantee the robustness in presence of undesirable associated inputs "time-delay and faults occurring during the process". In view of this, the control problems have been at the forefront of this study, where control approaches and strategies have been proposed for HS in this thesis. Besides that, an overview of hybrid systems modeling is presented by given a description for hybrid systems.

2. *Relevant works in literature*

Based on the fundamental definitions above, researchers faced difficulties in the modelling and control of hybrid systems, due to different terminology in various areas that use mixed nature of dynamic systems. For that reason, there has been much works studying modeling and control of hybrid systems (Benveniste,1990; Anstaklis,1995; Branicky,1995; Branicky,1998; Bemporad,1999; Antsaklis,2000; Heemels,2001; Lygeros,2003; Gueguen,2004; Lincoln,2004; Aihara,2010; Praveen Kumar Reddy,2019). To achieve dynamic performances in control theory and control engineering; a switching between different controllers "called Hybrid Control approach" is done to guarantee the dynamic performances under specific hierarchical structure, this aspect is raised in literature in several studies (Amarasinghe,2007; Tsai,2007; De Souza Júnior,2014; Zheng,2018; Jasso-Fuentes,2018; Oberdieck,2015). Consequently, the most common issue of HS researches is the study of optimization problem to compute the optimal control; which led to presenting and proposing several studies and control approaches in this aspect (Usman,2016; Zhu,2015; Zhang,2007; ShahidShaikh,2004; De Jager,2013; Mignone,2002; Goncalves,2000; Taringoo,2012; Potocnik,2004; Borrelli,2005; Potocnik,2008; Zahaf,2020).

Among these control designs, we have witnessed the growing of the Model based on Predictive Control "MPC", which is considered one of the most popular strategies in the field of control theory and automation. Basically, the prediction of future dynamics from an explicit paradigm of the systems is the basic concept of Model Predictive Control; meanwhile, the designated optimal control is based on optimization problem of the specified cost function; various studies in this notion are presented in (Richalet,1978; Clark,1987; Morari,1994; Scokaert,1999; Mayne,2000; Camacho,2004). Aimed to guarantee the reliability of MPC design under terminal constraints, authors in (Maciejowski,2002; Xia,2008) have proposed and discussed control strategies for ensuring robustness performances; therefore, a new control approach in terms of Linear Matrix Inequalities "LMIs" was introduced, called Robust Model Predictive Control "RMPC" by (Kothare,1996), followed by (Vesely,2009) to increase systems robustness efficiency. These fruitful results in control theory were enhanced over the years, and led to the implementation of the MPC strategy in control of hybrid systems for its reliability and adaptability to the complex behavior (Potocnik,2008; Bemporad,2000; Lazar,2006; Altin,2018; Camacho,2010). This complexity is increasing in real applications due to the undesirable associated inputs as time-delay, that results more conservatism to handle with terminal constraints. This latter led researchers to study delayed systems behavior performances based on MPC (Hu,2004; Ding,2007; Bououden,2016; Siroupour,2006; Bobal,2013; Rebel,2011). In addition to time-delay in hybrid systems, the hierarchical framework of HS is considered as an extra factor to increase the control difficulty; therefore, extended studies in the presence of time-delay were proposed to establish a reliable control design (Phat,2010; Li,2009; Lien,2020; Zahaf,2020).

Pursuing the goal of designating the optimal control, unexpected faults occurred in real applications process additionally to time-delay, which might result in more difficulties for the dynamic performances control. These faults are considered as undesirable associated inputs in real engineering systems. This situation results to appear the Fault Tolerant Control (FTC) scheme in control theory in the last decades. Mostly, fault tolerant control involves the conception and design of specific control strategy, that is able to tolerate with the actuators, sensors and process faults; while the requirement performances are ensured.

Therefore, FTC strategies have been widely used for the compensation of the faulty hybrid systems (Zahaf,2019a; Zahaf,2019b; AitLadel,2021; Zhao,2005; Rodrigues,2006; Yang,2009; Wang,2017). Generally, there are two types of FTC: a passive and active approaches. The passive FTC focuses on control robustness against occurred faults based on a fixed control scheme, which leads to more conservative conditions to handle with faulty systems and influences the control reliability to deal with all kinds of faults. Besides, the active FTC is mainly used for the online optimization strategies, by implementing fault diagnosis and tolerable control mechanism for the reconfiguration process, to compensate the undesirable behavior and preserve the specified objectives; some different FTC approaches are presented in (English,1998; Bader,2017; Zhai,2016; Youssef,2017; Lin,2018; Li,2018; Bounemeur,2018; Zhai,2019). The reconfiguration mechanism is based on specific techniques as observation and estimation, to maintain the specific desired performances. Since the accessibility to state variables vectors cannot always be guaranteed with accurate values in real applications, the knowledge of this stage about the information accumulation is an important key for the system reconfiguration design. Basically, a robust AFTC strategy is derived from a reliable systems modelling and based on a reconfiguration mechanism, reconfigurable controller and diagnosis stage. This last is related to the aspect of observability, that is based on the concept of the observers' design. The notion of observability is related to the states reconstruction of complex systems behavior, which started with (Kalman,1960) who proposed an estimation approach for a particular class of nonlinear systems based on Kalman Filter, passing through the state reconstruction design using Luenberger observer (Luenberger,1971), which led increasingly its use to improve the systems behavior diagnosis in last decades (Patton,1989; Frank,1990; Gertler,1998; Patton,2000; Isermann,2006). Recently, the employment of observability and estimation techniques are of a significant importance in control of hybrid systems, due to its adaptive behavior to keep high control performances (Li,2011; Zahaf,2017; De la Sen,2000; Bemporad,2000; Pettersson,2006; Di Benedetto,2009; Yu,2011; Orani,2011; Shim,2011; Tanwani,2014).

So, several researchers have proposed to design an estimation scheme based on MPC for SS in presence of time-delay (Aminsafaee,2019; Taghieh,2020). To extend the reliable control design, a significant approach to study faults occurring with additional time-varying delay for hybrid systems is proposed in (Zahaf,2020).

Generally speaking, the reliability of each proposed control approach is related to the fact of whole system stability. Therefore, the stability performance is analyzed and enhanced by researchers, to according to the paradigm, terminal constraints and undesirable associated inputs. Thus, choosing an appropriate stability technique remains the key point to establish necessary and sufficient conditions, such as the Lyapunov functional, for ensuring the hybrid system stability. There have been several existing researches about HS stability, based on different kinds of stability concepts as "Lyapunov-Like". In this context, we mention studies of the stability theory for hybrid dynamical systems (Branicky,1998; Hui,1998; Hespanha,1999; DeCarlo,2000; Hetel,2007; Naghshtabrizi,2008; Goebel,2012; Minh,2013; Philippe,2017; Wang,2021). Besides that, a set of stability conditions in terms of LMIs to ensure robustness properties is proposed by (Pettersson,2002; Xu,2008; Oishi,2010), included stability study for the discrete-time switched systems in (Kundu,2017); where further stability analysis was given for HS models of robotics by (Singh,2013; Kolathaya,2017). Moreover, to study the stability in presence of undesirable associated inputs, authors in (Hetel,2006; Xu,2008) analyzed and proposed a control scheme to handle with the fact of hybrid systems stability with time-delay, these analyses extended to establish sufficient conditions for HS using Lyapunov-Krasovskii Functional (Zong,2018; Ding,2018; Ghaemi,2019). In spite of the fruitful results in hybrid systems stability, it remains not investigated thoroughly in presence of faults and time-delay.

Starting Point for this Dissertation

Several control approaches with different designs were presented to scientific community, which led to improve the control reliability of HS in face of undesirable associated inputs. Through understanding the probable scenario in the real engineering applications, it is often that both issues of time-delay and faults occurring in actuators, sensors and process faults hinder high performances of the dynamics system. The question to be answered is: what can be proposed as solution to deal with time-delay and faults occurring in hybrid system under terminal constraints?

3. *Contribution of this Dissertation*

The aim of this work is to design a robust hybrid fault tolerant optimal predictive control scheme (HFTPC) for some classes of nonlinear hybrid systems, subjected to faults occurring and time delay, to reconfigure the controller and compensate the continuous dynamic based on our published works (Zahaf,2020). To improve dynamic performances and decrease the conservatism to deal with undesirable associated inputs (time-delay and faults), few different approaches are proposed based on predictive control theories coupled with an online estimation mechanism at each sampling time using an observer (Zahaf,2017). In fact, introducing time delay and faults into a hybrid system model results in a dynamic system converted into a strict-feedback model; where, the modelling aspect is raised in this thesis by presenting a new description of hybrid systems models after a careful examination. Therefore, the possible design goals are as follow:

1. Achieve the control optimality for hybrid systems,
2. Provide more relaxed conditions to ensure the observability concept,
3. Fasten faults compensation and disturbance rejection.,
4. Compensation of Time-Delay,
5. Stability of the closed loop hybrid systems,
6. Low sensitivity to process variations (transition) of sub-systems.

Thus, in order to compute the optimal solution while the performances of HS are ensured, we can summarize the contributions of this dissertation in the following keys:

- Two strategies (aspects) are analyzed: hybrid control and control of hybrid systems with faults and time-delay.
- Hybrid fault tolerant predictive control (HFTPC) design to compute an optimal hybrid control for some classes of HS, based on online optimization of the objective cost function using *min-max* formulation in terms of LMIs, under faults and time-delay. The optimization problem is made by combining two components, to provide necessary and sufficient conditions to compute the optimal solution; the first one is a robust MPC to cope with time-varying delay as continuous dynamics, while the second part is the robust stable hybrid fault tolerant predictive control to handle actuator, sensors, process faults and external disturbances as discrete dynamics.
- The new proposed control scheme (HFTPC) allows simultaneous reconstruction of time-varying and faults of hybrid system, based on an augmented system that includes state variables, faults and different estimated errors. This new presentation allowed us to design a reliable controller without considering FDI scheme due to the proposed faults estimator and the new control law.
- The new proposed control law has two features: an estimated state and error dynamics of faulty HS, with using the estimated faults to reconfigurable the robust optimal control, and then compensate the undesirable behavior.
- To show the efficiency and testing the validity of this dissertation contribution, we propose two approaches of fault-tolerant control based on predictive control theories to compute the optimal control. In the first approach, we use Quadratic Programing “QP” method (as classical optimization) of model predictive control. The second approach HFTPC is

introduced to decrease the computation burden for designating the optimal control based on *min-max* optimization criterion, by deriving a dependent less conservative conditions in term of LMIs.

- In this thesis, the developed approaches for the faulty constrained HS with time-delay are established by ensuring: the closed loop feedback robustness and the stability conditions based on Lyapunov theories "Lyapunov function and Lyapunov-Krasovskii function".

Dissertation Outline

The present dissertation is structured in fourfold: Introduction, Part I and Part II with two chapters for each part, then a conclusion and perspectives.

After introducing the requisite background and thesis contributions; Chapter One presents an overview by focusing on different paradigms on HS modelling in literature. It also presents a framework description for hybrid systems raised in this work. Then an analysis of control scheme for this classes in literature is mentioned, followed by a presentation of control optimality problem.

The Second chapter is devoted to the principal techniques and strategies in control theory, that be used for the problem reformulation and optimization stage in the next chapter to compute the optimal control. The raised concepts, in this chapter, focus on predictive control theories, fault tolerant control of HS, hybrid systems stability and useful optimization tools as LMIs.

Chapter Three presents the fruitful results of this dissertation, some effective computational analytical techniques as solutions to compute the optimal control are developed for HS over this chapter, to handle with different undesirables' associated inputs (Time-Delay and Faults coming from actuator and/or sensor and process system). Firstly, model predictive control combined with an observer is presented to define necessary conditions for the computation of optimal solution control. Then, two strategies to compute the optimal controls are presented by dividing the optimization problem on two stages. Firstly, the new robust fault tolerant optimal predictive control (HFTPC) is presented, based on less conservative optimization conditions in terms of LMIs, with different kinds of faults (actuators, sensors) for delayed hybrid systems under constraints. Followed by the classical optimization method QP of model predictive control.

In Chapter Four, we consider some classes of constrained hybrid systems, inspired by problems in the area of control applications according to the presented description: Industrial (Hybrid Actuators Manipulator arm), Hybrid Network, and Academic Research Hybrid Model (Inverted Pendulum: Fuzzy modelling), to show the effectiveness, robustness and outperforming of the developed and proposed strategies in this dissertation.

Finally, we conclude the dissertation and we present perspectives for future research.

Part I

Introduction on Hybrid Systems

Chapter I

An Overview of Hybrid Systems: Modeling and Control

This chapter is devoted to study and examine the complex and mixed systems entitled hybrid systems. Motivated by presenting a description of the existing hybrid system models; after a careful examination of related works and previous contributions in this field, a paradigm description for HS as unified framework is presented. In addition, we present an overview of the principal concepts in control theory to achieve the optimality control of hybrid systems.

I.1 Introduction

The *hybrid* concept refers generally to an interaction of at least two different dynamic natures. Over the last decades, several studies introduced two different aspects for the *hybrid* concept: hybrid systems and hybrid control approaches. Hybrid control scheme is described as a hierarchical design to manage the separate control modes as discrete dynamic in the controller to meet the system behavior performances, where the sub-control modes describe the continuous dynamic. Basically, this general definition covers a big range of existing control system. It is often that a discrete dynamic is in the form of a scheduler or a supervising algorithm within the controller. Recently, control systems in complex engineering applications and hybrid systems normally contain discrete dynamic in the controller. In some cases, the system dynamic behavior has control scheme as discrete dynamic resulting from the modelling stage. This situation is a definition of the transition or switching function for some classes of the hybrid systems. In fact, hybrid systems refer to the existing of an interaction of continuous-time dynamics "continuous-valued" and discrete event dynamics "discrete variables"; So, the continuous dynamics describes the physical and mathematical dynamic relations between states, inputs, outputs and undesirables associated inputs of the studied systems, defined as modeled framework for hybrid system in general. Besides that, the discrete dynamics manage the continuous dynamics through decisions rules, logic variables and supervising algorithms.

In broad terms, modelling and control of hybrid systems faced difficulties since the significant employing of the hybrid systems in our modern style life, especially in the industrial field for its impact on the economic cost. The latter motivated researchers to propose different unified frameworks for the modelling problems of hybrid systems, and consequently followed by proposing several control approaches.

In this chapter, we present and discuss various aspects of related studies concerning paradigms classification and control scheme of hybrid systems as an overview on HS. This latter, motivated us to suggest a description of hybrid systems based on existing real applications after a thorough study.

I.2 Hybrid Systems: Terminologies and Definitions

Based on preceded definitions of hybrid systems, the most common definition is an interaction between continuous dynamics and discrete dynamics. It describes different aspects as: modelling, classification categories and control approaches. First, let us present the basic definitions of dynamical systems and its dynamic behavior to agree about unified definition for hybrid systems.

Basically, two classes of dynamical systems exist, according to the mathematical models and the equations that describe the evolution of the system behavior;

a. Linear Systems "Linear Models": the system behavior is described by linear mathematical model or linear differential equation.
b. Nonlinear Systems "Nonlinear Models": the nonlinear mathematical model or nonlinear differential equation is describing the system behavior.

On the other hand, the controlled dynamical system is classified on three aspects, according to time-domain models or the set of times where the system behavior evolves:

a. System with Continuous Time: contains an infinite set of values in a set of times *t,* that can also be defined as a connected subsets of the real line. Where $t \in \mathbb{R}$ is used to denote continuous time in the mathematical model as an ordinary differential equation to describe the evolution of the system behavior. As an example, the state space representation of linear dynamic system as a mathematical model in case of CT is defined as:

$$\begin{cases} \dot{x}(t) = A(t)x(t) \ + B(t)u(t) \\ y(t) = C(t)x(t) + D(t)x(t) \end{cases} \tag{1.1}$$

b. System with Discrete Time: uses $k \in \mathbb{Z}$ to denote discrete time, where a finite set of values is considered in a set of times defined as a subset of the integers. A difference equation is among mathematical model that describe the system behavior. Thus, State space representation is presented as the mathematical model:

$$\begin{cases} x(k+1) = A(k)x(k) \; + B(k)u(k) \\ y(k) = C(k)x(k) + D(k)x(k) \end{cases} \tag{1.2}$$

c. *Systems with Hybrid Time:* the basic definition leads to a combination of evolution of the system behavior in continuous and discrete time? To be more specific, the evolution of systems is over continuous time with discrete instants.

Generally, the dynamic system behavior was thoroughly studied based on the influence of engineering applications development. In real applications, the system dynamic is classified according to the type of their state, wherein, three classes of dynamic system are described:

a. *Continuous Dynamic:* we define $x \in \mathbb{R}^n$ to denote the state of a continuous dynamical system, if the state takes values in Euclidean space $\mathbb{R}^n$ for some $n \geq 1$. More general, continuous dynamic is in which the system behavior changes continuously over time according to state variables.
b. *Discrete Dynamic:* is one in which the system behavior changes according to state variables, only at a discrete set of times. Thus, we define p to denote the state of a dynamical system, if the state takes values in countable or finite set $\{p_1, p_2, \dots\}$, then, the dynamic system is a discrete dynamic.
c. *Hybrid Dynamic:* defines that there is an interaction of states, where the part of state takes values in $\mathbb{R}^n$ while another part takes values in a finite set. Basically, hybrid dynamic is a continuous dynamic system behavior according to states variables that change continuously over time, until something happens as a new input rules or supervising algorithm instruction to move to another continuous dynamic system. This transition is a discrete dynamic.

Based on above definitions, the hybrid systems can be defined as hybrid time systems “discrete and continuous time systems” and hybrid state systems. Given that, it is necessary to describe some frameworks directive for HS taking into consideration the related works.

I.3 Modelling of Hybrid Systems

Design, modelling and analysis of hybrid systems are in general more difficult than the study of the continuous or discrete systems separately, since each dynamic affects the other dynamic behavior and vice versa. However, there have been several studies to discuss and study these concepts. To better have an understanding of hybrid dynamics, we consider the definitions from (Lygeros,2008) of different modelling systems for most range of engineering applications:

- *Electrical Circuits:* the continuous phenomena such as the charging of capacitors … etc. are interrupted by switches opening and closing, or transistors (diodes) going on or off.
- *Mechanical Systems:* the continuous motion may be interrupted by collisions.
- *Chemical Process Control:* the continuous evolution of chemical reactions is controlled by valves and pumps.
- *Embedded Computation:* a digital computer interacts with a mostly analogue environment, also in signal communication the discrete dynamic is processed by continuous computation.

According to (Lygeros,2008), all these systems are convenient to a hybrid model. The discrete components (switches, valves, computers, etc.) introduce instantaneous changes "a discrete dynamic" in the continuous components (charging of capacitors, chemical reactions, etc.) "the continuous dynamic”.

Starting from previous examples, to describe a classification and modelling framework of hybrid systems that are raised in several studies, we introduce the general HS model as follows:

Definition 1.1: A Hybrid Dynamic System (HDS) is described as follows:

$$\mathcal{H} = \mathbb{f}(\mathcal{D}, \mathcal{C}, \mathcal{U}_{\mathcal{D}}, \mathcal{U}_{\mathcal{C}}, \mathcal{Y}, \mathcal{F}, \mathfrak{U}, \mathcal{T}, \mathcal{S}, \mathcal{R}, Init, Inv) \tag{1.3}$$

Where

- $\mathcal{D} = \{1 \ldots p\}$ is the finite set of discrete dynamics;
- $\mathcal{C}$ is the set of continuous dynamics;
- $\mathcal{Y}$ is the set of continuous outputs;
- $\mathcal{U}_{\mathcal{D}}$ describes the set of discrete inputs;
- $\mathcal{U}_{\mathcal{C}}$ defines the set of continuous inputs;
- $\mathcal{F}: \mathcal{D} \times \mathcal{C} \times \mathcal{U}_{\mathcal{D}} \times \mathcal{U}_{\mathcal{C}} \times D \times \mathfrak{U} \longrightarrow \mathcal{C}$ represents the set of vector fields for each sub-systems (modes);
- $\mathfrak{U}$ denotes the set of continuous model uncertainties;
- $Init \subseteq \mathcal{D} \times \mathcal{C}$ is the set of initial states;
- $Inv: \mathcal{D} \longrightarrow 2^{\mathcal{C}}$ assigns to each mode an invariant set;
- $\mathcal{T}: \mathcal{U}_{\mathcal{D}} \times \mathrm{F}_{\mathcal{D}} \longrightarrow \mathcal{D} \times \mathcal{D}$ is the set of discrete transitions between sub-systems (modes);
- $\mathcal{S}: \mathcal{T} \times \mathrm{F}_{\mathcal{D}} \longrightarrow 2^{\mathcal{C}}$ denotes ability set related to each transition $(j, j') \in \mathcal{T}$ (smooth switching from mode j *to* j');
- $\mathcal{R}$ is the set of reset maps;

The HDS (1.3) is a general model of usual definitions in literature, that describes the evolution in time of the values of a set of continuous and discrete variables (Branicky,1998; Antsaklis,2000; Lygeros,2003; Zhao,2005). In chapter 2, an extension of HDS is presented in presence of different undesirable associated inputs.

In this regard, we ask what are the range of HS that (1.3) is able to cover?

I.3.1 Different Classes of Hybrid Systems

Based on accurate works of HS, different views for HS modelling are raised in literature (Benveniste,1990; Anstaklis,1995; Branicky,1995; Branicky,1998; Bemporad,1999; Antsaklis,2000; Heemels,2001; Lygeros,2003; Gueguen,2004;

Lincoln,2004; Yang,2009; Aihara,2010; Wang, 2017; Praveen Kumar Reddy,2019), where a general classification of a wide range of HS is given as a framework, according to the extended dynamics of the purely continuous dynamics of complex systems in real-world applications. Therefore, our focus in this thesis is to ensure the stability and achieve the optimality in case that the continuous dynamics are given in discrete-time systems "continuous-time systems", as follows:

$$\begin{cases} \dot{x}(t) = f(x(t), t) \\ x(k+1) = f(x(k), k) \end{cases} \tag{1.4}$$

Roughly speaking, hybrid systems are classified into four phenomena according to (1.4) based on (1.3) in most of studies in the literature as follows:

- *Dynamics with Autonomous Switching,*
- *Dynamics with Autonomous State Jumps "Autonomous Impulses",*
- *Dynamics with Controlled Switching,*
- *Dynamics with Controlled State Jumps "Controlled State Jumps".*

Nevertheless, the author in (Branicky,1995) proposed another classification as unified paradigm for hybrid systems:

- *General Hybrid Dynamical Systems,*
- *Hybrid Dynamical Systems,*
- *Switched Systems,*
- *Continuous Switched Systems.*

Moreover, a class of hybrid systems is introduced by (Bemporad,1999) for modelling a broad class of system applications, which can be approximated by some appropriate approximation techniques to obtain piecewise linear functions. This new class is entitled mixed logical dynamical (MLD) systems.

Regardless of the aforementioned classifications of hybrid systems; as an example, which category we can classify the hybrid system raised in (Zahaf,2020)? After a careful examination of existing studies in the field of HS, especially which focuses on the modelling aspect, we identify a new description of HS.

I.3.2 Description of Hybrid Systems: A Viewpoint

In this part, we describe hybrid systems according to their behavior, structure and phenomena that they exhibit. We explore the description as follows.

I.3.2.1 Switched Systems

This class of HS has a topology of multi-model "sub-systems" or variable structure. These modes are a simple continuous portions of the HS, where we can talk here, generally, about linear models. SS describes the fact that the vector field f that occurs in (1.4) is changing discontinuously. The switching between modes (sub-systems) can be related to some specification functions or higher process such as algorithms, controller and operator "human and computer"; where the SS is considered as controlled SS in this case. Also, it can depend on some factors and functions as time and state, which can be considered as autonomous SS. In the aim to spot the difference between the existing SS in real applications, a class of HS (1.3) based on (1.4) are considered as switched systems that take the next form:

$$\begin{cases} x(k+1) = f_\sigma(x(k), u_\sigma, h_\sigma) \\ y(k) = g_\sigma(x(k), u_\sigma) \end{cases} \tag{1.5}$$

According to switching function h_σ, we distinguish four types of switched systems:

A. Hybrid Systems with Time-Dependent Switching

This class of HS is considered as autonomous SS, while the switching between different continuous modes is according to time functions $h_\sigma(k)$, that the switching occurs at predefined interval or instant time. In this way, we cite some works that raised this topic (Karabacak,2020; Zhao,2012; Yang,2010; He,2016).

B. Hybrid Systems with State-Dependent Switching

The switching occurs whenever the continuous state hits some given boundaries, surfaces or satisfy constraints. Also, this class is classified as autonomous SS; where researches concerning the study of this class are given in various aspects as in (Yang,2019; Li,2020; Leth,2015).

C. Hybrid Systems with Impulsive and Stochastic Switching

This is the third class of autonomous SS; where in the impulsive switching, the dynamic behavior is abruptly changed at each switching instant due to the impulse effect. While a random process governs the switching action in stochastic switching; examples of relevant works are given in (Gao,2019; Gao,2019; Wu,2016).

D. Hybrid Systems with Discrete Specifications Switching

This class of HS is considered as controlled SS, since the switching function $h_\sigma(k)$ takes into consideration the control input of discrete specification of the studied problem. For that, the continuous dynamic is globally convergent and stable whatever sub-system is activated. Where $h_\sigma(k)$ can take the form of algorithms, specific rules, controller scheme and operators such as human or computer; few examples of recent works for this class are mentioned as follow (Zhang,2020; Ren,2019; Zhang,2020; Yang,2020).

I.3.2.2 Continuous Switched Systems

This class of HS is subject to additional constraints, which allow to the switched sub-systems agree at the switching time. we distinguish two types of this class of HS:

A. Sequential Hybrid Systems

The switching of sub-systems in SHS is described that the output of i^{th} mode is the input of *(i^{th}+1)* mode. A clear example of CSS is the injection molding process that is studied in (Wang, 2017). Thus, this class of HS is widely used in industrial field.

B. Auxiliary Hybrid Systems

The switching process is based on the complementary execution of the engineering application; where the first dynamic is considered as principal continuous dynamic, while the other dynamics are considered as sub-system (subroutine) to be activated in case of incapability of the principal continuous dynamic, to meet the required performances. Examples for this category is shown in regulation speed of blades in Wind Turbine (assistance by Motor in case of weak Wind Speed), and in hybrid vehicles (i.e., activation of petrol consumption in case that the tank of GPL tends to empty); as an example, the power management for heavy duty hybrid vehicle (Barelli,2020) and the aircraft dynamics …

I.3.2.3 Embedded Hybrid Systems

In this class of hybrid systems, the different continuous dynamics are completely fusional and integrated, which appear that no continuous dynamic can be used without the others to meet the required performances. For example, this class of HS, a model of EHS is investigated in (Zahaf,2020).

I.3.2.4 General Hybrid Dynamical Systems

In this class, some behavioral properties of hybrid systems are subject to modelling systems itself. Therefore, we assume Γ is an ordered set with the least upper bound property of an HS (Branicky,1995) to a modelled wide range of HS, where the reachability, accessibility and stability are guaranteed for the global hybrid systems. We can cite under this class of HS the following unclassified systems:

A. Piecewise Affine Systems (PWA),

B. Mixed Logical Dynamical (MLD).

C. Can we Add Fuzzy Control Systems (FCS) as hybrid systems?

A scientific debate about the classification of Fuzzy control systems is raised, the key point is whether FCS can be considered as HS or not?

In his discussion, (Branicky,1995) considered the fuzzy control systems as class of HS, he justified his view based on two points:

- The control scheme is given by finite rule base ***If-Then***, that is related to the finite symbols of the hybrid model. the control scheme of FCS is considered as discrete dynamic for HS.

- Second point, FCS is producing continuous areas; this continuity is based on each transition between multi-models for definite area, this transition process can be considered as hybrid systems with state-depend switching. It is easy to notice that the multi-models of fuzzy systems can be combined to construct arbitrary piecewise linear functions with state-depend switching function.

Fuzzy control systems principles are extensively studied in the literature, which goes beyond the scope of this dissertation. However, we present as techniques in *Appendix A* useful structures and representation for uncertain nonlinear systems (state-space representation); to spot how can we derive some frameworks of HS.

I.4 The Main Features to be Guaranteed in the Modelling of Hybrid Systems

Broadly speaking, a careful and precise formulation in the modelling process is the first key to design a robust and reliable control system. This step remains as manifold to provide a precise model, that meets the required hybrid system performances. Since the requirements of the HS is more conservative than ordinary system, it's worthy to take into consideration the next aspects in the stage of mathematical modelling:

I.4.1 Observability of Hybrid Systems

The concept of observability refers to the conditions studied that allow to infer the state of dynamical systems from measurements of output behavior. In hybrid systems, observability has two manifolds: for discrete dynamic and continuous dynamic. Thus, the observability issue is the main subject search for many studies (Chaib,2005; Di Benedetto,2009; Yu,2011; Arbib,2020), these studies continued to enhance the sufficient conditions, as some proposed solutions to improve the observability for HS (Bempoead,2000; Shim,2011; Tanwani,2014). Meanwhile, the reachability of HS to a desired space or a set point is related to strength of the observability conditions (Petreczky,2010).

I.4.2 Reachability of Hybrid Systems

Reachability refers to the set of points computed by the control design, that belongs to the set defined as trajectory for general hybrid dynamical system $\mathcal{H}$. Thus, the reachability is related to the designed controller; studies in this subject are raised in the literature as (Lygeros,1999; Zhendong Sun,2002).

I.4.3 Controllability of Hybrid Systems

Controllability is the ability of the control design to compute the input control for general hybrid dynamical system $\mathcal{H}$ at any time t for any activated sub-system (mode) j. The correlation between controllability and observability for hybrid systems is so important due to the specifications of HS, this concept is in the heart of several investigations for a wide range of hybrid systems (Bempored,2000; Xie,2004; Ji,2008; Liu,2008; Lin,2020) to derive sufficient and necessary conditions, for strict complete controllability of both discrete and continuous dynamics for various class of hybrid systems; besides the ability to compute an admissible input control law for the continuous dynamic.

I.5 Controller Design Basics for Hybrid System

Usually the aim behind designing a controller is satisfying the constraints and meeting the required system performances, even in the presence of the undesirable associated inputs (as time-delay, disturbances and faults). Generally speaking, in control theory and engineering applications, guarantee the stability of the studied systems is the main objective of all research, through designing a robust and reliable control scheme. In addition to the reliability, robustness and stability features of the controller for ordinary dynamic systems, the Zeno phenomenon is a harmful event for hybrid systems, that should be avoided for any proposed control design in all classes of HS.

I.5.1 Zeno Phenomena

A Zeno phenomenon (refers to the philosopher Zeno "500-400 B.C.") is described as an infinite number of switching or discrete transitions in a finite time interval. It can lead to a common problem for most frameworks of HS raised in the previous section, by losing stability of equilibriums and the emergence of unexpected and meaningless solutions for most cases of hybrid systems.

Definition 1.2: *(Zeno Phenomena)* A Hybrid Dynamic System (1.3) is called Zeno if $\lim_{p\to\infty} t_p = t_\infty < \infty$, and if there exists $(\mathcal{D}_0, \mathcal{C}_0) \in Init$ such that all executions in $\mathcal{H}^{\infty}_{(\mathcal{D}_0,\mathcal{C}_0)}$ are Zeno executions.

Generally, Zeno phenomena is arising as a consequence of:

- *Modelling process*: since our aim is making strong and robust modelling frameworks that cover a wide class of hybrid systems, some additional mathematical manipulation to improve the mathematical model can produce the Zeno phenomena over the running time which affect the HS behavior.

- *Weak Controller Scheme*: a weak synthesis of the control design can reflect the stability and reliability of HS. Here, we mean as controller scheme the discrete dynamic and the control law for continuous dynamic.

Therefore, the practical solution to avoid the Zeno phenomena is providing a careful and strong modelling framework, in addition to designing a reliable controller scheme (for both Discrete and Continuous Dynamics), in order to avoid the prosthetic solutions that can provide useful results as well as it can be useless (Johansson,1990; Or,2011; Dashkovskiy,2017) in the aim to ensure the stability of hybrid system and meet the required performances.

I.5.2 Stability of Hybrid Systems

Stability of HS is a more sensitive criterion than ordinary systems, due to the handling of additional constraints to avoid Zeno phenomenon. To discuss the aspect of stability in the next chapters, we recall the fundamentally concepts of stability theory. Basically, stability refers to a systems conditions or property that meets its equilibrium position based on control input, even when it is subject to undesirable associated inputs. Formally, the equilibrium points x_{jeq} represent the real solutions of the different sub-systems.

Definition 1.3: Recall the hybrid dynamic (1.4) based on (1.3), the function f is a locally Lipschitz function, if for every x in $\mathcal{C}$ there exists a $x^* \in \mathcal{X}$ such that f is Lipschitz continuous in $\mathcal{X}$, where $\mathcal{X}$ is subset of the set of continuous dynamics $\mathcal{C}$. Equivalently, if $\mathcal{C}$ is convex set, then f is locally Lipschitz function if and only if it is Lipschitz continuous on every sub-system.

Definition 1.3 generalizes the concept of stability; in several studies, the equilibrium point is considered as the origin ($x_{jeq} = 0$). However, the concept of stability leads to the Lyapunov stability theory, where Lyapunov theory refers to loss energy of the state dynamic (trajectory) evolution over time in defined space, where $V(x)$ depends on the system state. We distinguish the stability kinds.

Definition 1.4: Recall the hybrid dynamic (1.4) based on (1.3), with $x_{jeq} = x^* \in \mathcal{X}$ the equilibrium point, if there exists a function $V\big(x_j(k), u_j(k)\big): \mathbb{R}^n \to \mathbb{R}$ such that $\delta \left\|x_j(k)\right\| \leq V\left(x_j(k), u_j(k)\right) \leq \varepsilon \left\|x_j(k)\right\|, \ \forall\, x_j \in \mathcal{X} \ \subset \mathbb{R}^n$

We say the system (1.4) is:

A. Stable if

$$\Delta V\left(x_j(k)\right) = V\big(x_j(k+1), u_j(k+1)\big) - V\left(x_j(k), u_j(k)\right) \leq 0 \ \ \forall\, x_j \in \mathcal{X}, \ x_j \neq 0$$

B. Asymptotically stable if

$$\Delta V\left(x_j(k)\right) \leq -\alpha \left\|x_j(k)\right\|, \qquad \alpha \in \mathbb{R}, \qquad \forall\, x_j \in \mathcal{X}, x_j \neq 0$$

C. Globally Asymptotically Stable if x_{jeq} is stable and $\forall\, x_j \in \mathbb{R}^n$ such that

$$\lim_{k \to \infty} x_j(k) = x_{jeq}$$

In fact, to ensure the stability of HS, it is required to meet the conditions for:

A. The hybrid system (1.3) is stable (asymptotically or global asymptotically depends on the controller), based on stability of each sub-system that is related to necessary and sufficient conditions to guarantee the stability (feasible and optimal solution).

B. The ability that the discrete transitions between sub-systems (modes) make the system stable “Avoid Zeno Phenomena”.

To derive necessary and sufficient conditions, several works were presented to guarantee the stability of HS based on Lyapunov function (Branicky,1998; Hui,1998; Hespanha,1999; DeCarlo,2000; Hetel,2007; Naghshtabrizi,2008; Goebel,2012; Minh,2013; Philippe,2017; Wang,2021), Even in the presence of undesirable associated inputs (Hetel,2006; Xu,2008).

Obviously, the stability performances are related to the reliability and robustness of the controller.

I.5.3 Control Design for Hybrid Systems

Control design (scheme) or the design of controllers, refers to strategies and techniques for controlling the behavior of any system, using its input variables such as different dynamics for HS. Since the appearance of the concept of hybrid systems, different control approaches have been proposed for various classes of HS, depending on whether the variables are available. However, the sampled-data controller is a control system based on periodic sampling. This feature is adequate for many controlled switched systems; authors in (Hauroigne,2011; Wang,2019) investigate the impact of employing the sampled data controller for switched systems; wherein, an optimal state feedback control law for switched affine systems is computed based on the Fillipov solutions (Pattino,2009) as an adaption of the control law parameters. This approach of the adaptive control consists of an online identification of the control law at each sampling time, a model reference adaptive control is considered in (Elzaghir,2018) for manipulating the hybrid electric vehicle, extending the implementation of adaptive control for same application in (Chen,2018) based on fuzzy models as HS. The hybrid systems are taken for investigation using the model predictive control, which is basically a set of algorithms depending on the controlled system mainly. A summary of some perspectives and techniques of MPC for HS is presented by (Camacho,2010). Therefore, a hybrid model predictive control is introducing to meet the high performances of HS based on a mixed-integer programming using the Quadratic program algorithms (Marcucci,2020); in order to reduce the conservatism to compute the input control based on the mixed-integer programming using

Lagrange dual function; an LMI approach based on model predictive control is proposed in (Nodozi,2020) to guarantee the stability of the HS. Moreover, a robust model predictive control in presence of faults and time delay is presented in (Zahaf,2020), by relaxing the optimization conditions of the defined cost function, by introducing the LMIs technique to convert the constrained faulty hybrid system to a convex optimization problem, where the computation of the optimal solution is based on *min-max* method to decrease the computation burden. This approach is presented in chapter 3 as theoretical results and validated in chapter 4 through simulations' results.

I.6 Conclusion

This chapter has presented a description of hybrid systems modelling after a precise and careful examination of the related works throughout the preparation of this thesis; where basic definitions and related concepts of hybrid systems were clearly presented. Yet, the modelling classification and the control strategies were raised in the literature as an overview on HS, to synthesis a reliable control design to cope with time-delay and faults. Therefore, we devote in the next chapter a thorough discussion on the fault tolerant control aspect based on existed control strategies, by focusing on the model predictive control.

Chapter II

Hybrid Systems with Undesirable Associated Inputs: Control, Estimation and Synthesis

In this chapter, we present the useful techniques and the principal concepts in control theory, that are employed for the estimation and control of hybrid systems with undesirable associated inputs which are defined as faults, disturbances and time-delay. In addition, an optimal control is required to define an appropriate approach synthesis in order to achieve the control optimality for hybrid systems.

II.1 Introduction

Over the last years, there has been a considerable scientific research in the field of control for hybrid systems and hybrid control; wherein, many techniques are presented to be employed in several control designs for faulty hybrid systems. Among these, the Model Predictive Control (MPC) has become one of the most famous advanced control techniques used in the industrial and automatic engineering; due to its tolerance for different types of systems, satisfaction of imposed constraints and handling undesirable associated inputs (time-delay, disturbances and faults). Therefore, to deal with undesirable behavior, especially faults occurring requires an efficient control design. Accordingly, a fault tolerant control (FTC) strategy is designed for HS through predictive control along this thesis, where the reliability and robustness of FTC strategy are quite important for complex behavior. Hence, we shall describe the relations between FTC and predictive control as a control strategy, to maintain high performance control for HS in presence of faults and time-delay, which is not investigated thoroughly in the literature and it be the main topic of our study.

So, this chapter is devoted to present different strategies and principal concepts raised in control theory to design a reliable FTC, which is used in the computation of the optimal control for faulty hybrid systems. Meanwhile, an overview and a synthesis of main concepts: MPC and FTC with undesirable associated inputs (Faults and Time-Delay) are presented as well as stability analysis.

II.2 Fault-Tolerant Control for Hybrid Systems

II.2.1 Significant Impact of Fault Tolerant Control for HS

For several years, works and research in fault tolerant control and hybrid system have been investigated and developed separately. Thus, the FTC study for HS is not thoroughly investigated, despite of numerous fruitful results in both aspects. However, the complex engineering systems that have a hybrid nature can be modeled based on defining frameworks presented in the previous chapter. Basically, Fault-Tolerant Control (FTC) was taken into consideration in last decades in real application for its reliability in control process. The FTC concept is defined as a control system with fault-tolerant capability to compensate the undesirable behavior.

The main goal to design an FTC strategy is to preserve the specified process and required performances of the system under study, and give agents (or controlling automat) sufficient time to compensate the faulty behavior or apply alternative scenario to avoid application control crisis (Chen and Patton,1999) against the undesirable inputs. Thus, the FTC is concerned with the interaction between a given plant and a controller term (Controller term refers to control approach in general sense).

Generally speaking, Fault is an undesirable associated inputs in engineering applications, that changes the behavior of a system so such that the system is not able to meet the desired performances. The fault can occur in an actuator, sensor and a system (Fig. 2.1). It may be caused by many possibilities and is considered as losses of information in connection system, internal event, error in design, human operator wrong action … etc. A brief analysis of the faults influences on the system behavior based on graphical interpretation is given in (Blanke,2006). This situation can lead to systems failure. It is worthy to distinguish between fault and failure: the failure refers to the total breakdown of system, while fault is denoting malfunction that can be compensated. This compensation or tolerable action should be preceded by diagnosis of malfunction, usually the fault diagnosis (FD) stage consists of three tasks:

- ***Fault Detection***: this step is so important for any practical system; the main task is to make a logical rule to determine whether there is a wrong matter in system functionality or it is fine.
- ***Fault Isolation***: to locate the fault and make it out in the redesign of the controller depending on the nature of the system.

- ***Fault Identification and Fault Estimation:*** this step is the ability to estimate the nature and the limits of faults through a reconfiguration design of faults. This stage is one of the important key for designing a robust reliable control scheme for the reconfiguration/or reconstruction mechanism.

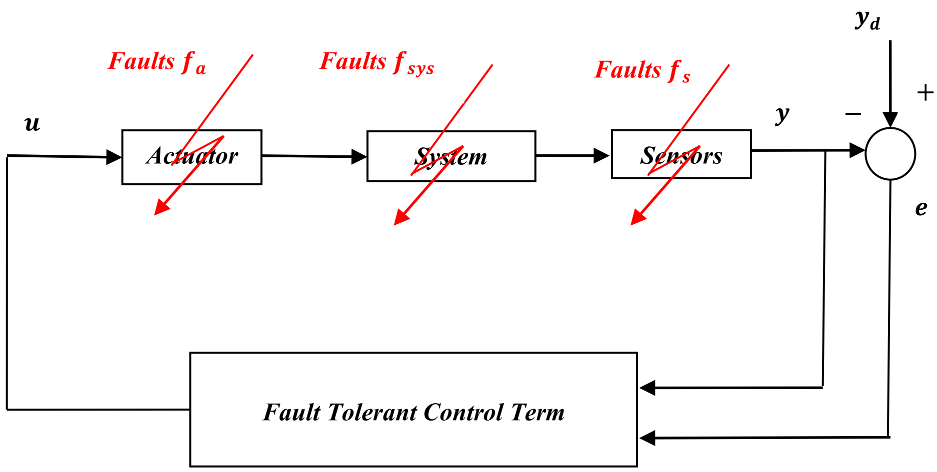


Fig. 2.1. Fault Tolerant Control Concept

Basically, fault diagnosis is the primary stage for designing a fault tolerant control scheme. Undoubtedly, at least one of the fault diagnosis tasks is involved in FTC strategy. Throughout this thesis, fault detection and identification stages are considered based on an adequate estimation approach, due to its smooth usage and employment to redesign a robust controller, that is able to adapt to the faulty situation and meet the satisfying objectives, compared to classical fault diagnosis. The faulty situation can appear in the model plant over redesigning the FTC scheme, by including the constraints on the input control "u" and output "y" in the modelling of the faulty system; Thus, faults are considered as constraints in this conceptualization (Blanke,2006). If this concept is true, I expect we should ask what is Fault …

II.2.2 Classification of Fault

As aforementioned, faults can occur in actuators, sensors or systems. Whereas the impact of a fault is an undesirable behavior of the system. It is significant to distinguish between Faults, Disturbances and Model Uncertainties.

From the basic definitions in the literature, faults are represented as Additive or Multiplicative Faults.

- *Additive Faults* are usually represented as additional external input (or Signals). They are similar to *Disturbances* that are represented as unknown input signals. Usually, the additive faults will change the mean value of state variables.
- *Multiplicative Faults* depend on fault size that are multiplied with system state or the input; where *Model Uncertainties* change the model parameters as multiplicative factor. Mostly, the multiplicative faults will change the variance or covariance of state variables.

To recognize the differences of the impact of each input event, the obtained results from the literature show that *Disturbances* and *Model Uncertainties* are handled by an appropriate task as filtering or robust control design. Besides, to deal with faults requires a conception of FTC mechanism that can tolerate the faults to an acceptable level to maintain system performances.

Usually, to design a reliable FTC reconfiguration mechanism requires a better Knowledge about the type and pattern of faults. As a result, many researches on the classification of faults depending on the different factors are presented in (Chen and Patton,1999; Blanke,2006; Bošković,2003; Bounemeur,2018). Subsequently, to extend these classifications of faults for HS, it is worthy that the next question to be asked is;

Are the aforementioned faults concepts valid for HS?

Because our concern in this thesis is the FTC approach for HS, let us consider the aforementioned classification of faults for HS in a general case. To check the validity of these concepts by applying for different dynamics and frameworks, we introduce a general HS model as follows:

Definition 2.1: A Hybrid Dynamic System (HDS) with undesirable associated inputs is described as follows:

$$\mathcal{H} = \mathbb{f}(\mathcal{D}, \mathcal{C}, \mathcal{U}_{\mathcal{D}}, \mathcal{U}_{\mathcal{C}}, \mathcal{Y}, \mathrm{F}_{\mathcal{D}}, \mathrm{F}_{\mathcal{C}}, T_{\mathcal{D}}, T_{\mathcal{C}}, \mathcal{F}, D, \mathfrak{U}, \mathcal{T}, \mathcal{S}, \mathcal{R}, Init, Inv) \tag{2.1}$$

Where

- $\mathcal{D} = \{1 \dots p\}$ is the finite set of discrete dynamics;
- $\mathcal{C}$ is the set of continuous dynamics;
- $\mathcal{Y}$ is the set of continuous outputs;
- $\mathcal{U}_{\mathcal{D}}$ describes the set of discrete inputs;
- $\mathcal{U}_{\mathcal{C}}$ defines the set of continuous inputs;
- $\mathrm{F}_{\mathcal{D}}$ defines the set of faults of the discrete dynamics;
- $\mathrm{F}_{\mathcal{C}}$ defines the set of faults of the continuous dynamics;
- $T_{\mathcal{D}}$ denotes the time-delay of the discrete dynamics;
- $T_{\mathcal{C}}$ denotes the time-delay of the continuous dynamics;
- $\mathcal{F}: \mathcal{D} \times \mathcal{C} \times \mathcal{U}_{\mathcal{D}} \times \mathcal{U}_{\mathcal{C}} \times D \times \mathfrak{U} \longrightarrow \mathcal{C}$ represents the set of vector fields for each sub-systems (modes);
- D denotes the set of continuous disturbances;
- $\mathfrak{U}$ denotes the set of continuous model uncertainties;
- $Init \subseteq \mathcal{D} \times \mathcal{C}$ is the set of initial states;
- $Inv: \mathcal{D} \longrightarrow 2^{\mathcal{C}}$ assigns to each mode an invariant set;
- $\mathcal{T}: \mathcal{U}_{\mathcal{D}} \times \mathrm{F}_{\mathcal{D}} \longrightarrow \mathcal{D} \times \mathcal{D}$ is the set of discrete transitions between sub-systems (modes);
- $\mathcal{S}: \mathcal{T} \times \mathrm{F}_{\mathcal{D}} \longrightarrow 2^{\mathcal{C}}$ denotes ability set related to each transition $(j, j') \in \mathcal{T}$ (smooth switching from mode j *to* j');
- $\mathcal{R}$ is the set of reset maps;

The above HDS is a general model and an extension of usual definitions in the literature (Branicky,1995; Branicky,1998; Lygeros,2003; Zhao,2005; Yang,2009). This model scales up all parameters and undesirable associated inputs that can occur in practical applications. Thus, this paradigm is more general than the existing works which collect all kinds of undesirable associated inputs: faults, disturbances, model uncertainties and time-dely.

As a description in (Yang,2009), the FTC objectives for HS in this thesis are devoted to two required aspects:

- Continuous dynamic performances achievement, e.g., the global stability of the origin of the HS, compensation of faulty states and output performances achieved (Regulation/Tracking problem).
- Discrete dynamic specification purpose guarantees the desired performances, i.e., satisfying imposed constraints on discrete modes, e.g., the switching rule.

Therefore, the above mentioned description and classification of faults cannot be exactly significant in Hybrid Systems. Since HS is a combination between discrete dynamic and continuous dynamic. Thus, further faults types, nature and their classification, can be classified as follows Fig. 2.2:

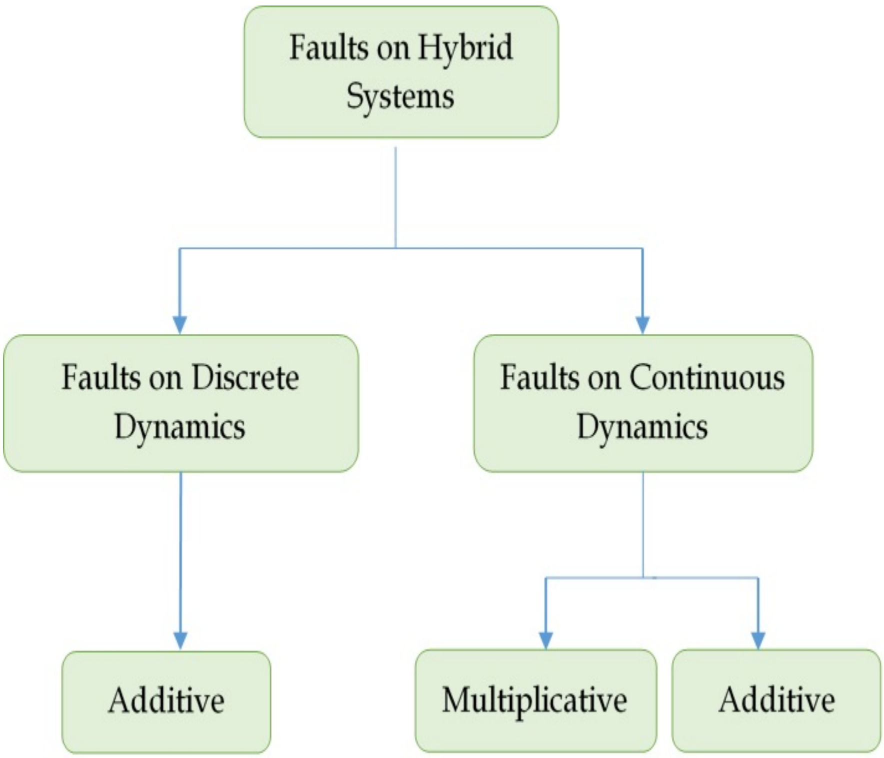


Fig. 2.2. Faults Classification for Hybrid Systems

To be more specific about the classification of different occurring faults, Table 2.1 elaborates all faults as peer manner, kind, type and nature.

Table 2.1. *Classification of Faults for HS*

Faults Manner	Faults Kind	Faults Type	Faults Nature
Faults on Continuous Dynamics	Actuator/Sensor	Additive	- *Bias* - *Drift* - *Loss of Accuracy*
		Multiplicative	- *Loss of effectiveness*
Faults on Discrete Dynamics	/	Additive	- *Bias* - *Loss of Accuracy*

For more details and description about the types and nature of faults, readers can refer to (Bounemeur,2018).

From the above discussion, to deal with faulty HS cases, it is basically vital to achieve the continuous dynamic performances based on an efficient FTC scheme. Therefore, a class of hybrid discrete-time system (2.1) can take the next form:

$$\begin{cases} x(k+1) = h_\sigma(x_\sigma, u_\sigma, f_\sigma, D_\sigma, f_\sigma, T_\sigma) \\ \quad\quad y(k) = g_\sigma(x_\sigma) \end{cases} \tag{2.2}$$

Where

- $x_\sigma \in \mathcal{C}$ is the state variables of continuous dynamic;
- $u_\sigma \in \mathcal{U}_\mathcal{C}$ is the inputs control of continuous dynamic;
- $f_\sigma \in \mathrm{F}_\mathcal{C}$ is the faults in continuous dynamic;
- $D_\sigma \in D$ is the disturbances in continuous dynamic;
- $\mathcal{u}_\sigma \in \mathfrak{U}$ is the continuous model uncertainties;
- $T_\sigma \in T_\mathcal{C}$ denotes the time-delay introduced in continuous dynamic;
- $\sigma(k): [1, \infty) \longrightarrow \mathcal{D}$ denotes the logical function or rule for the transition of sub-systems (modes). Generally, in the literature $\sigma(k)$ refers to switching function for Switched systems, which is not the case in this

thesis, where it is considered as general transition function for different frameworks of HS.

Hence, the state-space representation of the hybrid discrete-time systems is described in this thesis as follows:

$$\begin{cases} x(k+1,j) = A_{(k,j)}(v)x(k,j) + A_{(k,j)d}(v)x(k-T_{dk},j) + B_{(k,j)}(v)u(k,j) + \\ \qquad F_a f_a(k,j) + d_{\mathrm{d}} D_{\mathrm{d}}(k,j) \\ y(k,j) = C_{(k,j)}(v)x(k,j) + +F_s f_s(k,j) \\ \sigma(k) = \eta(x(k,j), u(k,j)) \end{cases} \tag{2.3}$$

Where k and j are time step and subsystem index; $A_{(k,j)}(v)$, $A_{(k,j)d}(v)$, $B_{(k,j)}(v)$ *and* $C_{(k,j)}(v)$ are state matrices of sub-system j, $x(k,j) \in \mathrm{S}^n, u(k,j) \in \mathrm{S}^n$, $f_a(k,j) \in \mathrm{S}^1, f_s(k,j) \in \mathrm{S}^1$, $D_{\mathrm{d}}(k,j) \in \mathrm{S}^1$ and $y(k,j) \in \mathrm{S}^n$, denote the state, input, actuator faults vector, sensor faults vector, disturbances and output of the process at time step k in the j^{th} subsystem. T_{dk} is a time-delay. $\sigma(k)$ represents general transition "switching" function for some classes of hybrid systems.

From the above HDS model (2.3), an efficient and reliable FTC strategy is required. Namely discussed, based on fruitful results of fault tolerant control design in the literature, an FTC can be designed as an active (AFTC) or passive (PFTC) approach depending on the aspect of incapability of the systems to face faults occurring (Fig. 2.3). A comparative study for both approaches was presented in (Jiang,2012).

The Passive FTC is intended to deal with faults as preliminaries of potential malfunctions of the system, neither a reconfiguration mechanism nor an estimation of faults is required. Usually to maintain the systems performances; a synthesize of a robust controller design or redundancies tasks for basic faults are enough to handle with normal conditions and the pre-considered faults. Thus, the PFTC focuses on the robustness of the control systems, without striving to achieve the optimal performance for any occurred fault, by considering a single robust controller, which turns the designed controller scheme to be more conservative for different kinds of faults.

Besides that, an Active FTC approach mainly introduces an online reconfiguration mechanism in real time that based on three aspects: controller reconfiguration scheme, reconfigurable controller and diagnosis scheme. The effectiveness of these elements for designing reliable AFTC approach is based on the ability to set up the existing control scheme, to maintain the system behavior at required performances level through an efficient and smooth design. This can be realized by optimizing sufficient and necessary conditions to achieve the optimal solution, for a large part of fault cases (considered-fault cases and free-fault) based on a significant impact of diagnosis scheme, which can provide information about the occurred faults with minimal uncertainties at an appropriate time, which in turn leads to follow functional steps for an efficient AFTC strategy.

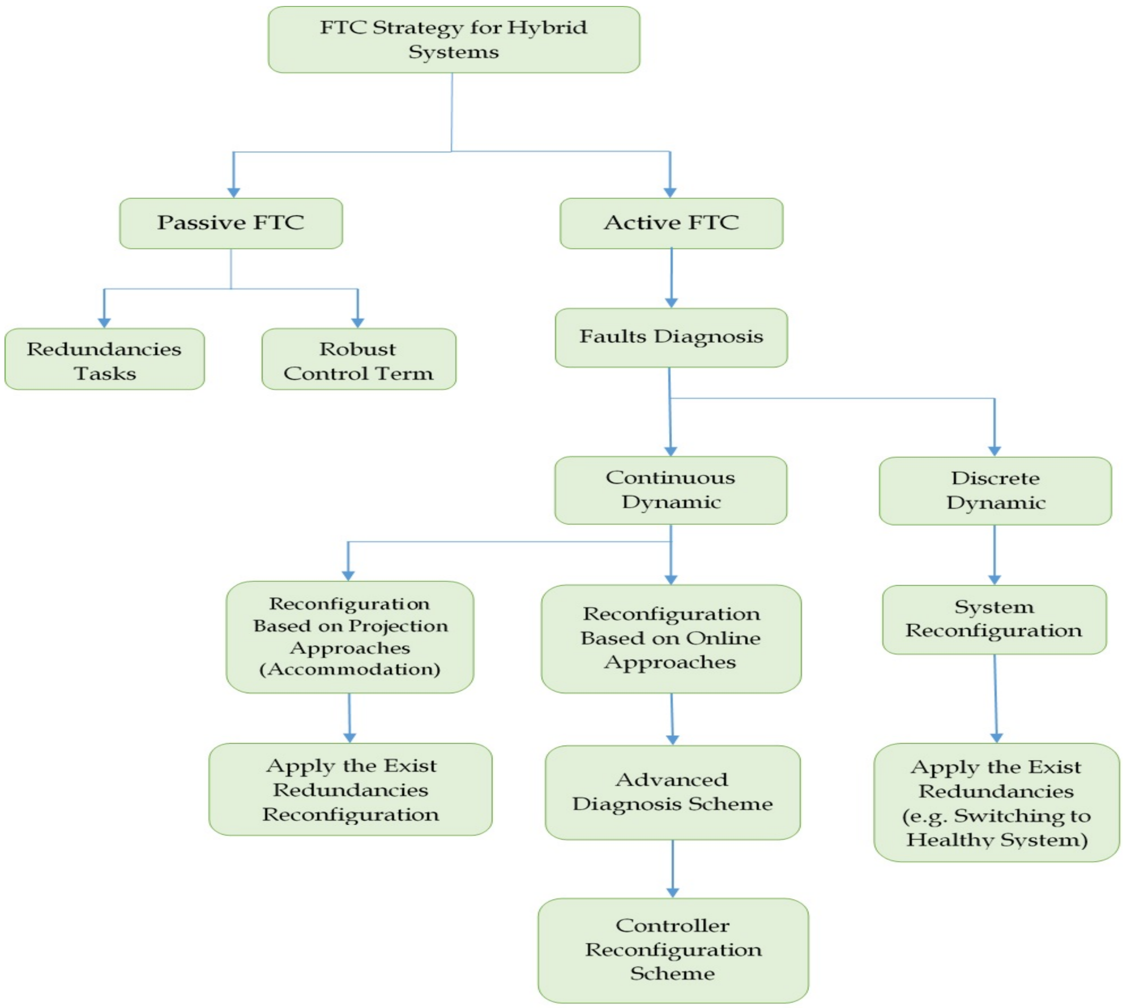


Fig. 2.3. Classification of FTC Approaches for Hybrid Systems

II.2.3 Designing of a Reliable Active FTC Approach

As mentioned above, an AFTC compensates the faults impact based on efficient controller by synthesizing an online reconfiguration mechanism. Therefore, before raising the relevant works of an AFTC design for reconfiguration mechanism, an overview concerning related concepts and techniques for faults diagnosis is necessary for the synthesis of AFTC approach.

II.2.3.1 Faults Diagnosis

Usually, FD is a highly required strategy for modern complex control systems. Thus, to design a reliable FTC approach, the FD stage consists of three aspects: detection, isolation and estimation (Identification) of faults. To set up this approach, many strategies have been developed and proposed in this purpose (Chen and Patton,1999; Patton,2000; Isermann,2005; Blanke,2006); wherein FD can be classified as: Signal Processing methods and methods based on Model. Therefore, we mention below the principal definitions for FD approaches that are raised in the literature:

A. ***Parity Relation Approach:*** is basically founded to generate the residual (parity vector) from the input and output information system over finite range time. Unfortunately, this approach has not received enough attention due to its conservative conditions, readers can refer to (Mironovski,1980; Chen-Zhang,1990; Gertler,1995; Gertler,1997).
B. ***Parameter Estimation Approach:*** is considered one of the important FD scheme. It is based on identification techniques, where the parameter vector is estimated based on the change of the system behavior (Bakiotis,1979; Isermann,1984; Zolghadri,1996; Isermann-Balle-,1997).
C. ***Diagnosis based on Direct Synthesis of Filters:*** can be classified into two categories: the first is concerned with the fault estimation (Stroustrup-Niemann,2002; Henry-Zolghadri,2006) and the second focuses on the generation of residual vector (Ding2000; Jaimoukha,2006). These

approaches are based on the direct realization of diagnosis filters to ensure the robustness performances and system stability.

D. ***Observer-based Fault Diagnosis:*** basically, these approaches are based on Luenberger observers, to generate residual vector and estimate the possible measured data of the system starting from accumulated information (Frank,1990; Chen-Patton,1999). Several researches based on this concept are introduced in the literature.

In the existing works, the implementation of each definition is subject to the requirement of the studied system. Whereas, in this thesis, observer-based FD approach is considered for state estimation, where some brief definitions about the state estimation and the impact to apply observers in FTC strategies are presented.

II.2.3.1.1 State Estimation

In the design of controllers scheme, availability of the information $x(k, j)$ at time k for subsystem j is very important task, to update the controller parameter in reconfiguration mechanism at $k+1$ for HS, the procedure is similar to the other nature of systems. In reality, the aspect that all state variables are measurable is not always true; as a consequence, there should be an approach to describe the systematic relation between the state variables as first part and the input and output as a second part, using an appropriate specific state-space realization. So, the alternative scenario is to provide an estimation mechanism, to estimate the state variable $x(k, j)$ from the feedback channel information's of the closed loop system, and to approximate the unknown state-variables to the desired values. This concept led to appear the notion of observers in control engineering context; that allowed an extension of the observer implementation to deal with a noisy environment, which we can use it as a filter to reduce the effect of noise on the measurement. As a result, the mechanism of estimation depends on the system to be studied. Since our concern in this thesis is to design a reliable FTC scheme that requires a full acknowledgment of the measured states, we present here the basic ideas about observers.

II.2.3.1.2 Observers for Hybrid Dynamics Systems

In FD structure, state observers take an important consideration for estimation tasks. Indeed, many FD approaches are designed in FTC based on the observer's concept to generate fault-sensitive residuals. An observer for HS is constructed based on a mathematical model that describes the inputs system $u(k,j)$, outputs system $y(k,j)$ and estimated states $\hat{x}(k,j)$. To present the observer mathematical representation, we recall model (2,2) as discrete-time dynamic systems representation for a class of hybrid systems. Thus, the state observer representation which is based on (2.2) can be described as follows:

$$\begin{cases} z(k+1) = l_\sigma(z_\sigma, u_\sigma, f_\sigma, D_\sigma, f_\sigma, T_\sigma) \\ \hat{x}(k) = p_\sigma(z_\sigma, u_\sigma, f_\sigma, D_\sigma, f_\sigma, T_\sigma) \end{cases} \tag{2.4}$$

Such that the state estimation error $e(k) = x(k) - \hat{x}(k)$ tends asymptotically to zero:

$$\|e(k)\| = \|x(k) - \hat{x}(k)\| \to 0 \qquad when \qquad k \to \infty \tag{2.5}$$

The main objective to design an observer is to define the functions $l_\sigma(z_\sigma, u_\sigma, f_\sigma, D_\sigma, f_\sigma, T_\sigma)$ and $p_\sigma(z_\sigma, u_\sigma, f_\sigma, D_\sigma, f_\sigma, T_\sigma)$ for different cases in presence of undesirable associated inputs, in order to ensure the convergence of the state estimation error to zero. Generally, there is a key point in presence of undesirable inputs, where it is not possible to reconstruct the state $x(k)$ of the system from the inputs and outputs; which is not the case in this thesis. Since we assume that the conditions under which it is possible to establish the conditions of existence of a reliable observer exist, this is called the observability notion.

A. *Observability Concept:*

The observability concept is a study that involves the determination of the conditions where the state of the system $x(k)$ can be uniquely determined, which is obtained from a set of k observations of the output. A state variable system is said to be completely observable if: for any sample time k_i, there exists a sample time $k_{i+1} > k_i$, such that a knowledge of the output *y(k)* and input *u(k)* in the time interval $k_i \leq k \leq k_{i+1}$ is sufficient to determine the initial state $x(k_i)$ and as a consequence, $x(k)$, for all k between k_i and k_{i+1}.

In the context of this thesis, necessary and sufficient conditions for a linear discrete-time HS to be completely observable can be deduced from the calculation of the rank of the Kalman criterion. Consequently, if the observability matrix

$$L_h = \begin{bmatrix} C_{(k,j)} \\ C_{(k,j)}A_{(k,j)} \\ \vdots \\ C_{(k,j)}A_{(k,j)}^{n-1} \end{bmatrix} \tag{2.6}$$

has rank n, where n is the dimension of the state variable of each sub-system. Then, the HS (2.2) is completely observable. The issue of observability for HS is investigated in the last years for its importance of hybrid dynamics controllability (De la Sen,2000; Bemporad,2000; Di Benedetto,2009; Yu,2011; Medina,2008; Tanwani,2014).

B. Structure of an Observer:

The recent structure of many observers for nonlinear systems in the literature started with (Thau,1973), that is basically designed and used Lyapunov techniques to extend the Luenberger observer (Luenberger,1971). An observer in discrete-time for HS according to the representation of (Thau,1973) is presented as follow:

$$\begin{cases} \hat{x}(k+1,j) = A_{(k,j)}\hat{x}(k,j) + f(\hat{x}(k,j),u(k,j)) + L(y(k,j) - \hat{y}(k,j)) \\ \quad \hat{y}(k,j) = C_{(k,j)}\hat{x}(k,j) \end{cases} \tag{2.7}$$

To ensure that the designed observer is reliable to implement to FTC scheme, it should define the gain matrix *L*, such that the state estimation error $e(k) = x(k) - x\hat{}(k)$ tends asymptotically to zero.

In the next chapter, we will define the necessary and sufficient conditions to determine the observer gain *L*, which is used for the computation of the optimal control. Therefore, readers can refer to (Ichalal,2009).

Continuously, to analyze different efficient techniques that are used for the reconfiguration mechanism of the controller, it is worthy to distinguish the different controller approaches that can be employed in AFTC strategy design.

II.2.3.2 Controller Designs for Active Fault Tolerant Control

In AFTC mechanism, the most important key is how to adopt an adequate controller in reconfiguration scheme. In this regard, several works were presented based on different controllers' strategies to achieve one purpose, maintain high performances of the system in case of faulty behavior event. Therefore, we cite some controller approaches that are applied in the design of an AFTC.

A. ***Eigenstructure Assignment Approach:*** this approach is raised in (Andry,1983; Konstantopoulos,1996; Tsui,1999; Wang,1999; Zhang,2000) by assigned Eigenvalues and Eigenvectors to handle robustly with occurred faults. The main idea is based on assigning the most important eigenvalues of the healthy systems and faulty systems, then minimize the error between the corresponding eigenvectors of the closed loop, e.g. using l_2. The advantage of this approach is to ensure the stability of faulty systems. Besides, the main handicap is to deal with model uncertainties and these are related to detection and estimation of faults.
B. ***Pseudo-Inverse Approach:*** is based on synthesis of feedback control law to obtain a faulty dynamics behavior approximately similar to the healthy dynamics (Gao,1991; Gao,1992; Staroswiecki,2005a; Ciubotaru,2006; Tohidi,2016). Meanwhile, the stability issue is not guaranteed for this approach.
C. ***Multiple-Models Methods:*** is based on control of nonlinear systems on functional area, where it is modeled for finite linear models based different equilibrium points. Therefore, the fault tolerant control scheme for this approach consists of computing the n controllers, that cover all linear-systems possibilities through a weighted combination of control law. Where finite linear models describe the original systems in nominal behavior

conditions and also for the faulty behavior, readers can refer to (Aubrun,1993; Theilliol,2002; Theilliol,2003). This class of AFTC controllers is divided into two categories: *Multiple Model Switching and Tuning (MMST)* (Boskovic,1998) and *Interacting Multiple Model (IMM)* (Zhang,2001).

D. ***Adaptive Control Approach:*** This approach is widely employed in control theory. As result, several AFTC strategies used an analytical model based on adaptive approach, where an online adaption of the control law is to identify and estimate at each step time the new adaption gains. This approach is mainly split in two folds: direct (Wang,1993; Labiod,2005; Essounbouli,2006; Labiod,2016) and indirect (Chen,1996; Essounbouli,2006; Labiod,2006; Bounemeur,2018).

E. ***Model Based Predictive Control:*** Basically, this approach is used to minimize a defined optimization problem of weighted quadratic criterion using a defined cost function, in order to compute online the optimal control at each sampling time. The optimization problem includes the error dynamics for trajectory tracking "desired output for regulation", the control input and imposed constraints. The main advantage of this approach is handling with different control constraints, which leads to employ this feature to deal with occurring faults for designing an FTC strategy. Due to its ability to modify online the control input parameters to maintain the required performances of the closed-loop system. These notions are involved for both aspects: academic (Maciejowski,2000; Camacho,2010; Yang,2012; Raimondo,2013; Yang,2015) and industrial (Kerrigan,1999; Maciejowski,2003; de Almeida,2010; Ferranti,2019; Tao,2020).

In this thesis, our concern is the study of an AFTC based on predictive control theory for hybrid systems. Throughout our research in the literature, it seems that this topic is not thoroughly investigated (Ocampo,2009; Wang,2016). Therefore, why model predictive control is not thoroughly discussed for designing an active fault tolerant control?

II.3 Fault-Tolerant Predictive Control for Hybrid Systems

The opinions about employing the MPC approach in FTC are differentiated. Some searchers doubt the reliability question of MPC; they are rather lean on that AFTC problems require an online fast and ultimate power of computing, due to the computation burden for designating the optimal control, which is not always the case for MPC; that, only the first element of input control vector has relevance for next iteration. In addition to the weakness recognition of fault model and its impact on the systems, and the inability to deal with all kinds of faults.

To answer this question, we start first by an overview about the MPC and its favorable features to adapt for HS.

II.3.1 Predictive Control Scheme for Hybrid Systems

Generally speaking, the concept of MPC is not a specific control strategy but rather a set of algorithms, that explicitly uses a system model in an optimization problem to be solved; in order to compute the optimal control sequence, satisfy the imposed constraints and improve the system performances. These latter are based on formulating sufficient and less conservative conditions through a specific optimization criterion on a finite or infinite horizon at each step-time. This specific aspect of the optimization criterion is related to the terminal constraints and undesirable associated inputs as time-delay, disturbances and faults occurring.

Historically, Model (Based) Predictive Control (MPC/MBPC) concepts were introduced in control theory due to Richalet in 1978 (Richalet,1978), and were generalized to the industrial field by Clarke in 1987 (Clarke,1987). Over the years, the MPC design is enhanced with the increase of systems complexity, due to is well-suited for the control of constrained systems, at which the outputs and inputs constraints are directly underlying on the optimization problem. With the aim to keep up high systems performances, several studies and control approaches are proposed for this concept (Morari,1994; Scokaert,1999; Camacho,2004). In order to achieve the robustness performances, a new sight of optimization problem in term of LMIs was proposed based on sufficient conditions for constrained systems (Kothare,1996). Followed by numerous researches, to increase the MPC efficiency and ensure the stability of different paradigms for many applications based on nonlinear systems modelling techniques (Maciejowski,2002; Hu,2004; Wada,2006; Scholte,2008; Vesely,2009; Belarbi,2007; Xia,2010; Khairy,2010). Due to difficulty control of HS by using the classical control scheme; the effectiveness and robustness are mainly the target of an objective control design to maintain hybrid systems performances; where the MPC is considered one of the well-suited strategies for the control of HS (Potocnik,2008; Bemporad,2000; Lazar,2006; Li,2009; Camacho,2010; Phat,2010; Altin,2018; Lien,2020; Zahaf,2020).

In fact, based on the above numerous researches and those that are not mentioned in the present thesis, Predictive control boils down using an explicit model to predict the system behavior at each sampling-time, at least over a definite horizon called the prediction horizon, by choosing the best cost decision over the optimization problem to designate the input control by respecting the imposed constraints. Usually, only the first input control element has been taken from the computed input control vector, to use it in the next sampling period of the optimization problem and to compute again the new input control with new parameters obtained from the system. This procedure is then repeated: it is the principle of the sliding or receding horizon (Figure 2.4).

Basically, the model predictive control denotes a general framework of a various generic names of predictive control (Maciejowski,2002), we mention below as examples of these plethora names:

- Sequential Open Loop Optimization (SOLO),
- Dynamic Matrix Control (DMC),
- Quadratic Dynamic Matrix Control (QDMC),
- Model Algorithmic Control (MAC),
- Extended Prediction Self-Adaptive Control (EPSAC),
- Predictive Functional Control (PFC),

- Generalized Predictive Control (GPC),

In addition to this unified description, Model-Based Predictive Control (MBPC) is considered as a different designation to denote different variants of predictive control. Generally, predictive control is constituted from the next basic elements:

- Prediction Model,
- Cost function to minimize the imposed constraints,
- An optimization algorithm to compute the control input.

Each element has several considerable optimization options which results into several optimization techniques. So, this leads to a variety of predictive control algorithms.

II.3.1.1 Prediction Model

Two components are the main elements of the prediction model: the first one is the mathematical equations that describes the relation between the inputs and outputs of the modelling process; besides, constraints, faults, disturbances and modelling errors are the second element. As a result, there are many paradigms of predictive control based on the prediction model:

- Linear Predictive Control based on: State Space Model, Transfer Function… etc.
- Non-Linear Predictive Control based on Nonlinear State Model … etc.

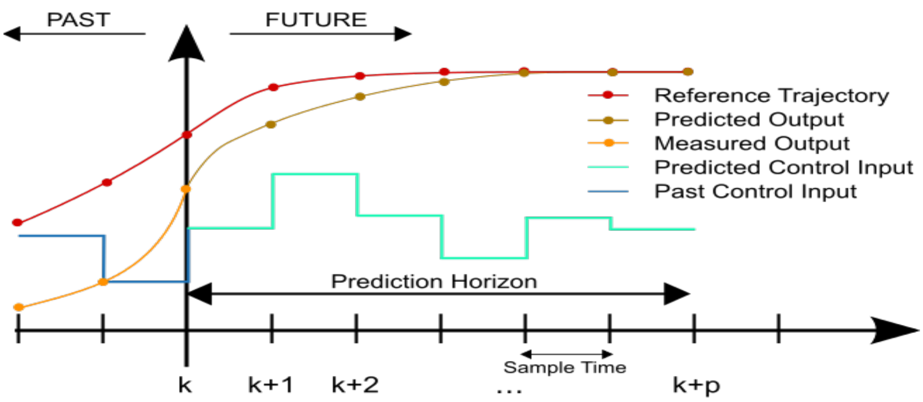


Figure 2.4: Model Predictive Control Principals

II.3.1.2 Cost Function (Performance Criterion)

The cost function penalized the big changes between the controlled predicts outputs $\hat{y}(k+t|k)$ and the reference trajectory $y_{ref}(k+t|k)$, in addition to the input control vector $\Delta u(k) = u(k) - u(k-1)$. Usually cost function is given by the next expression:

$$J = \sum_{i=1}^{\infty} \Big[\big(y_{ref}(k+i) - \hat{y}(k+i|k) \big)^T Q \big(y_{ref}(k+i) - \hat{y}(k+i|k) \big) + u_i(k+i-1)^T R u_i(k+i-1) + \Delta u_i(k+i-1)^T S \Delta u_i(k+i-1) \Big] \quad (2.4)$$

$y_{ref}(k)$: ***i****th* Reference output variable. (Desired State)

$\hat{y}(k)$: ***i****th* Controlled output variable (measured State)

u_i : ***i****th* Control Input.

Δu_i : ***i****th* Step control Input

$Q_{\hat{y}}$: Weighting coefficient or Matrix reflecting the relative importance of Controlled variables.

R_{u_i} : Weighting coefficient or Matrix penalizing the big changes of Control Input.

$S_{\Delta u_i}$: Weighting coefficient penalizing the big changes of Step Control Input.

Note: Q is definite positive matrix, R and S are semi definite positive matrices.

Since that, only the first element of the Input control vector is applied in next computation of optimal control at each step-time, we consider:

$$\Delta u(k+i-1) = 0, \qquad i > 1 \quad (2.5)$$

II.3.1.3 Input Control

The input control is obtained from the optimization problem, where the prediction model is underlying to the cost function to satisfy the imposed constraints, undesirable associated inputs (time-delay, disturbances and faults) and ensure the stability of system. The computation of general prediction model is presented in ***Appendix B.***

II.3.2 Optimization Techniques and Stability of Constrained Predictive Control for Hybrid Systems

The model to be used in control design scheme is considered as a discrete-time state-space representation. In a general case, we assume that the underlying plant is a multi-input and multi-output system. For SISO or MISO systems, it is easier to design a control scheme by a simple manipulation compared with MIMO plant, which can be considered as hybrid systems for many cases, due to the similarity of the mathematical representation. The described discrete-time state space model is:

$$\begin{cases} x(k+1,j) = A_{(k,j)}(v)x(k,j) + B_{(k,j)}(v)u(k,j) \\ \quad y(k,j) = C_{(k,j)}(v)x(k,j) \end{cases} \tag{2.6}$$

Where $x(k+1,j)$ is the state variable, $y(k,j)$ is the system output and $u(k,j)$ is the input control, j represents the j^{th} sub-system (model), for simplicity of state matrices e.g., $A_{(k,j)}(v) = A_{(k,j)}$ in the rest of this thesis.

Obviously, the main objective of predictive control is to bring the predicted states as close as possible to the reference signal over a prediction horizon. Therefore, we distinguish two aspects of optimization based predictive control: classical optimization methods for model based predictive control and robust predictive control using *min-max* optimization method.

II.3.2.1 Optimization Problem Based on Model Predictive Control

Optimization problem using model predictive control is based on redundancy solutions depending on the systems nature, constraints and undesirable associated inputs. In existed works, an analytical solution is obtained if the study model is linear and there are no constraints (Clarke,1987), the problem is cast as a quadratic problem in the presence of constraints, where a set of algorithms is proposed as an efficient solution (Maciejowski,2002).

On the other hand, the optimization problem becomes non convex if the model is nonlinear with or without constraints, which increases the complexity of the solution procedures and time consuming (Mayne,2000). For some cases, authors in (Scokaert,1999) raised that only a feasible and stable solution may be sought for a class of nonlinear systems. Therefore, to set up the steps to obtain an optimal solution if it is possible, we recall the cost function (2.4), which can be described as:

$$J = \sum_{i=1}^{N_h} \Big[\big(y_{ref}(k+i) - \hat{y}(k+i|k) \big)^T Q \left(y_{ref}(k+i) - \hat{y}(k+i|k) \right) + u_i(k+i-1)^T R u_i(k+i-1) \Big] \tag{2.7}$$

In addition to the prediction model (B.8) in ***Appendix B***, which applies for computing an input control u_i that minimizes the cost function J in quadratic criterion, we obtain:

$$J = \sum_{i=1}^{N_h} \Big[\big(y_{ref} - \Psi_h x(k) + \Lambda_h u_i \big)^T Q \big(y_{ref} - \Psi_h x(k) + \Lambda_h u_i \big) + {u_i}^T R u_i \Big] \tag{2.8}$$

From the literature, (2.8) has two cases of solution synthesis:

II.3.2.1.1 Solutions of MPC without Constraints

The solution is being analytical. At first, we take the first derivative of the cost function (2.8) with respect to the vector of inputs u_i,:

$$\frac{\partial J}{\partial u_i} = -2{\Lambda_h}^T Q^T \left(y_{ref} - \Psi_h x(k) \right) + 2\big({\Lambda_h}^T Q \Lambda_h + R^T \big) u_i \tag{2.9}$$

Following steps in (Clarke,1987; Wang,2009) to obtain the optimal solution of J, necessary condition is obtained as: $\frac{\partial J}{\partial u_i} = 0$ (2.10)

The optimal solution for the control input is defined as:

$$u_i = \big({\Lambda_h}^T Q \Lambda_h + R^T \big)^{-1} {\Lambda_h}^T Q \left(y_{ref} - \Psi_h x(k) \right) \tag{2.11}$$

Finally, many algorithms are available to solve this kind of optimization problems, which is basic to guarantee the optimality solution, wherein the set of feasible solutions of the cost function is a convex problem.

Since the most engineering applications are a constrained problem, the solution of the optimization problem becomes a quadratic programming problem.

II.3.2.1.2 Solutions of MPC with Constraints

The optimization based quadratic programing methods requires a considerable effort to completely understand the relevant theory and algorithms, which have been extensively studied in the literature

$$J = \frac{1}{2}\tilde{x}^T P \tilde{x} + \tilde{x}^T R \tag{2.12}$$

$$W\tilde{x} < Z \tag{2.13}$$

Where P, R, M and Z are compatible matrices and vectors in the quadratic programming problem, where P is symmetric and positive definite matrix, and where W and Z are representing the upper and lower energy limits, respectively.

Therefore, we present the essential computational algorithms, methods and conditions to better understand the essence of quadratic programing problems optimization, which is extensively raised with descriptive examples in (Wang,2009).

A. Quadratic programing for Equality Constraints

The obtained solution from the minimization with equality constraints consists of the number of constraints which should be less than or equal to the number of decision variables (*i.e.,* $\tilde{x}$ *in 2.12*). Since the reformulated of the quadratic programming problem to a simplest problem is the way to find a feasible solution, by defining a constrained minimum of a positive definite quadratic function with linear equality constraints. That each linear equality constraints defines a hyperplane, in addition to the positive definite quadratic functions that are represented as hyperellipsoids.

Therefore, the existed feasible solution from the minimization of the cost function without any additional variable is obtained which satisfies the constraints. Besides that, in case where the number of equality constraints is greater than the number of decision variables, we face the situation called infeasible, where there is no feasible solution that can satisfy the constraints.

In this method, the most used technique is based on ***Lagrange Multipliers*** represented by Lagrange expression in the cost function as follows:

$$J = \frac{1}{2}\tilde{x}^T P\tilde{x} + \tilde{x}^T R + \lambda^T (W\tilde{x} - Z) \tag{2.14}$$

Since that (2.14) is subject to (2.13) with additional variable λ, the objective is to satisfy the objective function (2.14) as in the original cost function (2.12).

B. *Quadratic programing with Inequality Constraints*

Contrary to the QP with equality constraints, the minimization with inequality constraints requires that the number of constraints should be larger than the number of decision variables. The general idea about inequality constraints (2.13) can take the form of inactive constraints and active constraints during the iteration procedure. For the active constraints case, the solution of the QP problem is more labors and requires efficient set of algorithm compared with inactive constraints case. The most used methods for QP with inequality constraints are *Kuhn-Tucker conditions* and *Active set methods*.

- ***Kuhn-Tucker Conditions***: This method is based on the formulation of *Lagrange multipliers,* by underlying the active and inactive constraints in the cost function, that necessary conditions are derived for the QP optimization problem based on *Kuhn-Tucker conditions*. Since the set of active constraints is related to the *Lagrange multipliers* vector λ, let us define the set of active constraints S_{AConst} to distinguish it from inactive constraints set. Then the necessary conditions can be described as follows:

$$P\tilde{x} + R + \sum_{i \in S_{AConst}} \lambda_i \, W_i^T \tag{2.15}$$

$$W_i\tilde{x} - Z_i \leq 0 \qquad i \in S_{AConst} \tag{2.16}$$

$$W_i\tilde{x} - Z_i = 0 \qquad i \notin S_{AConst} \tag{2.17}$$

$$\lambda_i \geq 0 \qquad i \in S_{AConst} \tag{2.18.a}$$

$$\lambda_i = 0 \qquad i \notin S_{AConst} \tag{2.18.b}$$

- ***Active Set Methods***: This method is based on actively defined set. This means that in each iteration of an algorithm, only the actual active constraints are considered for the optimization problem as a subset of the constraints, which allow them to be treated as the active set with relaxed conditions. This technique can allow us to obtain an optimal solution for QP problem, where the current solution using i^{th} subset of constraints is a feasible solution. This leads to improve the feasible solution over the prediction horizon to the optimal solution. We can differ two cases of solution based the Active Set method that is related to *Lagrange multipliers* vector λ_i: if all $\lambda_i \geq 0$ then the value point is a local solution to the original problem. On the other hand, if there exists a $\lambda_i < 0$ then the cost function can be decreased by relaxing constraint i. Also, it is necessary to take into consideration the unapplied constraints at each step time to avoid big penalizing of input control computation, since each solution is defined by the algorithm which must be feasible; where these unapplied constraints can occur while moving on new working area over the prediction horizon.

For further understanding of the difference between the QP with inequality constraints and equality constraints, a short comparison for both methods is introduced in (Wang,2009). The main difference comprises that the optimization through the inequality constraints is less conservatism than the case of equality constraints, due to its advantage which depends on the linearity feature of the inequality constraints that obtained based on additional decision variables. Moreover, the optimal solution in the case of inequality constraints it could be obtained, compared with the other case which considers the feasible solution as enough solution.

C. *Quadratic programing Based on Primal-Dual Method*

The *Primal Methods* is considered as a basic aspect of active techniques, as the *Active Set method;* where the computation of the solution is related to the decision variables (Primal Variables). It is clear that the computation load is quite large in the case of many constraints, which can happen in the active set methods; since it requires the identification of the active constraints along with the optimal decision variables, as well as the programming complexity task. Therefore, the inactive constraints can be ignored in the computation of the solution at each iteration, which is identified systematically based on the *Dual Method*. This method can lead to designating the optimal solution of the constrained minimization problems. The Primal-Dual method uses the Lagrange multipliers, where the original problem optimization is reformulated to a dual problem to be optimized. Thus, the dual problem is also a quadratic programming problem with λ as the decision variable, the optimization of dual objective function minimized by Lagrange multipliers is described as follows:

$$\min_{\lambda \geq 0}\left(\frac{1}{2}\lambda^T S\lambda + \lambda^T H + \frac{1}{2}Z^T P^{-1} Z\right) \tag{2.19}$$

$$\text{subject to } \lambda \geq 0$$

The matrices S and H are given by:

$$S = W^T P^{-1} W \tag{2.20}$$

$$H = Z + W^T P^{-1} R \tag{2.21}$$

The obtained dual objective function (2.19) is derived from (2.14) with some mathematical transformation using (2.20), (2.21) and additional variables. To solve the dual problem that can be easier than the primal problem, due to the transformation of constraints. Many techniques are proposed to handle this optimization problem; As an example of this algorithms, we cite the *Hildreth's Quadratic Programing Procedure* and *Closed-Form Solution of* λ^*, as proposed methods to solve the optimization of dual problem. Both procedures are based on the optimization of the decision variable λ. For more details about mentioned methods, readers can refer to (Wang,2009).

As a brief conclusion, the above solutions of the optimization problems based model predictive control is not competent, especially in the case that the study problems is with constraints; it is clearly that the main issue is the large computation burden which cannot always guarantee the optimality solution. while finding a feasible solution is enough for the optimization problem over the prediction horizon.

II.3.2.2 Robust Predictive Control Based on LMIs

The optimization of robust predictive control is introduced by (Kothare,1996) to handle with plant model uncertainties, in that the robustness is ensured. The main idea of this optimization method is based on the reformulation of the optimization problem to a convex problem, at which an upper bound on the "worst-case" objective function is minimized subject to the input and output constraints. So, the obtained constrained convex problem is reformulated in terms of Linear Matrix Inequalities (LMIs).

The LMIs technique makes it possible to solve and synthesize the control of several problems and nonlinear systems, due to the existence of efficient methods of solution. The LMI is basically the version of semi positive definite (SDP) algorithms, which are convex problems (El Ghaoui,1997). Based on an interior point method for linear programming, an efficient procedure as a solution of LMI was introduced in (Karmakar,1984). The direct integration of the interior point method with linear matrix inequalities was made by (Nesterov,1994) as an improvement of this technique, which allows to generalize the LMIs as programming algorithms in Matlab (Gahinet,1995). The real trend of involving LMIs in control systems started with (Boyd,1994), due to the soft underlying of the constraints and to decrease the conservatism optimization problems in terms of linear conditions. Therefore, it is necessary to highlight this concept of LMIs.

II.3.2.2.1 The Linear Matrix Inequalities

This part presents basic definitions and some useful lemmas and assumptions that allow the reformulation of the matrix inequalities in order to make them in a linear form.

Definition 2.1: (Boyd,1994)

A linear matrix inequality is a matrix inequality in the following form:

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0 \tag{2.22}$$

with $x(t) = [x_1(t), \dots, x_m(t)]^T$ *is the vector of the variables to be found, and symmetric matrices defined as* $F_i = F_i^T \epsilon R^{n*n}$ *,* $i = 0, \dots, m,$ *are given matrices. For an existing solution of inequality (i.e., Eq. LMI1),* $F(x)$ *must be positive definite, i.e., all eigenvalues are positive. So, LMI (i.e., Eq. LMI1) is a convex constraint for* $\boldsymbol{x}$*, and the set* $\{x|F(x) > 0\}$ *is convex. That allows to consider that (i.e., Eq. LMI1) is a diagonal matrix if* $F_1(x) > 0, F_2(x) > 0, \dots, F_m(x) > 0$*, and these constraints are convex. Thus, (2.22) is equivalent to the next expression:*

$$\begin{bmatrix} F_1(x) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & F_m(x) \end{bmatrix} > 0 \tag{2.23}$$

A. <u>Basic Problems of LMIs</u>

There are three types of convex optimization problems encountered in the form of LMI:

- ***Feasibility Problem:***

Is based on how to find a vector $x(t) = [x_1(t), \dots, x_m(t)]^T$ such as $F(x) > 0$. This problem is generally solved by finding the vector $x(t) = [x_1(t), \dots, x_m(t)]^T$ that minimizes the scalar $\boldsymbol{\gamma}$ such as :

$$\boldsymbol{-F(x) < \gamma * I} \tag{2.24}$$

If the minimum value of $\boldsymbol{\gamma}$ is negative, then the problem will be feasible.

- ***Eigenvalue Problem EVP :***

The idea is to minimize the worst case "upper value" eigenvalue of a symmetric matrix with constraint in term of LMI:

$$\begin{gathered} \textit{Minimize}\ \lambda \\ \textit{subject to constraints} \begin{cases} \lambda I - A(x) > 0 \\ B(x) > 0 \end{cases} \end{gathered} \tag{2.25}$$

- ***General Eigenvalue Problem (GEVP) :***

The idea also is to minimize the worst case "upper value" generalized eigenvalue of a pair of matrices with respect to the constraints in term of LMI:

$$\begin{gathered} \textit{Minimize}\ \lambda \\ \textit{subject to constraints} \begin{cases} \lambda B(x) - A(x) > 0 \\ B(x) > 0 \\ C(x) > 0 \end{cases} \end{gathered} \tag{2.26}$$

Usually, decreasing the conservatism of the optimization problem requires making all LMIs in the linear form, which is not often the case for all formulated convex problems. Therefore, some useful lemmas as tools for transformations and manipulation of LMIs, for relaxation conditions are presented in ***Appendix C***.

II.3.2.2.2 The Design of Robust Predictive Control Based on LMIs Using Min-Max Method

As aforementioned, the formulation of MPC scheme in LMIs form was introduced by (Kothare,1996). That allows reformulating any optimization problem to a convex problem in term of linear matrix inequalities (LMIs), and deriving sufficient conditions for the computation of a feasible solution using the formulation *min-max* as optimization method. The optimization approach *min-max* permits to modify the minimization of the nominal cost function to an oriented optimization problem, using the cost function of the worst case "cost function with maximum undesirable inputs values". The *min-max* formulations are considered as linear programming "LP", to decrease the computation burden compared with QP methods. However, the standard cost function that is used in optimization over infinite horizon for robust model predictive control based on LMIs is described as:

$$\min_{u(k+i/k)} \max_{i>0} J_\infty(k) \tag{2.27.a}$$

$$J_\infty(k) = \sum_{i=0}^{\infty} \left[\hat{X}(k+i) + U(k+i)\right] \tag{2.27.b}$$

$$\begin{cases} \hat{X}(k+i) = x^T(k+i/k)Q_0x(k+i/k) \\ U(k+i) = u^T(k+i/k)R_0u(k+i/k) \end{cases} \tag{2.27.c}$$

According to (2.27), the stability of the system is related to the computed input control, wherein the necessary and sufficient conditions are obtained after the reformulation of the studied system to a convex problem, based on the optimization *min-max* method; by including the constraints and undesirable inputs in the cost function using the Quadratic Lyapunov function. To guarantee that the proposed control scheme is able to stabilize the closed loop systems.

II.3.2.2.3 The Guarantees of the Stability of Robust Predictive Control

The stability of the closed loop system is hardly ensured when designing an off-line predictive controller; where tuning the parameters in the problem formulation is quite a solution for this issue. This solution is a nominal stability, that is only valid in case of healthy systems without any constraints or undesirable inputs, which is not the case in real and recent applications. However, the dynamic performances are very sensitive and challenging concerning the stability using predictive control scheme, which is being more challenging for on-line reconfiguration mechanism in case affected by undesirable events and existing imposed constraints. Based on the discussion in (Maciejowski,2002) about the influence of some features of using the optimization *min-max* method over an infinite horizon with constraints; we present the following concepts as relevant points to ensure stability of systems using predictive control.

A. Do the Constraints Support or Destroy the Stability?

According to the study of (Maciejowski,2002), to achieve forces the state variables to take a specific value at the end of the prediction horizon, using a stabilizable control scheme is done by adding some terminal constraints. The controller stabilizes the constrained system by deriving some sufficient conditions by optimizes the cost function over infinite horizon using Lyapunov function.

B. *Infinite Horizons of MPC and Stability*

This concept is also analyzed by (Maciejowski,2002) derived from existed works. The differences are about the obtained solution in optimization problems, whether using an infinite or a finite horizon. The main difference concerns the obtained optimal solution that is computed from the optimization problem. The result over infinite horizon is the same as the performed optimal trajectory from the start; while in the case of finite horizon, the new computed optimal solution over a new finite horizon is completely different from the one computed at the earlier step according to (Maciejowski,2002); where he shows a particular case for which closed-loop stability can be established despite the use of a finite horizon.

For a complete stability concept, a stabilizable controller is performed from the optimization problem using necessary conditions based the Lyapunov function. However, the stability that is based on Lyapunov function is the ability of the designed controller to carry on the system to the equilibrium point. The next definition approaches the concept of stability as:

- The equation $x(k+1) = f\big(x(k), u(k)\big)$ has an equilibrium point (fixed value) at state x_0 and input $x_0 = f(x_0, u_0)$.
- According to the definition of (Maciejowski,2002), for the nonlinear system, one has to consider the stability of a particular equilibrium point, rather than the system. (Some equilibria may be stable, others unstable). In the sense of Lyapunov, an equilibrium point (x_0, u_0) is stable if a small perturbation of the state or input results in a continuous perturbation of the subsequent state and input trajectories. More precisely, given any $\varepsilon > 0$, there is $\beta > 0$ (which depends on $\varepsilon > 0$ and the system) such that if $\|[{x_0}^T, \quad {u_0}^T]\| < \varepsilon$ then $\|[{x_k}^T, \quad {u_k}^T]\| < \beta$, for all $k > 0$. Then we can say that the system is *Asymptotically Stable* if

$\|[x_k{}^T, \quad u_k{}^T]\| \to 0\ as\ k \to \infty$. Extension of this notion to closed loop system with feedback control law is presented in Lyapunov definition.

- *Lyapunov's Theorem:*

  If there is a function $V(x,u)$ which is positive-definite, namely such that $V(x,u) \geq 0$ and $V(x,u) = 0$ only if $(x,u) = (x_0, u_0)$ and has the decreasing property along any trajectory for $x(k+1) = f\big(x(k), u(k)\big)$ as follows:

  $$\|[x_{i+1}{}^T, \quad u_{i+1}{}^T]\| < \|[x_i{}^T, \quad u_i{}^T]\| \Longrightarrow V(x_{i+1}, u_{i+1}) \leq V(x_i, u_i), \text{ for } i > 0$$

  Then (x_0, u_0) is a stable equilibrium point. If, in addition $V(x(k), u(k)) \to 0\ as\ k \to \infty$. The closed loop system is Asymptotically Stable.

Based on the previous concepts, (Kothare,1996) proposed sufficient and necessary conditions to compute a feasible solution, that can stabilize an uncertainty system, which is reformulated from the beginning to a convex problem without constraints, and optimized through *min-max* formulation in terms of LMIs. The obtained conditions are presented as follows:

$$\min_{\gamma,P} \gamma \tag{2.28.a}$$

$$\begin{bmatrix} 1 & x^T\,(k/k) \\ x\,(k/k) & Q \end{bmatrix} > 0 \tag{2.28.b}$$

$$\begin{bmatrix} Q & (A_iQ - B_iY)^T & Q^T {Q_0}^{1/2} & Y^T {R_0}^{1/2} \\ (A_iQ - B_iY) & Q & 0 & 0 \\ {Q_0}^{1/2} Q & 0 & \gamma I & 0 \\ {R_0}^{1/2} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{2.28.c}$$

II.3.3 Validity of the Robust Stable Predictive Control for Constrained Hybrid Systems using Fault-Tolerant Control

Over the last parts we presented different approaches for optimization of predictive control. The constrained HS with undesirables' associated inputs is not thoroughly investigated in this chapter, since it requires an in-depth synthesis to deal with many aspects in one approach.

Therefore, to answer the question of the validity of using robust predictive control in terms of LMIs for faulty HS with undesirable associated inputs and constraints, the next chapter tries to raise the design of AFTC scheme in different stages and cases of faults.

II.4 Conclusion

In this chapter, the necessary concepts were presented as keys for designing a robust reliable AFTC scheme; in additions to an overview on the predictive control theory. Different phases to compute the input control using a specific cost function and prediction model were analyzed, for nominal cases, defined parameters and basic conditions. This was preceded by a description of the main stages and steps to design a reliable active fault tolerant control for hybrid and complex systems, where an elaboration of undesirable associated inputs, fault diagnosis scheme and controller's strategies concepts are introduced. These controllers, in this thesis, are namely a predictive control using optimization approaches based on *min-max* formulation in term of LMIs.

Part II

Optimal Fault Tolerant Control Based on Predictive Control Theory of Constrained Hybrid Systems

Chapter III

Robust Optimal Active Fault Tolerant Predictive Control for Hybrid Systems with Time-Delay: Theoretical Results

In this chapter, based on the main results in our published works, the existence of optimal and near-optimal controls for the hybrid systems control problem are established with constraints and undesirable associated inputs (time-delay and faults), by passing from fault diagnosis to the robust fault tolerant control based on predictive control theory. First, an optimization framework based on model predictive control combined with an observer is presented, as a coupled observer-MPC as an online approach to define necessary conditions for the computation of optimal solution. Therefore, the proposed HFTPC control design is presented in terms of LMIs, by deriving these conditions to ensure the robust stability of the overall closed-loop hybrid system, that are composed of estimated state and estimation error dynamics. Furthermore, the stability and robustness are ensured and derived from a less conservative theoretical results in terms of LMIs by using a Lyapunov-Krasovoskii candidate function, to satisfy the required performances that underlie the cost function associated with the minimization problem "min-max method". Then, aiming to show the outperformance of the proposed HFTPC compared to a standard quadratic programming method (QP) with MPC in actuators/sensors fault scenarios, the QP is used to derive a reliable algorithm for fault diagnosis and fault-tolerant control to define stabilizable optimal control law. Reported simulation results show the effectiveness of the proposed approach.

III.1 Introduction

In our best knowledge, dealing with faults occurring and time-delay issues at the same time for some classes of HS is not raised in the literature. Thus, the fault tolerant predictive control for hybrid systems with time-delay was not investigated until now. In this chapter, a new insight is presented to compute the optimal control; two schemes have been proposed for fault-tolerant based on predictive control theories, that combined with the state observer of the continuous dynamic, known as coupled observer-MPC as an online approach to guarantee the observability conditions (Zahaf,2017). Relying on this aspect, some different objectives designs are raised in this thesis, the obtained results are presented for different cases: time delay, actuators faults, sensors failures and for both issues. Mainly, we converted the studied system to an augmented problem using the states variables and output tracking error variables of the original process. Besides, the computation of the proposed optimal control design is based on *min-max* optimization criterion in terms of LMIs; in which, two control schemes are combined. The first is a robust MPC investigated to cope with time-delay, and the second component is introduced to deal with faults occurring (actuators, sensors and process faults) and external disturbances; that allowed implementing a reliable control scheme of the dynamic system, the proposed control design is a Hybrid Fault-Tolerant Predictive Control (HFTPC) (Zahaf,2020), which does not only guarantee the convergence and tracking performances of the system, but also offers more degrees of freedom to design the controller. However, the Lyapunov-Krasovskii (L-K) theory is employed to manage the time-varying delays, due to its ability to reflect the original state-space system. In addition, less conservatism on stability conditions, in terms of LMIs, is obtained to synthesize the controller and then the computation of the optimal solution, to guarantee the robust asymptotical stability of the constrained closed-loop system. These conditions are obtained by the minimization of the studied problem subjected to specific constraints at each sampling instant; then, the control action is computed by solving an online constrained optimization problem (OCOP) that minimizes the upper bound of the “worst-case” performance index, in terms of LMIs. In order to highlight the efficiency of the proposed HFTPC approach, we present the classical optimization method of model predictive control "QP" to compute a feasible solution as a comparison study in the next chapter.

Therefore, this chapter is divided into four sub-sections. In the first part, preliminaries and definitions are presented. Then, the next part is devoted to the study of the observability concept for hybrid systems based on coupled observer-predictive control theory in terms of LMIs. The third section shows the proposed approach of fault-tolerant based model predictive control for HS using QP optimization techniques. A conclusion ends this chapter, which is preceded by a presentation of the obtained results for fault-tolerant predictive control with constraints for delayed hybrid systems in term of LMIs; while the robustness is ensured based on less conservative conditions derived from the *min-max* optimization problem using Lyapunov's functions.

III. 2 Background and Preliminaries

In this section, some concepts and definitions of relevant dynamics models are presented. In which the state space representation is obtained from an appropriate modelling approach, depending on the nature and the category of the system. So, the problem formulation and the proposed control approaches raised in this chapter are set up for the classes of discrete-time hybrid systems. Basically, we define the following healthy discrete-time hybrid system in the ideal behavior case:

$$\begin{cases} x(k+1,j) = A_{(k,j)}x(k,j) \ + B_{(k,j)}u(k,j) \\ y(k,j) = C_{(k,j)}x(k,j) \\ \sigma(k) = \eta\big(x(k,j),u(k,j)\big) \end{cases} \tag{3.1}$$

Where $k \; and \; j$ are time step and sub-system index; $A_{(k,j)}, B_{(k,j)} \; and \; C_{(k,j)}$ are state matrices of sub-systems j, $x(k,j) \in S^n, u(k,j) \in S^1$, and $y(k,j) \in S^1$ denote the state, input and output of the HS at time step k for the j^{th} subsystem, $\sigma(k)$ represents the general transition "switching" function for some classes of HS.

In real applications, HS in (3.1) is a representation of variety of hybrid systems, it can be MIMO or MISO paradigms, with logical rules for the discrete dynamic.

For simple notation, (3.1) can be represented throughout this thesis as follow:

$$\begin{cases} x(k+1) = A_j x(k) + B_j u(k) \\ y(k) = C_j x(k) \\ \sigma(k) = \eta\big(x(k,j), u(k,j)\big) \end{cases} \tag{3.2}$$

Where, the classical feedback control law is defined as:

$$u(k) = K_j x(k) \tag{3.3}$$

Definition 3.1.

Based on our description of HS in chapter one, the transition function σ(k) between sub-systems is according to the studied framework of hybrid system.

Based on real control applications, in many cases, it is not often that (3.3) is able to deal with the undesirable associated inputs at reasonable time, due to incapability of the system to identify the outputs variables in the feedback channel. To compensate this issue, a reconfiguration of the output states is widely used in control theory based on the concept of state estimation. To estimate unmeasurable states or unknown variables, several conditions are proposed to build a reliable observer, that has the ability to estimate an approximate state values that are required for computing the control input.

On that wise, we try to design a control law for system (3.2), in order to compensate the faults effects, provide a stable trajectory tracking for the state variables and eliminate the undesirable influences of unknown inputs. Thus, to estimate the state in the feedback channel, an observer is designed for the uncertain hybrid discrete-time system as:

$$\begin{cases} \hat{x}(k+1) = A_j \hat{x}(k) + B_j u(k) + L\big(y(k) - \hat{y}(k)\big) \\ \quad \hat{y}(k) = C_j \hat{x}(k) \end{cases} \tag{3.4}$$

Where, the control estimation to be generated on the controller side is:

$$u(k) = K_j \hat{x}(k) = F_j G^{-1} \hat{x}(k) \tag{3.5}$$

Through this manuscript the error dynamics are defined as:

$$e(k) = x(k) - \hat{x}(k) \tag{3.6}$$

From (3.2), (3.4) and using (3.5), (3.6), a classical problem formulation is obtained in (3.7),

$$\begin{cases} \hat{x}(k+1) = (A_j + B_j K_j)\hat{x}(k) + LC_j e(k) \\ \hat{y}(k) = C_j \hat{x}(k) \\ e(k+1) = (A_j - LC_j)e(k) \end{cases} \tag{3.7}$$

The obtained problem in (3.7) is implemented in many studies for a reconfiguration mechanism based on the results in (Kalman,1960; Luenberger,1971); in which in the next section, it is used to provide necessary conditions for the reconstruction of the uncertainties of the hybrid systems.

Assumption 3.1.

The pair of each subsystem $(A_{(k,j)}, B_{(k,j)})$ is completely controllable and the pair $(A_{(k,j)}, C_{(k,j)})$ is completely observable if and only if the set $(K_{(k,j)}, L)$ is stabilizable.

To be more precise, the values of the discrete dynamic is not predicted, thus the following assumption is useful.

Assumption 3.2

The future values of the discrete dynamic function $\sigma(k)$ (the General transition "Switching" function for different framework of HS) cannot be predicted; however, its instantaneous value is known.

In order to optimize and compute the optimal control input based on robust predictive control theory, we consider the following problem by minimizing the given objective function in an infinite horizon as follows:

$$\min_{u(k+i/k)} \max_{i>0} J_{\infty}(k) \tag{3.8. a}$$

$$J_{\infty}(k) = \sum_{i=0}^{\infty} [X(k+i) + U(k+i)] \tag{3.8. b}$$

$$\begin{cases} X(k+i) = x^T(k+i/k) Q_0 x(k+i/k) \\ U(k+i) = u^T(k+i/k) R_0 u(k+i/k) \end{cases} \tag{3.8. c}$$

The motivation to use (3.8) is related to the *min-max* problem known as a solution of the global optimization problem, and is translated from the optimization of the worst case scenario. In fact, the worst case is defined as a maximum in the uncertain space and the optimal design corresponds to the minimum of all the maxima in the design space. Moreover, the maximization is over the set $\Omega 1$ and set $\Omega 2$, and corresponds to choosing that time-varying space $[A(k+i)\ B(k+i)] \in \Omega 1,\ and\ [A+B_p \Delta C_q\ \ B+B_p \Delta D_{qu}] \in \Omega 2,\ i \geq 0.$, which, if used as a 'model' for predictions (where Ω1 is a healthy time-varying space and Ω2 is faulty time-varying space). Would lead to the largest or "worst-case" value of $J_\infty(k)$ (Upper bound) among all plants in Ω1 and Ω2. In this manuscript, we proposed a control scheme by using (3.8) in order to designate the optimal solution. As a result, we address the studied problem by first deriving an upper bound on the robust performance objective, where derivation of the upper bound considers a quadratic function. Then, we minimize this upper bound with the proposed control law in worst-case at each sampling time k, by converting the studied problem to a convex optimization problem in terms of linear matrix inequalities (LMIs). Further details are found in (khothare,1996).

The guaranteed cost function $J(k)$ related to the Lyapunov function is obtained, if the following stability is satisfied:

$$V\big(x(k+i+1/k)\big) - V\big(x(k+1/k)\big) \leq -[X(k+i) + U(k+i)] \tag{3.9}$$

The objective cost function (3.8.c) is related to the required performances objectives to achieve the optimality solution, in addition of the dynamic framework for each studied system.

III.3 Observer-Based Model Predictive Control for Hybrid Systems

The concept of observability is a very sensitive criterion, that involves the computation of a feasible solution, based on provided sufficient and necessary conditions using model predictive control strategy. Thus, to obtain predicted (estimated) states from a predictive controller require a full inference of the output behavior of the closed loop system; which is not the case for several applications. We have discussed in chapter one and chapter two the significant impact of the observability for both aspects: robust control of closed loop and diagnosis stage for fault tolerant control for wide classes of HS.

An observer-based robust model predictive control scheme is devoted in this section, to present necessary conditions in terms of LMIs, using the estimation of uncertain states of sub-systems for HS. The idea of this approach is deriving necessary and sufficient conditions to designate the optimal control based on one observer for $\boldsymbol{p}$ sub-systems of HS (Zahaf,2017). In order to be extended later for the case that the time-delay is introduced and faults occurring.

Similar concept for designing a robust controller set up on state observer based on robust model predictive control is presented recently for switched systems (Hosseini-Pishrobat,2018; Khan,2019; Taghieh,2020), to derive necessary and sufficient conditions to compute the controller gain.

In order to have a full knowledge of the output behavior, the concept of involving an observer state was proposed (Zahaf,2017) to enhance the uncertain optimization problem associated with the robust constrained model predictive control, subject to the variations of transition rules for HS and unmeasurable states for sub-systems.

Therefore, we recall the system in (3.7), that can be expressed as:

$$\begin{cases} \hat{x}(k+1) = (A_j + B_j K_j)\hat{x}(k) + LC_j e(k) \\ e(k+1) = (A_j - LC_j)e(k) \end{cases} \tag{3.10}$$

To present unified representation of the previous system, the augmented problem and dynamic errors is constructed on the following extended state-space model:

$$\tilde{x}(k+1) = \tilde{A}(k)\tilde{x}(k) \tag{3.11}$$

It follows that the representation of the system with the augmented state-space model must now be solved, using an algorithm that satisfies the constraints.

Where The new system can be described as follows:

$$\tilde{x}(k) = [\hat{x}^T(k)\ e^T(k)\ \hat{x}^T(k-1)\ e^T(k-1)]^T \tag{3.12.a}$$

$$\tilde{A}(k) = \begin{bmatrix} \tilde{A}_1(k) \\ \tilde{A}_2(k) \\ \tilde{A}_3(k) \\ \tilde{A}_4(k) \end{bmatrix} = \begin{bmatrix} (A_j + B_j K_j) & LC_j & 0_n & 0_n \\ 0_n & (A_j - LC_j) & 0_n & 0_n \\ I_n & 0_n & 0_n & 0_n \\ 0_n & I_n & 0_n & 0_n \end{bmatrix} \tag{3.12.b}$$

$$\begin{array}{ll} \hat{x}(k+1) = \tilde{A}_1(k)\tilde{x}(k), & e(k+1) = \tilde{A}_2(k)\tilde{x}(k) \\ \hat{x}(k) = \tilde{A}_3(k)\tilde{x}(k), & e(k) = \tilde{A}_4(k)\tilde{x}(k), \end{array} \quad (3.12.\text{c})$$

The Eq. (3.10) is the novel state-space model that includes the state variables and the dynamic errors of process. That allow to enhance the control of the dynamic response of the system and manage the dynamic errors. This feature will be used to design the proposed controller, that does not only have guarantees the convergence and tracking performances, but also increases degrees of freedom compared with conventional fault-tolerant control approach.

So as to achieve less conservatism of the LMIs formulation and avoid the problem of Bi-Linearity, Assumption 3.1 is a useful tool.

Assumption 3.3.

For any matrices ***W, V*** and a symmetric matrix ***Z > 0***, the following statements are equivalent and hold:

1. $(W+V)^T Z(W+V) > 0$ (3.13.a)
2. $-W^T ZW - V^T ZV < 2(W^T ZV + V^T ZW)$ (3.13.b)

Proof:

$$(W+V)^T Z(W+V) > 0 \Rightarrow \quad W^T ZW + V^T ZV + (W^T ZV + V^T ZW) > 0$$

$$-W^T ZW - V^T ZV < (W^T ZV + V^T ZW)$$

To be sure that inequality is always verified and the first term is always bound, we can write:

$$-W^T ZW - V^T ZV < \alpha\,(W^T ZV + V^T ZW), \quad where \quad \alpha > 1 \text{ to be determined}$$

Note 3.1:

In (Zahaf,2017) we take $\alpha = 2$.

In order to optimize and compute the optimal observer gain and the optimal control based on robust predictive control scheme, we recall the cost function (3.8.a) and (3.8.b) which minimize the given objective cost function in an infinite horizon as follows:

$$\begin{cases} X(k+i) = \hat{x}^T(k+i/k)Q_0\hat{x}(k+i/k) \\ U(k+i) = u^T(k+i/k)R_0u(k+i/k) \end{cases} \tag{3.14}$$

An estimation of uncertain states for some classes of hybrid system is proposed, the observer gain is obtained by optimizing the set of LMIs in Theorem 3.1.

The main results are presented in the next theorems;

Theorem 3.1.1:

Given $\gamma > 0$, the robust state observer (3.4) for the HS defined by (3.2) is able to stabilize the estimated state of hybrid system; if there exists a positive define matrix $Q > 0$; H, F_j *and* G satisfying the following LMIs:

$$\min_{Q,F_j,G,H} \gamma \tag{3.14. a}$$

Subject to

$$\begin{bmatrix} G^T\tilde{A}_3^T + \tilde{A}_3G - P^{-1} & * & * & * & * \\ \frac{1}{4}(A_jG + B_jF_j)\tilde{A}_3 & Q & * & * & * \\ \frac{1}{4}H\tilde{A}_4 & 0_n & Q & * & * \\ Q_0^{1/2}\tilde{A}_3G & 0_n & 0_n & \gamma I & * \\ R_0^{1/2}F\tilde{A}_3 & 0_n & 0_n & 0_n & \gamma I \end{bmatrix} < 0 \tag{3.14. b}$$

Where the observer gain is given by:

$$L = HG^{-1}C_j^{-1} \tag{3.14. c}$$

Based on theorem 3.1.1 the concept of the observability is guaranteed for hybrid systems (3.2), while the robust control law is given based on sufficient and necessary conditions provided by theorem 3.1.2 for the computation; in order to ensure the stability of each activated mode of the continuous dynamic and then the HS stability. The proof of theorem 3.1.1 has the same steps to follow for the proof of LMI (3.14.c) in theorem 3.1.2, with an appropriate mathematical reformulation; it will be mentioned in the following section, in order to avoid recurrence.

Theorem 3.1.2 (Zahaf,2017)**:**

Consider the closed loop estimate system (3.10), where the input feedback controller is defined by (3.5), and based on the extended state observer, which meets the performance (3.4) to compute the hybrid optimal control problem. In that (3.10) is globally asymptotically stable if there exists a positive define matrix $Q > 0$; L, F_j *and* G satisfying the following convex optimization problem:

$$\min_{Q,F_j,G,L} \gamma \tag{3.15. a}$$

Subject to

$$\begin{bmatrix} -1 & x^T(k/k) \\ x(k/k) & Q \end{bmatrix} < 0 \tag{3.15. b}$$

$$\begin{bmatrix} G^T\tilde{A}_3^T + \tilde{A}_3 G - P^{-1} & * & * & * & * \\ \frac{1}{4}(A_j G + B_j F_j)\tilde{A}_3 & Q & * & * & * \\ \frac{1}{4}\tilde{L}\tilde{A}_4 & 0_n & Q & * & * \\ Q_0^{1/2}\tilde{A}_3 G & 0_n & 0_n & \gamma I & * \\ R_0^{1/2} F\tilde{A}_3 & 0_n & 0_n & 0_n & \gamma I \end{bmatrix} < 0 \tag{3.15. c}$$

$$\begin{bmatrix} u_{max}^2 & F_j \\ F_j^T & G^T + G - Q \end{bmatrix} < 0 \tag{3.15. d}$$

Proof:

Let us recall the Lyapunov function candidate for the optimization problem described in (3.9); with $Q > 0$ to be determined.

Taking the forward difference of $V(k)$ as $\Delta V(k) = V(k+1) - V(k)$, and with respect to time along the trajectory of the system, (3.10) yields:

$$V(\hat{x}(k+1/k)) - V(\hat{x}(k/k)) \leq -\left[\left(\hat{x}^T(k/k)Q_0\hat{x}(k/k)\right) + \left(u^T(k/k)R_0u(k/k)\right)\right] \tag{3.16. a}$$

That can be written as:

$$[\hat{x}^T(k+1/k)P\hat{x}(k+1/k)] - [\hat{x}^T(k/k)P\hat{x}(k/k)] < -\left[\left(\hat{x}^T(k/k)Q_0\hat{x}(k/k)\right) + \left(u^T(k/k)R_0u(k/k)\right)\right] \tag{3.16. b}$$

With substitution of $\hat{x}(k+1/k)$ by (3.10) and $u(k+i/k)$ by (3.5) we obtain:

$$\left[((A_j + B_jK_j)\hat{x}\,(k/k) + LC_je(k))^T P((A_j + B_jK_j)\hat{x}\,(k/k) + LC_je(k)) - [\hat{x}^T\,(k/k)\,P\hat{x}\,(k/k)]\right] < -\hat{x}^T(k/k)[Q_0 + K_j^T R_0 K_j]\hat{x}(k/k) \quad (3.16.\text{c})$$

That will be :

$$((A_j + B_jK_j)\tilde{A}_3 + LC_j\tilde{A}_4)^T P((A_j + B_jK_j)\tilde{A}_3 + LC_j\tilde{A}_4) - \tilde{A}_3^T P\tilde{A}_3 < -\tilde{A}_3^T Q_0\tilde{A}_3 - \tilde{A}_3^T K_j^T R_0 K_j\tilde{A}_3 \quad (3.16.\text{d})$$

We multiply in the left by G^T and by G in the right we get:

$$G^T\tilde{A}_3^T P\tilde{A}_3G + ((A_jG + B_jF_j)\tilde{A}_3 + LC_jG\tilde{A}_4)^T P((A_jG + B_jF_j)\tilde{A}_3 + LC_jG\tilde{A}_4) + G^T\tilde{A}_3^T Q_0\tilde{A}_3G \mp \tilde{A}_3^T F_j^T R_0F_j\tilde{A}_3 < 0 \quad (3.17.\text{a})$$

The term $G^{T\tilde{A}_3^T}P\tilde{A}_3G$, can be written as follows:

$$\left(G^T\tilde{A}_3^T - P^{-1}\right)P\left(\tilde{A}_3G - P^{-1}\right) \geq 0 \Rightarrow$$

$$G^T\tilde{A}_3^T P\tilde{A}_3G - G^T\tilde{A}_3^T PP^{-1} - P^{-1}P\tilde{A}_3G + P^{-1}PP^{-1} \geq 0 \iff$$

$$G^T\tilde{A}_3^T P\tilde{A}_3G - G^T\tilde{A}_3^T - \tilde{A}_3G + P^{-1} \geq 0 \iff G^T\tilde{A}_3^T + \tilde{A}_3G - P^{-1} \leq G^T\tilde{A}_3^T P\tilde{A}_3G$$

$$G^T\tilde{A}_3^T + \tilde{A}_3G - P^{-1} \leq G^T\tilde{A}_3^T P\tilde{A}_3\,G \quad (3.17.\text{b})$$

We hold (3.17.b) in (3.17.a):

$$G^T\tilde{A}_3^T + \tilde{A}_3G - P^{-1} + ((A_jG + B_jF_j) + LC_je(k))^T P((A_jG + B_jF_j) + LC_je(k)) + G^TQ_0G + F_j^T R_0F_j < 0 \quad (3.18.\text{a})$$

We replace $P = \gamma Q^{-1}$ to the precedent inequality, we find:

$$(G^T\tilde{A}_3^T + \tilde{A}_3G - P^{-1}) + ((A_jG + B_jF_j)\tilde{A}_3 + LC_jG\tilde{A}_4)^T P((A_jG + B_jF_j)\tilde{A}_3 + LC_jG\tilde{A}_4) - G^T\tilde{A}_3^T Q_0\tilde{A}_3G - \tilde{A}_3^T F_j^T R_0F_j\tilde{A}_3 > 0 \quad (3.18.\text{b})$$

We put: $\tilde{L} = LC_jG$

Using the assumption 3.3 in this inequality we get:

$$(G^T\tilde{A}_3^T + \tilde{A}_3G - P^{-1}) + 1/2((A_jG + B_jF_j)\tilde{A}_3)^T P((A_jG + B_jF_j)\tilde{A}_3) + 1/2(\left(\tilde{L}\tilde{A}_4\right)^T P\left(\tilde{L}\tilde{A}_4\right)) + G^T\tilde{A}_3^T Q_0\tilde{A}_3G + \tilde{A}_3^T F_j^T R_0F_j\tilde{A}_3 < 0 \quad (3.18.\text{c})$$

Using generalized Schur's complement to (3.18.c), the LMI (3.15c) is obtained.

<u>Note</u>:

To obtain the LMI (3.14.b) it is enough to replace the observer gain L *by* $HG^{-1}C_j^{-1}$ *in equation (3.18.b).*

- To determine LMI (3.15.b), we recall the closed-loop system in (3.10) and consider the following Lyapunov function candidate:

$$V(x\,(k/k)) = \hat{x}^T\,(k/k)\,P\hat{x}\,(k/k) \tag{3.19.a}$$

For the stability performance we recall (3.9), we have:

$$V(\hat{x}\,(k+i+1/k)) - V(\hat{x}\,(k+i/k)) \leq -[X(k+i) + U(k+i)] \tag{3.19.b}$$

$$-V(\hat{x}\,(k/k)) \leq -J_\infty(k) \tag{3.19.c}$$

We can write it:

$$\max_{A_j,B_j,i>0} J_\infty(k) \leq V(\hat{x}\,(k/k)) \leq \gamma \tag{3.19.d}$$

While the problem of minimization becomes

$$\min_{Q,F_j,G,L} \gamma \tag{3.19.e}$$

With:

$$\hat{x}^T\,(k/k)\,P\hat{x}\,(k/k) \leq \gamma \Leftrightarrow -\gamma + \hat{x}^T\,(k/k)\,P\hat{x}\,(k/k) \geq 0 \tag{3.20.a}$$

Using Schur's complement to (3.20.a), we get (3.15.b).

- Next, we reformulate the input constraints in the LMI term, which is defined as follows:

$$u_{h,min} \leq u_h(k+i/k) \leq u_{h,max}\,,$$

$$\Rightarrow\; |u_h(k+i/k)| \leq u_{h,max}\,, \qquad for\; i \geq 0, h = 1,2,\dots,p \tag{3.21.a}$$

Where: $u_{max} = U,\; \|u(k+i/k)\|_{max} \triangleq \max_i u_i(k+i/k)$ (3.21.b)

We can write:

$$\max_{i>0} \|u(k)\|_{max} \geq \max_{i>0} \left\|F_j G^{-1} P\hat{x}(k)\right\|_{max} \tag{3.21.c}$$

We use again the LMI constraints, we obtain:

$$\begin{bmatrix} -u_{max}^2 & F_j G^{-1}) \\ (F_j G^{-1})^T & P \end{bmatrix} > 0 \tag{3.21.d}$$

By using Congruence property with full rank matrix $\begin{bmatrix} I & 0 \\ 0 & G^T \end{bmatrix}$ gives:

$$\begin{bmatrix} u_{max}^2 & F_j \\ F_j^T & G^T + G - Q \end{bmatrix} < 0 \tag{3.22}$$

End of proof.

The next algorithm summaries the steps of computing the optimal control based on sufficient and necessary conditions, by ensuring the stability of some classes of switched systems.

Algorithm 3.1.

Step 1: Initialize the model parameters by giving an admissible gain $H(0)$.

Step 2: According to the general transition "Switching" function, activate and apply the j^{th} sub-system.

Step 3: Solve the LMIs problem in Theorem 3.1.2 based on the optimal observer gain computed from theorem 3.1.1.

Step 4: The feasibility of the problem results in robustly asymptotically stabilizing matrices $\boldsymbol{K}$ and $\boldsymbol{L}$,

Step 5: Update the transition function for the $(j^{th} + 1)$ sub-system if it exists,

Step 6: Stop when the optimization problem (3.10), (cost function) results in robustly asymptotically stabilizing matrices $\boldsymbol{K}$ and $\boldsymbol{L}$; Otherwise, let $k=k+1$ and go back to Step 3.

❖ ***Has the observer (3.4) an Efficient Impact in Case of Faults Occurring?***

It is not often possible in real time applications to avoid the undesirable associated inputs. So, a reliable control scheme requires a full knowledge of the outputs behavior, which is not possible at all times.

In several control approaches, the control law formed from the estimated state is employed in the optimization problem; especially for the reconfiguration mechanism stage, that is fundamentally based on the estimation and observability concepts. In this context, the previous proposed observer scheme based on (3.4) is not always valid for the case of faults occurring and introducing time-delay.

Therefore, in this thesis, to estimate a full state in the feedback channel, an observer with unknown input is designed for the faulty uncertain hybrid discrete-time system, which is described as follow:

$$\begin{cases} \hat{x}(k+1) = A_j\hat{x}(k) + B_ju(k) + L\big(y(k) - \hat{y}(k)\big) + F_a\hat{f}_f(k) \\ \hat{y}(k) = C_j\hat{x}(k) + F_s\hat{f}_f(k) \\ \hat{f}_f(k) = F(y(k) - \hat{y}_f(k)) \end{cases} \tag{3.23}$$

An exact knowledge of the faulty system can enable the control scheme to be adapted to meet the required performances, as well as allow the reconfigurable system (as a new system) behaves as originally stated. Once the state variables and faults are estimated; a new control law is added to the nominal law to counteract the effect of the fault on the system.

The faulty uncertain hybrid system can be written as:

$$\begin{cases} \hat{x}(k+1) = A_j\hat{x}(k) + B_ju(k) + \big(LC_j + F_aFC_j\big)e(k) \\ \hat{y}(k) = C_j\hat{x}(k) + F_sFC_je(k) \\ \hat{f}_f(k) = F(y(k) - \hat{y}_f(k)) \end{cases} \tag{3.24}$$

Where K_j is the controller gain matrix and L is the observer gain matrix which can be designed by standard optimization methods. F_a, and F_s are the actuator and the sensor fault parameters, respectively; $\hat{y}$ is the sensors estimated measurement, $\hat{f}_f$ is the estimated output faults.

The observer (3.24) is derived for the HS from that proposed by (Ichalal,2008) for a class of fuzzy systems.

With the aim to design a reliable AFTC “Active Fault tolerant control” strategy, that has the ability to handle with large classes of faults and different range of faults, compared with the robust FTC "passive FTC" approach. This advantage of AFTC is based on the aspect of fault detection and isolation FDI to deal with faults. For this case, establishing an accurate cause and location of faults using (3.24) is so important to provide an efficient control input. Only with these features and a full knowledge of the dynamic behavior, we can make changes on the MPC algorithm with a reliable method to guarantee the stability of the faulty system, at the moment of dealing with faults. Furthermore, as compared to ordinary control strategies, the MPC rises up a new aspect in the field of fault-tolerant control. Since the advantages are very obvious, the main advantage of the MPC is the possibility to adapt the weights Q_0 and R_0 to make changes in the controller dynamics, in addition to control the variable weighting parameters of redundant control values in each optimization step. Also, the internal model of the process can be updated to adapt to any changes in real processing. Furthermore, the inclusion of constraints and control variables in the reformulated system is also a big advantage.

Therefore, designing an active fault-tolerant model predictive control as a control strategy for Hybrid systems is considered a new objective of HFTMPC research. The next points below sum up the advantages and reasons to employ the MPC to control hybrid system:

1 - Online handling of constraints and controller values in the system control process.
2 - Adaptation of changes in dynamics of the internal model due to faults in the hybrid system, actuators or sensors.
3 - Redundancy of the reconstruction values and actuators to accommodate actuator failures.
4 - Very good tracking performance for systems with large time delays or slow dynamics.
5 - Fault-tolerant capability within the MPC algorithm due to direct updates.

In the rest of this thesis, the mentioned observer scheme (3.23) is adopted according to the faults' kind "actuators, sensors and system faults"; in order to compute the optimal control, by implementing an adequate mechanism of estimation for the unknown outputs. This conceptualization is employed later to present a new insight for fault tolerant control using robust predictive control for constrained hybrid systems with time-delay.

Before that, the next part deals with the compensation of occurring faults using Quadratic Programming QP as a solution strategy, by combining the state observer and model predictive control "MPC" as the process control for the optimization problem.

III.4 Fault-Tolerant Based Model Predictive Control (MPC) for Hybrid Systems

Based on numerical optimization methods as QP, it is not always efficient to provide an optimal control law, which may not be easily to obtain. Thus, one has to improve algorithms and involve rigorous optimization methods. General optimization framework based on model predictive control (as Quadratic Programing, linear quadratic regulator… etc.) combined with an observer is not thoroughly investigated. A coupled of observer-MPC as an online approach for the parameter identification of the nonlinear systems is presented in (Flila,2008; Qian,2013), where a new approach of coupled state observer-MPC strategy to deal with occurred faults is proposed by (Zahaf,2019b), for a class of constrained HS. A fault-tolerant based model predictive control approach is proposed; the underlying of the state observer allow to design a new augmented system for the computation of the optimal control using QP method. The proposed approach that involves the state-space model of the nonlinear HS has less conservative conditions compared to the existing works.

From equation (3.24), we consider the next closed loop discrete-time hybrid system:

$$\begin{cases} \hat{x}(k+1) = (A_j + B_j K_j)\hat{x}(k) + \left(LC_j + F_a F C_j\right)e(k) \\ \qquad\qquad \hat{y}(k) = C_j \hat{x}(k) + F_s F C_j e(k) \end{cases} \tag{3.25}$$

The previous uncertain hybrid system can be written as:

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_p(k)\tilde{x}(k) + \tilde{B}_p \tilde{u}(k) \\ \qquad \tilde{y}_p(k) = \tilde{C}_p \tilde{x}(k) \end{cases} \tag{3.26.a}$$

$$\tilde{A}_p = \begin{bmatrix} (A_j + B_j K_j) & 0_n \\ C_j & 0_n \end{bmatrix},\ \tilde{B}_p = \begin{bmatrix} (L + F_a F)C_j & 0_n \\ F_s F C_j & 0_n \end{bmatrix},\ \tilde{C}_p = \begin{bmatrix} I & 0_n \\ 0_n & I \end{bmatrix} \tag{3.26.b}$$

The $\tilde{y}_p$ is the output of the augmented hybrid system. The control input *u(k)*, represents the desired energy of the current step *k*. The matrix $\tilde{A}_p$ represents the system matrix of the preview model while the matrix $\tilde{B}_p$ is the input in the presence of faults (Actuators and Sensors). The $\tilde{C}_p$ matrix is the weighting matrix used to construct performance of optimization following quadratic function J.

$$\min_{u(k+j/k)\ j>0} J_h(k) = \sum_{j=0}^{h} [\tilde{y}_P(k+j/k) - y_d(k+j/k)]^2 + \sum_{j=0}^{h} \lambda[\vec{u}(k+j/k)]^2 \quad (3.27.a)$$

$$\begin{cases} \hat{y}_{h,min} \le \hat{y}_h(k+j/k) \le \hat{y}_{h,max}\,, & j \ge 0, \quad h = 1,2,\dots,q \\ u_{h,min} \le u_h(k+j/k) \le u_{h,max}\,, & j \ge 0, \quad h = 1,2,\dots,p \end{cases} \quad (3.27.b)$$

Where :

$$\begin{cases} \tilde{y}_P(k) = [y(k+1)\ \ y(k+2)\ \ \dots\ \ y(k+N)]^T \\ \vec{u}(k) = [\tilde{u}(k)\ \ \tilde{u}(k+1)\ \ \dots\ \ \tilde{u}(k+N)]^T \\ \lambda = diag\big(\ \bar{\lambda}(k+1)\ \ \dots\ \ \bar{\lambda}(k+N)\big] \end{cases} \quad (3.28)$$

$[\tilde{y}_P(k+j/k) - y_d(k+j/k)]^2$, represents the weighting sum of future error between estimate states and desired trajectory.

$\lambda[\vec{u}(k+j/k)]^2$ represents the energy of input control.

The h and λ represents the prediction step and weighting factor for input, respectively, and k is the step of the present state.

Then, the predictive model $\tilde{y}_P(k)$ is expressed as follows :

$$\tilde{y}_P(k+1) = G\tilde{x}(k) + H\vec{u}(k) + Mu(k) \quad (3.29)$$

Where:

$$G = [\tilde{C}_p\tilde{A}_p\quad \tilde{C}_p\tilde{A}_p^2\ \ \dots\ \ \tilde{C}_p\tilde{A}_p^N]^T \quad (3.30.\text{a})$$

$$H = \begin{bmatrix} 0_n & 0_n & \dots & 0_n \\ \tilde{C}_p\tilde{B}_p & 0_n & \dots & 0_n \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_p\tilde{A}_p^{N-1}\tilde{B}_p & 0_n & \dots & 0_{n_0} \end{bmatrix} \quad (3.30.\text{b})$$

$$M = [\tilde{C}_p\tilde{B}_p\quad \tilde{C}_p\tilde{A}_p\tilde{B}_p\ \ \dots\ \ \tilde{C}_p\tilde{A}_p^{N-1}\tilde{B}_p]^T \quad (3.30.\text{c})$$

In equation (3.29), G, H and M are the augmented state system, input control and faults occurred (on actuators/sensors) matrices, respectively, of the state space model of hybrid system in (3.27).

On the other hand, the $\vec{u}$ term indicates the desired variation of energy, for the longitudinal control of h-step prediction horizon for hybrid system. For relaxation conditions on the online optimization, we hold the (3.29) in (3.27) with respect to the imposed constrains on the control input. Therefore, the cost function can be written as mentioned in (3.31); thus the performance index J (3.27.a) can be casted into a condensed QP problem of the form:

$$\min_{\vec{u},\ i>0} \frac{1}{2}\vec{u}^T P u + R^T \vec{u} \tag{3.31}$$

Subject to

$$W\vec{u} \leq Z \tag{3.32}$$

Where $u \in R^{n_u}$ is the vector of optimization variables, P is a symmetric and positive definite matrix, R is the linear cost, W and Z are the terms defining the inputs' constraints.

The next theorem summaries the main results.

Theorem 3.2 (Zahaf,2019b)**:**

For a given positive scalar $\bar{\lambda}$, symmetric positive definite matrices P and R. If it exists symmetric positive definite matrices H , M, weighting factor λ and K_j as such that meet the performance of (3.31) for computing the optimal control; where:

$$P = \lambda V^T V + H^T H \tag{3.33}$$

$$R = H^T (G\tilde{x}(k) + M u_{opt}(k)) \tag{3.34}$$

Then, the closed-loop hybrid systems (3.26.a) is called fault-tolerant asymptotically stable for any sub-system, satisfying the reliable control law with optimal cost.

Where W and Z are representing the upper and lower energy limits or the imposed constraints on output and input respectively.

$$W = [W_{upper}, \quad W_{lower}]^T, \qquad Z = [Z_{upper}^T, \quad Z_{lower}^T]^T \tag{3.35}$$

To deal with actuators and sensors failures the P and R matrices in (3.33) and (3.34), respectively, with:

$$V = \begin{bmatrix} v_{1,1} & v_{1,2} & 0 & \dots & 0 \\ 0 & v_{2,2} & v_{2,1} & \dots & 0 \\ 0 & 0 & 0 & \dots & \vdots \\ \vdots & \vdots & 0 & \ddots & v_{n-1,n} \\ 0 & 0 & 0 & \dots & v_{n,n} \end{bmatrix} \tag{3.36}$$

In this stage, to test the convexity and feasibility of (3.31) for the FTMPC, the definiteness of Matrix P has been evaluated. The quadratic function in (3.31) is convex if and only if matrix P is positive semi-definite. Besides, the quadratic function defined in (3.31) is strictly convex if and only if matrix P is positive definite. Then, the optimal control input can be computed based on the Quadratic Programming optimization using Active Set Methods.

The presented FTMPC approach is more efficient and less conservative optimization conditions compared to (Flila,2008; Qian,2013), due to the admissible response time.

However, due to the computation burden and the additive effort required by the QP with MPC (related to the computation of the matrices of the minimization problem), it can be exactly computed a-priori, whereas the QP cannot be guaranteed to maintain the convexity of the quadratic minimization problem. The possibility to certify the algorithm complexity and the required computational effort to compute the control law is of paramount importance for a real-time implementation of the algorithm in commercial control platform. Next, we propose an outperforming approach to improve the response time and meet the desired performances; in which have less conservative design based on online optimization of the objective cost function, using *min-max* formulation in terms of LMIs in next stage.

III.5 Synthesis of Robust Optimal Fault Tolerant Predictive Control for Hybrid Systems with time-varying delay: An LMI Approach

As aforementioned, this section is devoted to develop a robust mechanism for reconfiguration of the controller, to recover the desired performances in the presence of faults and compensate the time-delay, based on less conservative conditions with respect of the imposed constraints. These relaxed sufficient and necessary conditions guarantee the stability of HS; besides decreasing the computation burden in order to designate the optimal control at a reasonable response time.

We have discussed in chapter two that a diagnosis scheme, controller reconfiguration strategy and reconfigurable controller are the main elements to design a reliable AFTC strategy. In section 3.2 of this chapter, in order to infer the observability properties for some classes of HS, we presented a new strategy for the diagnosis stage based on the combination of state observer and robust model predictive control. This combination is used in the reconfiguration mechanism of the controller, which is setup on the ability of the controller itself to adopt updates according to new events "undesirable associated inputs" during the running process.

Basically, the reconfigurable controller design should be considered while the set-up of a control strategy from the beginning, by proposing a flexible controller scheme, that has the ability to adopt the updates and the reconfiguration of its parameters. In fact, proposing a reliable and flexible controller is a considerable point of our contributions in this dissertation. We proposed the controller law with two terms for dealing with different kinds of faults, which is basically composed of two features: an estimated state and error dynamics of faulty HS.

Before describing the new control prediction law presented in (Zahaf,2020); we first define the next control law to be generated on the controller side, which is formed on two terms; we propose a new control law u_{DD} to be added to the nominal control law to compensate the effect of the fault or failure on the system. Therefore, the total control law applied to the system is given by:

$$u_f(k) = \underbrace{u(k)}_{u_{CD}} + \underbrace{K_f(x(k) - \hat{x}_f(k))}_{u_{DD}} \tag{3.37}$$

Wherein, u_{CD} is the nominal control law defined in (3.5) and u_{DD} represents the additive control law to deal with the faulty behavior.

A quick examination of related works on control applications shows that using a controller with two terms is often used (Ichalal,2009; Wang,2017). This feature enables a robust adaption of the controller scheme with undesirable associated inputs as occurring faults and time-delay.

In view of that, the proposed control scheme in this thesis is mainly using the estimated states and the error dynamics of faulty HS, to compensate time-delay and reconfigure the robust controller to deal with faults.

According to (3.37), we distinguish two terms: u_{CD} and u_{DD} terms which refer to control law for continuous dynamic and discrete dynamics, respectively, as hybrid control law. We discuss in the rest of this work the possible form of u_{DD}, we propose two controller schemes to deal with different kinds of faults and time-delay.

III.5.1 Robust Optimal Fault Tolerant Predictive Control for Sensors Failures

This part deals with sensors faults and failures. Thus, it is assumed that all states of the hybrid system are not measurable; so to compensate the sensors fault/failure in the feedback channel, based on (3.23) an observer is designed as follow:

$$\begin{cases} \hat{x}_f(k+1) = A_j\hat{x}_f(k) + B_j u_f(k) + L(y(k) - \hat{y}_f(k)) \\ \hat{y}_f(k) = C_j\hat{x}_f(k) + F_s\hat{f}_f(k) \\ \hat{f}_f(k) = F(y(k) - \hat{y}_f(k)) \end{cases} \tag{3.38}$$

Where A_j, B_j and C_j are state matrices, L is the observer gain matrix which can be designed by optimization methods.

The new control prediction law to be generated on the controller side with two terms is based on (3.37), that is basically formed on the estimated faulty states as follows:

$$u_f(k) = u(k) + K_f(x(k) - \hat{x}_f(k)) \tag{3.39. a}$$

$$u(k) = K\hat{x}_f(k) \tag{3.39. b}$$

$$u_f(k) = u(k) + K_f e(k) \tag{3.39. c}$$

Where K and K_f are the controller gains matrices. Since the aim of (3.39) is approximating $\hat{x}_f(k)$ to $x(k)$, by defining the control law parameters, while $e(i|k)$ should approach zero, i.e., $e(\infty|k) = 0$.

The additive control law must be calculated so that the faulty system is as close as possible to the nominal system (desired performances). In other words, it must satisfy:

$$K_f\left(x(k) - \hat{x}_f(k)\right) = 0 \tag{3.39. d}$$

Defining control law parameters is converted to the optimization of the worst case scenario of occurring faults using *min-max* method, that is known as a solution of the global optimization for the convex problem. Therefore, we recall the cost function (3.7) that can be adapted and defined based on the desired performances of (3.38). Wherein, the related cost function for robust fault tolerant predictive control is defined by (3.39), with respect to the imposed output constraints (3.40.b) and the defined objective cost function (3.40.c) as follows:

$$\min_{u_f} \max_{i>0} J_\infty(k) \tag{3.40. a}$$

$$\hat{y}_{h,min} \leq \hat{y}_h(k+i/k) \leq \hat{y}_{h,max}\,, \quad i \geq 0, \quad h = 1,2,\dots,q \tag{3.40. b}$$

$$\begin{cases} \hat{X}(k+i) = \hat{x}_f{}^T(k+i/k)Q_0\hat{x}_f(k+i/k) \\ U(k+i) = u_f^T(k+i/k)R_0u_f(k+i/k) \end{cases} \tag{3.40. c}$$

The main objective in this section is to design an observer for the case of the faulty hybrid system, that is able to detect faults and use it to facilitate the reconfiguration procedure of the control scheme.

To set-up the necessary conditions of the closed loop discrete-time hybrid system, that is converted to a convex optimization problem (3.40); let us recall (3.6) and adapting (3.2) and (3.38), then we get:

$$\begin{cases} \hat{x}_f(k+1) = (A_j + B_jK)\hat{x}_f(k) + (B_jK_f + LC_j)e(k) \\ \qquad e(k+1) = \left(A_j - B_jK_f - LC_j\right)e(k) \\ \qquad e_f(k+1) = -LC_je(k) - FF_se_f(k) \end{cases} \tag{3.41}$$

The previous augmented system can be described as new system written as:

$$\tilde{x}(k+1) = \tilde{A}(k)\tilde{x}(k) \tag{3.42.a}$$

$$\tilde{A}(k) = \begin{bmatrix} \tilde{A}_1(k) \\ \tilde{A}_2(k) \\ \tilde{A}_3(k) \\ \tilde{A}_4(k) \\ \tilde{A}_5(k) \end{bmatrix} = \begin{bmatrix} (A_j + B_jK) & (B_jK_f + LC_j) & 0_n & 0_n & 0_n \\ 0_n & (A_j - LC_j) & 0_n & 0_n & 0_n \\ 0_n & LC_j & FF_s & 0_n & 0_n \\ I_n & 0_n & 0_n & 0_n & 0_n \\ 0_n & I_n & 0_n & 0_n & 0_n \end{bmatrix} \tag{3.42.b}$$

$$\tilde{x}(k) = [\hat{x}_f{}^T(k)\; e^T(k)\; e_f{}^T(k)\; \hat{x}_f{}^T(k-1)\; e^T(k-1)]^T \tag{3.42.c}$$

$$\begin{aligned} &\hat{x}_f(k+1) = \tilde{A}_1(k)\tilde{x}(k), \qquad && e(k+1) = \tilde{A}_2(k)\tilde{x}(k) \\ &e_f(k+1) = \tilde{A}_3(k)\tilde{x}(k), && \hat{x}_f(k) = \tilde{A}_4(k)\tilde{x}(k) \\ &e(k) = \tilde{A}_5(k)\tilde{x}(k), && \end{aligned} \tag{3.42.e}$$

In order to preserve the convexity feature of the studied optimization problem in term of LMIs, and ensure that the transition mathematical formulation remains bounded; the next useful assumption is used.

Assumption 3.4:

For any matrices W, V and a symmetric matrices $Z > 0$ and $R > 0$, we consider the problem of finding an appropriate matrix R, where the following statements are equivalent and hold:

1. $(W+V)^T Z (W+V) > 0$ (3.43.a)
2. $-W^TZW - V^TZV < (W^TZV + V^TZW) + R$ (3.43.b)

Proof:

$$(W+V)^T Z (W+V) > 0 \implies W^TZW + V^TZV + (W^TZV + V^TZW) > 0$$

$$\implies -W^TZW - V^TZV < (W^TZV + V^TZW) \tag{3.44}$$

To guarantee that inequality is always verified and the first term is always bounded, we can write: $-W^TZW - V^TZV < (W^TZV + V^TZW) + R$, if there exists an appropriate matrix R > 0.

End of Assumption.

The main results for computing the optimal control to handle with sensors faults are presented in the next theorem, based on relaxed conditions in set of LMIs.

Theorem 3.3 (Zahaf,2019a)***:***

Consider the closed loop estimate of a faulty hybrid system (3.38) and let the input feedback controller be defined by (3.39), which is based on the extended state observer and meets the performance (3.40) for computing the optimal control problem of hybrid system. It is globally asymptotically stable if there exists a positive define matrix $Q > 0$; Y, Y_f, L, F, F_s and G satisfying the following convex optimization problem:

$$\min_{Q,F,G,L} \gamma \tag{3.45. a}$$

Subject to

$$\begin{bmatrix} -1 & \hat{x}_f{}^T(k/k) \\ \hat{x}_f(k/k) & Q \end{bmatrix} < 0 \tag{3.45. b}$$

$$\begin{bmatrix} G^T\tilde{A}_3^T + \tilde{A}_3 G - P^{-1} & * & * & * & * \\ \frac{1}{4}(A_jG + B_jY)\tilde{A}_1 & Q & * & * & * \\ \frac{1}{4}\tilde{L}\tilde{A}_5 & 0_n & Q & * & * \\ Q_0^{1/2}\tilde{A}_1 G & 0_n & 0_n & \gamma I & * \\ R_0^{1/2}(Y\tilde{A}_4 + Y_f\tilde{A}_5) & 0_n & 0_n & 0_n & \gamma I \end{bmatrix} < 0 \tag{3.45. c}$$

$$\begin{bmatrix} W & C_j(\tilde{A}_4 + FF_s\tilde{A}_5) \\ (\tilde{A}_4 + FF_s\tilde{A}_5)^T C_j^T & P \end{bmatrix} > 0 \tag{3.45. d}$$

Proof:

For the LMIs (3.45.b) and (3.45.c), following steps in the proof of theorem 3.2 with respect and the adaption of the state-space model and the desired performances of the cost function (3.40).

- Where, the next inequality is the starting formula to obtain (3.45.b).

$$\max_{A_j,B_j,j>0} J_\infty(k) \le V(\hat{x}_f(k/k)) \le \gamma \tag{3.46}$$

- Also, the way to obtain (3.45.c), starts from the next inequality:

$$V(\hat{x}_f(k+1/k)) - V(\hat{x}_f(k/k)) \le -\left[\left(\hat{x}_f{}^T(k/k)Q_0\hat{x}_f(k/k)\right) + \left(u_f{}^T(k/k)R_0u_f(k/k)\right)\right] \tag{3.47}$$

To get the final form using the assumption 3.4, the final form can be written as:

$$(G^T\tilde{A}_1^T+\tilde{A}_1G-P^{-1})+1/2((A_jG+B_jF)\tilde{A}_1)^TP((A_jG+B_jF)\tilde{A}_1)+1/2\left(\left(\tilde{L}\tilde{A}_5\right)^TP\left(\tilde{L}\tilde{A}_5\right)\right)$$
$$+G^T\tilde{A}_1^TQ_0\tilde{A}_1G+\left(Y\tilde{A}_4+Y_f\tilde{A}_5\right)^TR_0(Y\tilde{A}_4+Y_f\tilde{A}_5)<0 \qquad (3.48)$$

Applying generalized Schur's complement to the previous inequality, the LMI (3.45.c) is obtained.

- Since the faults occurred on the sensors, it is worthy to impose some output constraints to meet the required performances by the proposed approach, thus:

$$y_{h,min}\le y_h(k+i/k)\le y_{h,max}\,,\Leftrightarrow |y_h(k+j/k)|\le y_{h,max}, \qquad (3.49.\text{a})$$
$$for\!: i\ge 0,\qquad h=1,2,\dots,q$$

Defining that $y_{max}=W$, where W is upper bound of the constraint

$$\|y(k+j/k)\|_{max}\triangleq \max_j y_j(k+i/k) \qquad (3.49.b)$$

Using (3.38), we can write:

$$\max_{j>0}\|y(k)\|_{max}\ge \max_{j>0}\left\|C_j\left(\tilde{A}_4+FF_s\tilde{A}_5\right)P^{-1}x(k)\right\|_{max} \qquad (3.49.c)$$

The relaxed condition for output constraint can be written as follows:

$$\begin{bmatrix} W & C_j(\tilde{A}_4+FF_s\tilde{A}_5) \\ (\tilde{A}_4+FF_s\tilde{A}_5)^TC_j^T & P\end{bmatrix}>0 \qquad (3.50)$$

End of proof.

Since the occurring faults during the dynamic process of the systems are not limited to sensors faults as has been discussed in chapter two. Thus, additionally to the aforementioned proposed solution to compensate and reconfiguration of the control behavior in the case of sensors failures. The next part deals with both issues of time-varying delay and actuators faults.

III.5.2 Robust Optimal Fault Tolerant Predictive Control for Hybrid System with Time-Varying Delay

This part is devoted for the study of the trajectory tracking of HS with actuators faults and time-delay. The main objective is to design a robust control scheme that deals and compensates the undesirable behavior of HS, influenced by faults and time-delay, as shown in Figure 3.1.

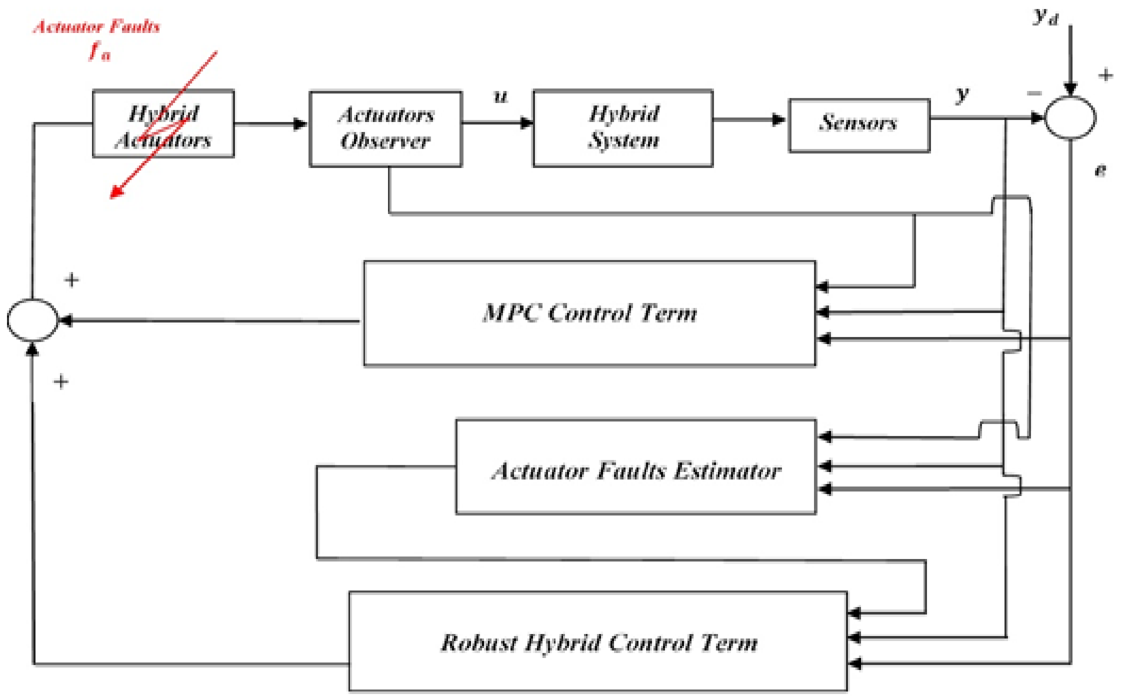


Fig. 3.1. *The Scheme of Hybrid Fault Tolerant Optimal Predictive Control (HFTPC)*

The main idea of this approach is to combine two strategies as hybrid control approach, using the *min-max* method to keep up the tracking performances of the desired trajectories, and preserve the stability conditions in the presence of faults. At first, to cope with time-delay using the obtained control term from robust MPC stage, in case that the HS is without faults for more fast-time processing, which is considered as continuous dynamic for the hybrid control. In the other case, faults occur in addition to time-delay, then necessary and sufficient conditions are obtained based on new proposed control law, to reduce the conservatism and decrease the computation burden as a discrete dynamic; in order to improve the consuming time in the compensation of the undesirable behavior compared with the classical MPC Approach.

Therefore, we consider the following uncertain discrete-time hybrid actuator system with time-varying delay represented as:

$$\begin{cases} x(k+1) = A_j x(k) + A_{jd} x(k-d_k) + B_j u(k) \\ y(k) = C_j x(k) \end{cases} \tag{3.51}$$

The faulty discrete-time hybrid actuator system with time-varying delay is described and represented as:

$$\begin{cases} x_f(k+1) = A_j x_f(k) + A_{jd} x_f(k-d_k) + B_j u_f(k) \\ \qquad\qquad\qquad\qquad + F_a f_a(k) \\ y_f(k) = \; C_j x_f(k) \end{cases} \tag{3.52}$$

In this part, it is assumed that the states of the hybrid actuators system are not measurable, so to estimate the state with fault and time delay in the feedback channel, an observer is designed:

$$\begin{cases} \hat{x}_f(k+1) = A_j \hat{x}_f(k) + A_{jd} \hat{x}_f(k-d_k) + B_j u_f(k) \\ \qquad\qquad + F_a \hat{f}_a(k) + L\left(y_f(k) - \hat{y}_f(k)\right) \\ \hat{y}_f(k) = C_j \hat{x}_f(k) \\ \hat{f}_a(k+1) = F\left(y_f(k) - \hat{y}_f(k)\right) \end{cases} \tag{3.53}$$

The new hybrid control prediction law to be generated on the controller side is with two terms based on (3.37); where u_{CD} represents the robust predictive controller of the delayed HS without faults using the estimated states obtained from the predictive control strategy as continuous dynamic, and u_{DD} refers to the computed control law term in the presence of faults as discrete dynamics. Therefore, the control strategy given in figure 3.1 is proposed in order to compute the hybrid control signal $u_f(k)$ such that:

- The closed loop system is stable
- The state of faulty system $x_f(k)$ converges asymptotically towards the reference state.

In presence of faults, the hybrid control to be generated on the controller side is based on the following proposed control law as:

$$u_f(k) = u(k) + K_f\left(x(k) - \hat{x}_f(k)\right) \tag{3.54. a}$$

$$u(k) = K\hat{x}(k) \tag{3.54. b}$$

$$u_f(k) = u(k) + K_f e_s(k) \tag{3.54. c}$$

Where A_j, A_{jd}, B_j and C_j are states matrices of hybrid actuators system, u, u_f are hybrid inputs and f_a faults vectors, $d_m \leq d_k \leq d_M$ is a time-varying delay, with d_m *and* d_M respectively. The lower and the upper bound of d_k, K, and K_f are the controller gains matrices, and L is the observer gain matrix, which can be determined by an appropriate optimization methods.

Similar to the optimization problem in case of sensors faults, we consider the worst case scenario of occurring faults on actuators to set up a convex optimization problem using *min-max* method. Thus, let us recall the cost function (3.7) that can be adapted and defined based on the desired performances of (3.51) and (3.53). Therefore, the related cost function for robust fault tolerant predictive control is defined by (3.55.a), with respect to the imposed output constraints (3.55.a) and the defined objective cost function (3.55.c) as follows:

$$\min_{u_f(k+i/k)} \max_{i>0} J_\infty(k) \tag{3.55. a}$$

$$u_{fh,min} \leq u_{fh}(k+i/k) \leq u_{fh,min}\,, \quad i \geq 0, \quad h = 1,2,\dots,q \tag{3.55. b}$$

$$\begin{cases} \hat{X}(k+i) = \tilde{e}^T(k+i/k) Q_0 \tilde{e}(k+i/k) \\ U(k+i) = u_f^T(k+i/k) R_0 u_f(k+i/k) \end{cases} \tag{3.55. c}$$

Throughout this chapter, the following assumptions are necessary for the stability synthesis of the HS, to derive less conservative computation conditions. For this reason, the mentioned Lemmas in *Appendix C* are made as useful tools for matrices transformation.

Assumption 3.5:

Assume $Z = Z^T \in S^{n\times n}$, $W \in \mathcal{R}^{n\times m}$ and $V \in \mathcal{R}^{m\times n}$, to be known matrices, and further assume that $rank\,(W) < n$ and $rank\,(V) < n$. Consider the problem of finding an appropriate matrix U, such that the following statements are equivalent and hold:

1. $W^T Z V + V^T Z W < 0$ (3.56. a)
2. $U + W^T Z W + V^T Z V < 0$ (3.56. b)

- ***Augmented System***

The augmented system of the dynamic error represented by $\tilde{e}(k+1)$ contains: the tracking error $e_s(k)$ in presence of faults, the dynamic error of the estimated state $e_{es}(k)$ and the dynamic error of the estimated faults $e_f(k)$ are given below. Therefore, we define the estimated dynamic errors as follow:

$$e_s(k) = x(k) - \hat{x}_f(k) \tag{3.57.a}$$

$$e_{es}(k) = x_f(k) - \hat{x}_f(k) \tag{3.57.b}$$

$$e_f(k) = f_a(k) - \hat{f}_a(k) \tag{3.57.c}$$

$$e_{ys}(k) = c_j e_s(k) \tag{3.57.d}$$

$$e_{yes}(k) = c_j e_{es}(k) \tag{3.57.e}$$

To set-up the necessary conditions of the optimization problem in order to compute the optimal control, it should be converted to a convex problem in terms of LMIs, we recall (3.51) and (3.52), we get:

$$\begin{cases} e_s(k+1) = \left(A_j - B_j K_f\right) e_s(k) + A_{jd} e_s(k-d_k) \\ \qquad -LC_j e_{es}(k) + (-F_a) e_f(k) \\ e_{es}(k+1) = \left(A_j - LC_j\right) e_{es}(k) + A_{jd} e_{es}(k-d_k) \\ \qquad +F_a e_f(k) \\ e_f(k+1) = -FC_j e_{es}(k) \end{cases} \tag{3.58}$$

The previous augmented system can be written as:

$$\tilde{e}(k+1) = \tilde{A}(k)\tilde{e}(k) \tag{3.59.a}$$

Where ;

$$\tilde{e}(k) = \left[e_s{}^T(k)\ \ e_s{}^T(k-d_k)\ \ e_{es}{}^T(k)\ \ e_{es}{}^T(k-d_k)\ \ e_f{}^T(k)\ \ \hat{x}_f^T(k)\right]^T \tag{3.59.b}$$

$$\tilde{A}(k) = \begin{bmatrix} \tilde{A}_1(k) \\ \tilde{A}_2(k) \\ \tilde{A}_3(k) \\ \tilde{A}_4(k) \\ \tilde{A}_5(k) \\ \tilde{A}_6(k) \end{bmatrix} = \begin{bmatrix} \left(A_j - B_j K_f\right) & A_{jd} & -LC_j & 0_n & -F_a & 0_n \\ 0_n & I_n & 0_n & 0_n & 0_n & 0_n \\ 0_n & 0_n & \left(A_j - LC_j\right) & A_{jd} & F_a & 0_n \\ 0_n & 0_n & 0_n & I_n & 0_n & 0_n \\ 0_n & 0_n & -FC_j & 0_n & 0_n & 0_n \\ 0_n & 0_n & 0_n & 0_n & 0_n & I_n \end{bmatrix} \tag{3.59.c}$$

$$\begin{cases} \tilde{A}_7(k) = [I_n 0_n 0_n 0_n 0_n 0_n], \\ \tilde{A}_8(k) = [0_n 0_n I_n 0_n 0_n 0_n], \\ \tilde{A}_9(k) = [0_n 0_n 0_n 0_n I_n 0_n], \end{cases} \quad (3.59.\mathrm{d})$$

Then, we can describe the different variables as follows:

$$\begin{cases} e_s(k+1) = \tilde{A}_1(k)\tilde{e}(k), & e_{es}(k+1) = \tilde{A}_3(k)\tilde{e}(k) \\ e_f(k+1) = \tilde{A}_5(k)\tilde{e}(k), & e_s(k-d_k) = \tilde{A}_2(k)\tilde{e}(k) \\ e_{es}(k-d_k) = \tilde{A}_4(k)\tilde{e}(k), & e_s(k) = \tilde{A}_7(k)\tilde{e}(k) \\ e_{es}(k) = \tilde{A}_8(k)\tilde{e}(k), & e_f(k) = \tilde{A}_9(k)\tilde{e}(k) \\ \hat{x}_f(k) = \tilde{A}_6(k)\tilde{e}(k), & \tilde{e}(k) = \omega_1 \xi(k) \end{cases} \quad (3.60.\mathrm{a})$$

Where ;

$$\omega_p \in R^{4n\times n}(p = 1, \ldots, 4) \text{ e.g. } \omega_1 = [I_n 0_n 0_n 0_n] \quad (3.60.\mathrm{b})$$

$$\xi(k) = [\tilde{e}^T(k)\tilde{e}^T(k-d_k)\tilde{e}^T(k-d_M)\tilde{e}^T(k-d_m)]^T \quad (3.60.\mathrm{c})$$

Our purpose is to present the necessary and sufficient stability conditions in terms of LMIs, to achieve the optimality of the hybrid state feedback control to guarantee the control performances using (3.55) for the faulty hybrid systems. This guarantee is ensured by analyzing the stability using the following Lyapunov-Krasovskii function (3.61). The scheme of the proposed robust hybrid fault-tolerant optimal predictive control (HFTPC) is shown in Figure 3.1.

Then, the following Lyapunov-Krasovskii function condidate is defined:

$$\begin{cases} V(k) = V_1(k) + V_2(k) + V_3(k) \\ V_1(k) = e_s^T(k) P e_s(k) \\ V_2(k) = \sum_{v=k-d_k}^{k-1} \tilde{e}^T(v) S \tilde{e}(v) \\ V_3(k) = \sum_{r=1-d_M}^{1-d_m} \sum_{s=k-1-r}^{k-1} \tilde{e}^T(s) V_0 \tilde{e}(s) \end{cases} \quad (3.61)$$

To guarantee the cost function $J(k)$ in (3.55) that is related to the global Lyapunov function (3.61), the following stability criterion should be satisfied:

$$V(\tilde{e}(k+i+1/k)) - V(\tilde{e}(k+1/k)) \leq -[\hat{X}(k+i) + U(k+i)] \quad (3.62.\mathrm{a})$$

As it is assumed that summation is up to ∞, i.e., $i \rightarrow \infty$, $\tilde{e}(i|k)$ should approach zero, i.e., $\hat{x}(\infty|k) = 0$. It yields:

$$J(k) \leq V\big(\tilde{e}(k/k)\big) \leq \gamma \tag{3.62.b}$$

The main findings of this part are presented in the next theorem.

Theorem 3.4 (Zahaf,2020)***:***

Consider the closed loop estimate of a faulty hybrid system with time-varying delay (3.59.a) and let the input hybrid feedback controller be defined by (3.54.c), which is based on the extended state observer and error dynamics, and which meets the performance (3.55.a) for faults and varying delay d_k, if there exists symmetric positive definite matrices Q, N, V, Y, Y_f, G, L, X, $Q_{l,\ l=1:6}$, $G_{l,\ l=1:6}$, and H satisfying the following convex optimization problem:

$$\min_{\gamma,Q,N,V,Y,G,L,X} \gamma \tag{3.63.a}$$

Subject to

$$\begin{bmatrix} -1 & {e_s}^T(k) \\ e_s(k) & -Q \end{bmatrix} < 0 \tag{3.63.b}$$

$$\begin{bmatrix} -u_{max}^2 & (Y+Y_f) \\ (Y+Y_f)^T & Q-G^T-G \end{bmatrix} \leq 0 \tag{3.63.c}$$

$$\begin{bmatrix} \Xi & * & * & * & * & * & * & * \\ 2(A_jG-B_jY_f)\tilde{A}_7\omega_1 & -Q & * & * & * & * & * & * \\ 2A_{jd}G\tilde{A}_2\omega_1 & 0_n & -Q & * & * & * & * & * \\ 2\tilde{G}\tilde{A}_8\omega_1 & 0_n & 0_n & Q & * & * & * & * \\ 2(-F_a)G\tilde{A}_9\omega_1 & 0_n & 0_n & 0_n & -Q & * & * & * \\ G\tilde{A}_7\omega_1 & 0_n & 0_n & 0_n & 0_n & Q & * & * \\ Q_0^{1/2}G\tilde{A}_1\omega_1 & 0_n & 0_n & 0_n & 0_n & 0_n & \gamma I & * \\ R_0^{1/2}(Y\tilde{A}_8+Y_f\tilde{A}_1)\omega_1 & 0_n & 0_n & 0_n & 0_n & 0_n & 0_n & \gamma I \end{bmatrix} < 0 \tag{3.63.d}$$

$$\Xi = Q1+Q2+Q3+Q4+Q5+Q6+(d+1)V_0+2(HG_1+H^TG_1^T)+(HG_2+H^TG_2^T) \\ -(HG_3++H^TG_3^T)-(HG_4++H^TG_4^T) \tag{3.63.e}$$

Where: $\tilde{G} = LC_jG$, and the feedback matrices K and K_f are given by $K = YG^{-1}$ and $K_f = Y_fG^{-1}$.

Proof:

We let the Lyapunov-Krasovskii functional candidate be (3.62) with $P > 0$, $S > 0$ and $V_0 > 0$ to be determined.

Taking the forward difference of $V(k)$ as $\Delta V(k) = V(k+1) - V(k)$, and with respect to time along the trajectory of the system, (3.59.a) yields:

1 / The difference of $V_1(k)$ is:

$$\Delta V_1(k) = {e_s}^T(k+1)Pe_s(k+1) - {e_s}^T(k)Pe_s(k) \quad (3.64.\mathrm{a})$$

$$= \left(\left(A_j - B_jK_f\right)e_s(k) + A_{jd}e_s(k-d_k) - LC_je_{es}(k) + (-F_a)e_f(k)\right)^T P\left(\left(A_j - B_jK_f\right)e_s(k) + A_{jd}e_s(k-d_k) - LC_je_{es}(k) + (-F_a)e_f(k)\right) - {e_s}^T(k)Pe_s(k) \quad (3.64.\mathrm{b})$$

$$= \tilde{e}^T(k){\tilde{A}_7}^T\left(A_j - B_jK_f\right)^T P\left(A_j - B_jK_f\right)\tilde{A}_7\tilde{e}(k) + \tilde{e}^T(k){\tilde{A}_2}^T{A_{jd}}^TPA_{jd}\tilde{A}_2\tilde{e}(k) + \tilde{e}^T(k){\tilde{A}_8}^T(LC_j)^TP\left(LC_j\right)\tilde{A}_8\tilde{e}(k) + \tilde{e}^T(k){\tilde{A}_9}^T(-F_a)^TP(-F_a)\tilde{A}_9\tilde{e}(k) + \Gamma(\tilde{e}, u, e_s, e_{es}, e_f) - \tilde{e}^T(k){\tilde{A}_7}^TP\tilde{A}_7\tilde{e}(k) \quad (3.64.\mathrm{c})$$

$$\Gamma(\tilde{e}, u, e_s, e_{es}, e_f) = \xi^T(k)\omega_1^T\left({\tilde{A}_7}^T\left(A_j - B_jK_f\right)^TPA_{jd}\tilde{A}_2 + {\tilde{A}_2}^T{A_{jd}}^TP\left(A_j - B_jK_f\right)\tilde{A}_7 - {\tilde{A}_7}^T\left(A_j - B_jK_f\right)^TP\left(LC_j\right)\tilde{A}_8 - {\tilde{A}_8}^T(LC_j)^TP\left(A_j - B_jK_f\right)\tilde{A}_7 + {\tilde{A}_7}^T\left(A_j - B_jK_f\right)^TP(-F_a)\tilde{A}_9 + {\tilde{A}_9}^T(-F_a)^TP\left(A_j - B_jK_f\right)\tilde{A}_7 - {\tilde{A}_2}^T{A_{jd}}^TP\left(LC_j\right)\tilde{A}_8 - {\tilde{A}_8}^T(LC_j)^TPA_{jd}\tilde{A}_2 + {\tilde{A}_2}^T{A_{jd}}^TP(-F_a)\tilde{A}_9 + {\tilde{A}_9}^T(-F_a)^TPA_{jd}\tilde{A}_2 - {\tilde{A}_8}^T(LC_j)^TP(-F_a)\tilde{A}_9 - {\tilde{A}_9}^T(-F_a)^TP\left(LC_j\right)\tilde{A}_8\right)\omega_1\xi(k) \quad (3.64.\mathrm{d})$$

$$= \xi^T(k)\omega_1^T\left({\tilde{A}_7}^T\left(A_j - B_jK_f\right)^TP\left(A_j - B_jK_f\right)\tilde{A}_7 + {\tilde{A}_2}^T{A_{jd}}^TPA_{jd}\tilde{A}_2 + {\tilde{A}_8}^T(LC_j)^TP\left(LC_j\right)\tilde{A}_8 + {\tilde{A}_9}^T(-F_a)^TP(-F_a)\tilde{A}_9 + \Gamma(\tilde{e}, u, e_s, e_{es}, e_f) - {\tilde{A}_7}^TP\tilde{A}_7\right)\omega_1\xi(k) \quad (3.65)$$

2 / The difference of $V_2(k)$ is:

$$\Delta V_2(k) = \sum\nolimits_{v=k+1-d_{k+1}}^{k} \tilde{e}^T(v)S\tilde{e}(v) - \sum\nolimits_{v=k-d_k}^{k-1} \tilde{e}^T(v)S\tilde{e}(v) \quad (3.66.\mathrm{a})$$

Applying *Lemma C.8,* we get:

$$\Delta V_2(k) = \tilde{e}^T(k)S\tilde{e}(k) - \tilde{e}^T(k-d_k)S\tilde{e}(k-d_k) \quad (3.66.\mathrm{b})$$

$$\Delta V_2(k) = \xi^T(k)\omega_1^T S\omega_1\xi(k) - \xi^T(k)\omega_2^T S\omega_2\xi(k) \quad (3.66.\mathrm{c})$$

$$\Delta V_2(k) = \xi^T(k)(\omega_1^T S\omega_1 - \omega_2^T S\omega_2)\xi(k) \quad (3.66.\mathrm{d})$$

3 / The difference of $V_3(k)$ is:

$$V_3(k) = \sum_{r=1-d_M}^{1-d_m}\sum_{s=k-r}^{k}\tilde{e}^T(s)V_0\tilde{e}(s) - \sum_{r=1-d_M}^{1-d_m}\sum_{s=k-1+r}^{k-1}\tilde{e}^T(s)V_0\tilde{e}(s) \quad (3.67.\mathrm{a})$$

$$= (d_k+1)\tilde{e}^T(s)V_0\tilde{e}(s) - \sum_{s=k-d_M}^{k-d_m}\tilde{e}^T(s)V_0\tilde{e}(s) \quad (3.67.\mathrm{b})$$

Utilizing *Lemma C.8* again:

$$= (d_k+1)\tilde{e}^T(k)V_0\tilde{e}(k) - \tilde{e}^T(k-d_M)V_0\tilde{e}(k-d_M) - \tilde{e}^T(k-d_m)V\tilde{e}(k-d_m) \quad (3.67.\mathrm{c})$$

$$\Delta V_3(k) = \xi^T(k)((d+1)\omega_1^T V_0\omega_1 - \omega_3^T V_0\omega_3 - \omega_4^T V_0\omega_4)\xi(k) \quad (3.67.\mathrm{d})$$

Substituting $\Delta V(k)$ in (3.62), we obtain:

$$\begin{aligned} &= \xi^T(k)\omega_1^T\Big(\tilde{A}_7{}^T\big(A_j - B_jK_f\big)^T P\big(A_j - B_jK_f\big)\tilde{A}_7 + \tilde{A}_2{}^T A_{jd}{}^T P A_{jd}\tilde{A}_2 + \tilde{A}_8{}^T(LC_j)^T P\big(LC_j\big)\tilde{A}_8 \\ &\quad + \tilde{A}_9{}^T(-F_a)^T P(-F_a)\tilde{A}_9 + \Gamma(\tilde{e},u,e_s,e_{es},e_f) - \tilde{A}_7{}^T P\tilde{A}_7)\omega_1\xi(k) \\ &\quad + \xi^T(k)(\omega_1^T S\omega_1 - \omega_2^T S\omega_2 + (d+1)\omega_1^T V_0\omega_1 - \omega_3^T V_0\omega_3 - \omega_4^T V_0\omega_4\Big)\xi(k) \\ &\quad \le -\tilde{e}^T(k)Q_0\tilde{e}(k) - u_f^T(k)R_0u_f(k) \end{aligned} \quad (3.68.\mathrm{a})$$

Then

$$\begin{aligned} &= \omega_1^T G^T\Big(\tilde{A}_7{}^T\big(A_j - B_lK_f\big)^T P\big(A_j - B_lK_f\big)\tilde{A}_7 + \tilde{A}_2{}^T A_{ld}{}^T P A_{jd}\tilde{A}_2 + \tilde{A}_8{}^T(LC_j)^T P\big(LC_j\big)\tilde{A}_8 \\ &\quad + \tilde{A}_9{}^T(-F_a)^T P(-F_a)\tilde{A}_9 + \Gamma(\tilde{e},u,e_s,e_{es},e_f) - \tilde{A}_7{}^T P\tilde{A}_7\Big)G\omega_1 \\ &\quad + \omega_1^T G^T SG\omega_1 - \omega_2^T G^T SG\omega_2 + (d+1)\omega_1^T G^T V_0G\omega_1 - \omega_3^T G^T V_0G\omega_3 \\ &\quad - \omega_4^T G^T V_0G\omega_4 + \omega_1^T G^T Q_0G\omega_1 + \omega_1^T\big(Y\tilde{A}_8 + Y_f\tilde{A}_1\big)^T R_0\big(Y\tilde{A}_8 + Y_f\tilde{A}_1\big)\omega_1 \\ &\quad \le 0 \end{aligned} \quad (3.68.\mathrm{b})$$

Based on *Assumption 3.5*, the previous inequality, leads to:

$$\omega_1^T\Big(2(\tilde{A}_7{}^T\big(A_jG - B_jY_f\big)^T P\big(A_jG - B_jY_f\big)\tilde{A}_7 + 2\tilde{A}_2{}^T G^T A_{jd}{}^T P A_{jd} G\tilde{A}_2$$
$$- 2\tilde{A}_8{}^T G^T (LC_j)^T P\big(LC_j\big)G\tilde{A}_8 + 2\tilde{A}_9{}^T G^T(-F_a)^T P(-F_a)G\tilde{A}_9)$$
$$- \tilde{A}_7{}^T G^T P G\tilde{A}_7\Big)\,\omega_1 + \Lambda(Q_i) + \omega_1^T G^T S G\omega_1 - \omega_2^T G^T S G\omega_2$$
$$+ (d+1)\omega_1^T G^T V_0 G\omega_1 - \omega_3^T G^T V_0 G\omega_3 - \omega_4^T G^T V_0 G\omega_4 + \omega_1^T G^T Q_0 G\omega_1$$
$$+ \omega_1^T\big(Y\tilde{A}_8 + Y_f\tilde{A}_1\big)^T R_0\big(Y\tilde{A}_8 + Y_f\tilde{A}_1\big)\omega_1 \le 0 \qquad (3.68.\mathrm{c})$$

Where :

$$\Lambda(Q_i) = Q1 + Q2 + Q3 + Q4 + Q5 + Q6 \qquad (3.68.\mathrm{d})$$

Applying Lemma C.5 on next term:

$$(\omega_1^T G^T S G\omega_1 - \omega_2^T G^T S G\omega_2 + (d+1)\omega_1^T G^T V_0 G\omega_1 - \omega_3^T G^T V_0 G\omega_3$$
$$- \omega_4^T G^T V_0 G\omega_4) \qquad (3.68.\mathrm{e})$$

We put : $G_p = G\omega_p,\ p = 1, \dots, 4$

And we get:

$$\Phi = (d_k+1)V_0 + 2(HG_1 + H^T G_1^T) + (HG_2 + H^T G_2^T) - (HG_3 + +H^T G_3^T)$$
$$- (HG_4 + +H^T G_4^T) \qquad (3.68.\mathrm{f})$$

By substituting $P = \gamma Q^{-1}$,

Using the Schur complement to (3.68.c), we get (3.63.d)

$$\begin{bmatrix} \Xi & * & * & * & * & * & * & * \\ 2(A_jG - B_jY_f)\tilde{A}_7\omega_1 & -Q & * & * & * & * & * & * \\ 2A_{jd}G\tilde{A}_2\omega_1 & 0_n & -Q & * & * & * & * & * \\ 2\tilde{G}\tilde{A}_8\omega_1 & 0_n & 0_n & Q & * & * & * & * \\ 2(-F_a)G\tilde{A}_9\omega_1 & 0_n & 0_n & 0_n & -Q & * & * & * \\ G\tilde{A}_7\omega_1 & 0_n & 0_n & 0_n & 0_n & Q & * & * \\ Q_0^{1/2}G\tilde{A}_1\omega_1 & 0_n & 0_n & 0_n & 0_n & 0_n & \gamma I & * \\ R_0^{1/2}(Y\tilde{A}_8 + Y_f\tilde{A}_1)\omega_1 & 0_n & 0_n & 0_n & 0_n & 0_n & 0_n & \gamma I \end{bmatrix} < 0 \quad (3.68.\mathrm{g})$$

- For the LMI (3.63.b), it is obtained by the following steps:

  The performance index $j(k)$ may admit an upper bound like γ. Then:

$$e_s{}^T(k/k)\,P\,e_s(k/k) \le \gamma \qquad (3.69.\mathrm{a})$$

Thus, the upper bound would be found from the minimization problem. The inequality (3.69.a) may be written as follows:

$$e_s{}^T\,(k/k)\,Q^{-1}\,e_s\,(k/k) \leq I \tag{3.69.b}$$

As a result, the inequality (3.63.b) would be obtained using Schur's complement lemma.

- For the input constraint LMI formulation, it is obtained as follows:

$$|u_h(k+i/k)| \leq u_{h,max}\,, \quad i \geq 0, \quad h = 1,2,\dots,q \tag{3.70.a}$$

$$\|u(k+i/k)\|_{max} \triangleq \max_i u_i(k+i/k) \tag{3.70.b}$$

$$\max_{i>0}\|u(k)\|_{max} = \max_{i>0}\|F\hat{x}(k)\|_{max} \tag{3.70.c}$$

$$\max_{i>0}\|u(k)\|_{max} \geq \max_{a\in\Re}|Fa|^2 \Leftrightarrow \tag{3.70.d}$$

$$\max_{i>0}\|u(k)\|_{max} \geq \left\|FP^{\frac{1}{2}}\right\|_2^2 \tag{3.71.a}$$

$$u_{max} \geq \bar{\sigma}\left[FQ^{-\frac{1}{2}}\right] \Leftrightarrow u_{max}^2 \geq Q^{-\frac{1}{2}}F^T F Q^{-\frac{1}{2}} \tag{3.71.b}$$

$$-u_{max}^2 + Q^{-\frac{1}{2}}F^T F Q^{-\frac{1}{2}} \leq 0 \tag{3.71.c}$$

Using Schur complement, we obtain:

$$\begin{bmatrix} -u_{max}^2 & (YG^{-1} + Y_f G^{-1}) \\ (YG^{-1} + Y_f G^{-1}) & -Q \end{bmatrix} \leq 0 \tag{3.71.d}$$

Multiplying the right by $\begin{bmatrix} I & O \\ 0 & G \end{bmatrix}$ and the left by $\begin{bmatrix} I & O \\ 0 & G^T \end{bmatrix}$, we get (3.63.c):

$$\begin{bmatrix} -u_{max}^2 & (Y + Y_f) \\ (Y + Y_f)^T & Q - G^T - G \end{bmatrix} \leq 0 \tag{3.71.e}$$

This ends the proof of theorem 3.5.

❖ ***How it Can Set-Up a Hybrid Control Approach to Deal with Actuators Faults and Time-Delay?***

Since the presented results in Theorem 3.4 are developed to deal with occurring faults and time-delay. We consider that HS is tuning according to (3.51). At the moment that faults occur, the AFTC strategy is activated for the computation of the optimal control to compensate the faulty behavior, based on the given conditions in theorem 3.4; where it is considered as a discrete dynamic since the faults happened in discrete instants.

However, at instant $t = t_f + i$, the reconfigurable controller is handling the fault effect. By the way, it is not required to have an explanation that the obtained controller from Theorem 3.4 have reliable specifications and robust performances. Our aim is applying an alternative scenario to avoid Zeno phenomena during the control process for HS. If there are no faults and only time-delay, where this latter is often happening in process control and applications, we consider that the control approach to deal with time-delay is only the continuous dynamic in the hybrid control strategy.

In this part, it is assumed that the states of the hybrid actuators system are not measurable at due time, influenced by time-delay of actuators or the transmission-delay. So, to estimate the state with time delay in the feedback channel, an observer is designed as follow:

$$\begin{cases} \hat{x}(k+1) = A_j\hat{x}(k) + A_{jd}\hat{x}(k-d_k) + B_ju_f(k) + L\big(y(k) - \hat{y}(k)\big) \\ \hat{y}(k) = C_j\hat{x}(k) \end{cases} \tag{3.72}$$

With the adaption of (3.58) with (3.51.) and (3.72) using (3.6), and using the control law of (3.54), taking into consideration that K_f represents the controller gain to be generated in case of time-delay. The necessary conditions of the closed loop discrete-time hybrid system with time-delay for the optimization problem is represented as follows:

$$\begin{cases} \hat{x}(k+1) = \big(A_j - B_jK_f\big)\hat{x}(k) + A_{jd}\hat{x}(k-d_k) - LC_je(k) \\ e(k+1) = \big(A_j - LC_j\big)e(k) + A_{jd}e(k-d_k) \end{cases} \tag{3.73}$$

The new augmented system related to time-delay can only be written as:

$$\tilde{x}(k+1) = \tilde{A}(k)\tilde{x}(k) \tag{3.74.a}$$

Where:

$$\tilde{x}(k) = [\hat{x}^T(k) \quad \hat{x}^T(k-d_k) \quad e^T(k) \quad e^T(k-d_k)]^T \tag{3.74.b}$$

$$\tilde{A}(k) = \begin{bmatrix} \tilde{A}_1(k) \\ \tilde{A}_2(k) \\ \tilde{A}_3(k) \\ \tilde{A}_4(k) \end{bmatrix} = \begin{bmatrix} (A_j - B_j K_f) & A_{jd} & -LC_j & 0_n \\ 0_n & 0_n & (A_j - LC_j) & A_d \\ I_n & 0_n & 0_n & 0_n \\ 0_n & I_n & 0_n & 0_n \end{bmatrix} \tag{3.74.c}$$

$$\tilde{A}_5(k) = [0_n 0_n I_n 0_n] \tag{3.74.d}$$

Then, we can describe the different variables as follows:

$$\begin{cases} \hat{x}(k+1) = \tilde{A}_1(k)\tilde{x}(k), & e(k+1) = \tilde{A}_2(k)\tilde{x}(k) \\ \hat{x}(k) = \tilde{A}_3(k)\tilde{x}(k), & \hat{x}(k-d_k) = \tilde{A}_4(k)\tilde{x}(k) \\ e(k) = \tilde{A}_5(k)\tilde{x}(k), & \tilde{x}(k) = \omega_1 \xi(k) \end{cases} \tag{3.74.e}$$

The main reason to provide additional conditions as a continuous dynamic for the control approach is avoiding that the LMI (3.63.d) is ***Nonsingular,*** at the time to compute the optimal control, in case of time-delay without occurring faults. In order to avoid Zeno phenomena for the control strategy, after dealing with actuators faults. Therefore, the next theorem is derived from theorem 3.4.

Theorem 3.5:

Consider the closed loop estimate of hybrid system with time-varying delay (3.59.a) and let the input hybrid feedback controller be defined by (3.54.c), which is based on the extended state observer and error dynamics, and which meets the performance (3.55.a) for faults and varying delay d_k, if there exists symmetric positive definite matrices Q, N, V, Y, Y_f, G, L, X, $Q_{l,\, l=1:6}$, $G_{l,\, l=1:6}$, and H satisfying the following convex optimization problem:

$$\min_{\gamma,Q,N,V,Y,G,L,X} \gamma \tag{3.75.a}$$

Subject to

$$\begin{bmatrix} -1 & e^T(k) \\ e(k) & -Q \end{bmatrix} < 0 \tag{3.75.b}$$

$$\begin{bmatrix} -u_{max}^2 & (Y+Y_d) \\ (Y+Y_d)^T & Q-G^T-G \end{bmatrix} \leq 0 \quad (3.75.\text{c})$$

$$\begin{bmatrix} \Psi & * & * & * & * & * & * \\ 2(A_jG-B_jY_f)\tilde{A}_3\omega_1 & -Q & * & * & * & * & * \\ 2A_{ld}G\tilde{A}_4\omega_1 & 0_n & -Q & * & * & * & * \\ 2\tilde{G}\tilde{A}_5\omega_1 & 0_n & 0_n & Q & * & * & * \\ G\tilde{A}_7\omega_1 & 0_n & 0_n & 0_n & Q & * & * \\ Q_0^{1/2}G\tilde{A}_1\omega_1 & 0_n & 0_n & 0_n & 0_n & \gamma I & * \\ R_0^{1/2}(Y\tilde{A}_8+Y_f\tilde{A}_1)\omega_1 & 0_n & 0_n & 0_n & 0_n & 0_n & \gamma I \end{bmatrix} < 0 \quad (3.75.\text{d})$$

$$\Psi = Q1+Q2+Q3+Q4+(d+1)V_0+2(HG_1+H^TG_1^T)+(HG_2+H^TG_2^T) \\ -(HG_3++H^TG_3^T)-(HG_4++H^TG_4^T) \quad (3.75.\text{e})$$

Where: $\tilde{G} = LC_jG$, and the feedback matrices K and K_f are given by $K = YG^{-1}\ and\ K_f = Y_dG^{-1}$.

Proof:

Following steps on the proof of theorem 3.4, then the main results in theorem 3.5 are obtained.

For a best visibility of the proposed hybrid control approach HFTPC, the next Algorithm summary the stages of the transition between control modes, as a discrete and continuous dynamic.

Algorithm 3.2.

Step 1: Initialize the model parameters by giving an admissible controller Gain,

Step 2: According to the general transition "Switching" function, activate and apply the j^{th} sub-system,

Step 3: Solve the LMIs problem in Theorem 3.5 to compute a feasible solution, as controller scheme for the continuous dynamic,

Step 4: The feasibility of the problem results in robustly asymptotically stabilizing matrices K, K_f and L,

Step 5: If a fault occurred, activate the discrete dynamic by replacing the set of LMIs to be solved in Step 3 by the LMIs in the theorem 3.4. Otherwise, move to the next step,

Step 6: Update the transition function for the $(j^{th} + 1)$ sub-system if it exists,

Step 7: Stop when the optimization problem (3.10), (cost function) results in optimality solution and robustly asymptotically stabilizing matrices K, K_f and L; Otherwise, let *k*=*k+1* and go back to Step 3.

To give a best visibility and show the effectiveness of the proposed control approach HFTPC (Theorem 3.4); Table 3.1 performs and presents a performances comparison and comparison study with similar and related works in the literature.

Table 3.1 *Comparison Study*

Our Proposed Control Approach HFTPC	Other Approaches	Corresponding References
The proposed control approach based on model predictive control is globally asymptotically stable based on Lyapunov-Krasovskii function.	The control strategy based on Model predictive control was analyzed without stability study	(Ishihara, 2018, Klancar,2007; Cavanini,2018)
The tracking desired position converges more smoothly to the origin due. While the closed-loop tracking error converges exponentially to the range set due. While the robustness is improved even in presence of faults.	The closed-loop system is uniformly ultimately bounded stable, and the tracking error converges exponentially only to a compact set, this is due to the conservative conditions in online optimization.	(Wu,2017; Ishihara,2018; Sun,2016; Klancar,2007)
No fault detection and isolation (FDI) loop is used, the controller deals directly with the occurrence of actuator faults, by updating the controller to compensate the undesirable effect.	- Fault detection and isolation (FDI) loop was introduced.	(Li,2018)
	- No Fault detection and isolation (FDI) loop were introduced.	(Wang,2017)
The hybrid system based on real physical hybrid actuators (Pneumatic and Electric).	- Considering the switching mode as a hybrid system	(Wang,2017)
The controller is adapting online to compensate the actuator fault's effect based on estimated actuator faults and error dynamics and estimated states, the reformulation of the constrained hybrid predictive problem in terms of LMIs has allowed us easily to both integrate MPC strategy and solve the optimization problem taking into account the constraints and actuator faults.	The controller is adapted online to compensate the fault's effect based on predictions states only.	(Ishihara, 2018; Klancar,2007; Cavanini,2018)
	The control law is adapted based on solving the optimization problems based on Algebraic Riccati Equation.	(Wang,2017)
Less conservative stable conditions.	Need more conditions to make the problem less conservatism and more time for the controller's computing time load.	(Wu,2017; Noda,2014 ; Sun,2016; Klancar,2007; Cavanini,2018)

III.6 Conclusion

In this chapter, we presented the theoretical results of fault-tolerant control based predictive control combined with the state observer of the continuous dynamic. Different theorems are presented based on the nature of the undesirable associated input "faults and time-delay". In this regard, a fault-tolerant based model predictive control for HS using QP optimization method is proposed to compute the optimal control. In order to improve the fault tolerant predictive control, different theorems are presented by minimizing an upper bound of the cost function in infinite horizon in terms of LMIs. However, the transformation of the nonlinear problem into a convex formulation has led to less restrictive conditions to reach the domains of validity, in addition to less conservative terms based on the constraints reformulation. The proposed HFTPC keeps up good tracking performances with acceptable dynamics. Moreover, unexpected inputs as faults do not induce a problem of infeasibility or instability, and the constraints remain respected. Therefore, new conditions in terms of LMIs subject to the constraints have been developed to obtain a robust HFTPC; wherein the convex optimization problem makes it possible to have reasonable computation times and to reach a unique global minimum of the cost function. The proposed strategy HFTPC with input and output constraints has shown its effectiveness in nominal process and in the presence of delays and faults. These proposed approaches are preceded by a study of the observability concept for hybrid systems based on coupled of observer-predictive control theory.

Chapter IV

Control of Hybrid Systems: Examples, Simulations and Discussions

In this chapter, we present and outline the reliability of the proposed control approaches in the chapter of the theoretical results, by simulation of results for some hybrid models systems raised in chapter 1. In this dissertation, ensuring the effectiveness and robustness of the controller to deal with undesirable associated inputs "faults and time-delay" is presented for validation through proposed tests according to our proposed framework of hybrid systems in chapter one. Thus, discussions are raised about the obtained results.

IV.1 Introduction

In this part, we present some illustrative examples with different hybrid models inspired from the framework described in chapter one, after an examination and analysis of this class. Since our focus is to show the ability of our proposed strategy, by designing a reliable RMPC approach applied on systems of hybrid nature to compute the optimal control of such problems; a specific control design has been studied and proposed in chapter three. This leads to an existence of an optimal control for hybrid systems, according to relaxed and less conservative conditions in terms of LMIs, in the presence of faults and time delay, such that the stability and the desired performances are ensured. To illustrate the results of chapter 3, we have divided our study into two distinct parts: the first case is without faults or time-delay, we consider an academic example of hybrid systems to verified the proposed approach concerning the observability. In the second part, we study an academic example with sensors failures, an inverted pendulum on cart and an industrial robot arm with hybrid actuators, wherein time-delay and faults on the system are considered. The obtained results of the robot arm show outperformance compared with that obtained from model predictive control scheme using QP method, and also the existing results in the literature.

The simulations were carried out using Matlab®, and for the computation of the control input, we have used the LMI toolbox.

IV.2 Observability of Hybrid Systems with State-Dependent Switching Framework: Servo Control for Network Systems

In order to illustrate the aim of this section, we use a servo control system to perform the proposed strategy by providing a new insight of the combination between the robust MPC and state observer, to design less conservative conditions to study the observability conditions for HS (Zahaf,2017). Therefore, we consider the hybrid discrete time model as follows:

We define some parameters for the optimization problem:

$$A_1 = \begin{bmatrix} 1.120 & 0.213 & -0.335 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & -0.053 & 0.1 \\ 0.8 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [0.0541 \quad 0.1150 \quad 0.0001]$$

The weighting matrices are: $Q_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_0 = 0.5,$

The initials conditions are: $\hat{x} = [0\ 0\ 0]^T$

The presented example has a switching nature, according the State-Dependent Switching rule. In this case, we consider that the state variables are unmeasurable. This example highlights the problem of observability for the hybrid systems with switching feature.

According to figures 4.1 and 4.2, the simulation results show the effectiveness of the proposed strategy raised in section 3.3 of chapter 3. Without faults, this approach is developed to estimate the unmeasurable states of the systems; since our observer can have multiple functions, depending on what we are implementing, as detection of faults and estimation of output behavior.

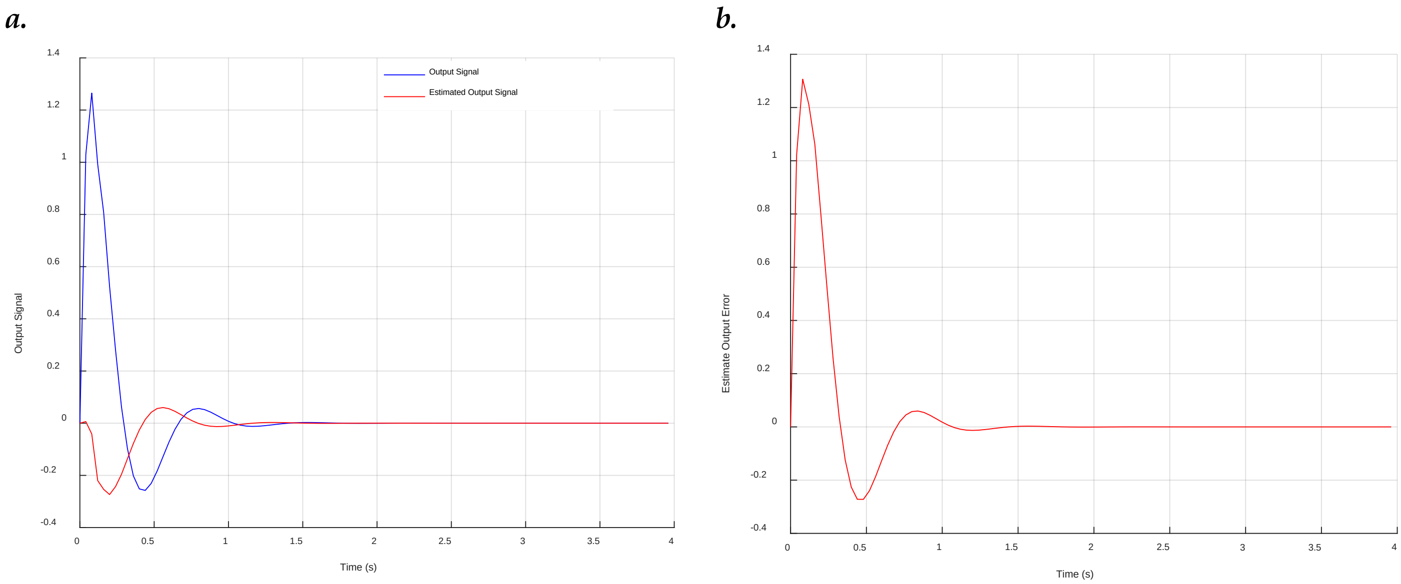


Fig. 4.1. *Evolution of the Network System:*

(a) *Output Signal and Estimated Output Signal.* ***(b)*** *Estimated Output Error.*

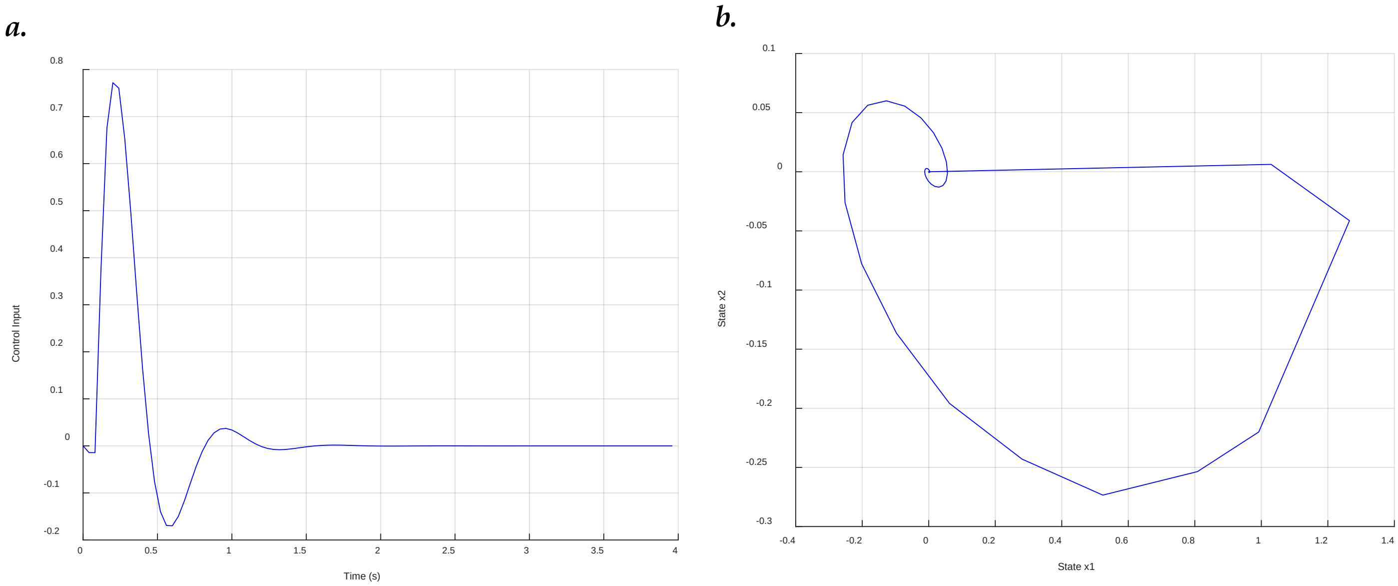


Fig. 4.2. Evolution of: (a) Control Input. (b) Switching Signal on State Space.

Figures 4.1.a and 4.1.b show up the output signal and the estimated error dynamics. We remark that the estimated system output converges asymptotically to the desired output system, with an acceptable and very small error. In order to present the efficiency of applying the control strategy from section 3.3 of chapter 3; that is basically established from the combination between robust MPC and observer.

Where the switching signal is presented on the state-space diagram to show the sensitivity of the transition between sub-systems. Figure 4.2.b shows up low sensitivity at transitions moments between sub-systems, while Figures 4.2.a show up the stabilizable controller derived from sufficient and necessary conditions.

IV.3 Continuous Switched Systems

This example is inspired from (Wang, 2017); similarly, to the injection molding application, we propose a CSS example with sensor. The main objective to use sensor is providing information's about the successful in reaching the intended action of each mode or not, to avoid the delay-processing. However, an expected fault is occurred on the sensor at t_i of the time processing which led to the failure of sensor; therefore, we apply the proposed FTC approach that based on a pertinent reliable estimation task and proposed control law in section 3.5.1 of chapter 3 "§3", to compensate this failures and keep-up the application processing at schedule time. Therefore, we consider the hybrid discrete time model as follows:

We defining some parameters for the optimization problem:

$$A_1 = \begin{bmatrix} 0.675 & 0.38 \\ 0.002 & 0.067 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.01 & 0 \\ 0.001 & 0.352 \end{bmatrix}, \quad C_1 = [1 \quad 0.5],$$

$$A_2 = \begin{bmatrix} 0.054 & 0.012 \\ 0 & 0.067 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.01 & 0.1 \\ 0.01 & 0.812 \end{bmatrix}, \quad C_2 = [1 \quad 0.5],$$

The weighting matrices are: $\mathrm{Q}_0 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad \mathrm{R}_0 = 1,$

The initials conditions are: $\hat{x} = [0\ 0]^T$

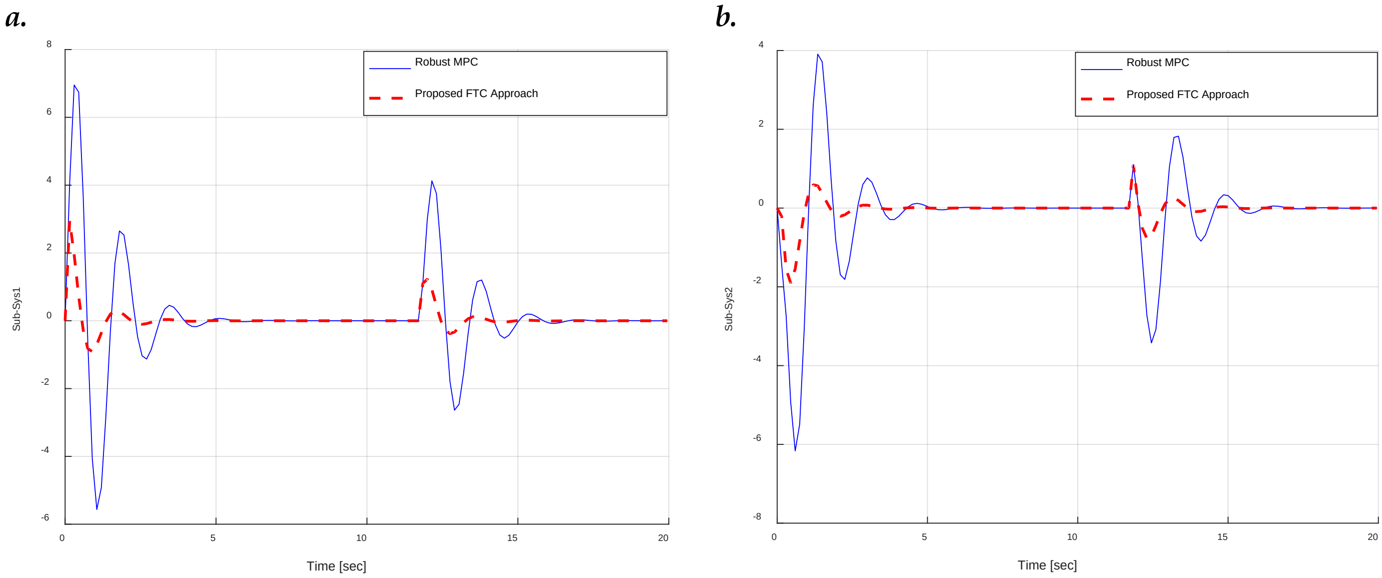


Fig. 4.3. Evolution of Continuous Switched System with Two Approach: Robust MPC and the Proposed FTC Approach to Cope Sensor Failures. (a) Evolution of the Mode 1 "Sub-Sys1". (b) Evolution of the Mode 2 "Sub-Sys2".

Figures 4.3.a, 4.3.b, show the evolution of the output dynamic of each mode represented by its sub-system. The applied control input in Figures 4.4.a has an undergoes a loss of accuracy of 55% due to a fault on sensor, for each i^{th} sub-system for the j^{th} batch start-up; followed by a complete failure "can called freezing" in Figures 4.4.b affected by an intense fault at time $t = 11.7$ $[s]$, which corresponds of a successive faults on the sensor. This faulty behavior is compensated and controlled by the control input u obtained from the optimized problem using the proposed FTC approach for sensors failures in chapter 3. With noting that the instant of the fault occurrence is assumed to be unknown during the simulation. Meanwhile the estimation and compensation of the faulty behavior is done better than the free compensation of sensor failures using the robust MPC. So, the proposed FTC scheme is applied to compute the control law, in order to reduce the effect of the sensor fault/failure on the system based on the estimation task and the proposed control law.

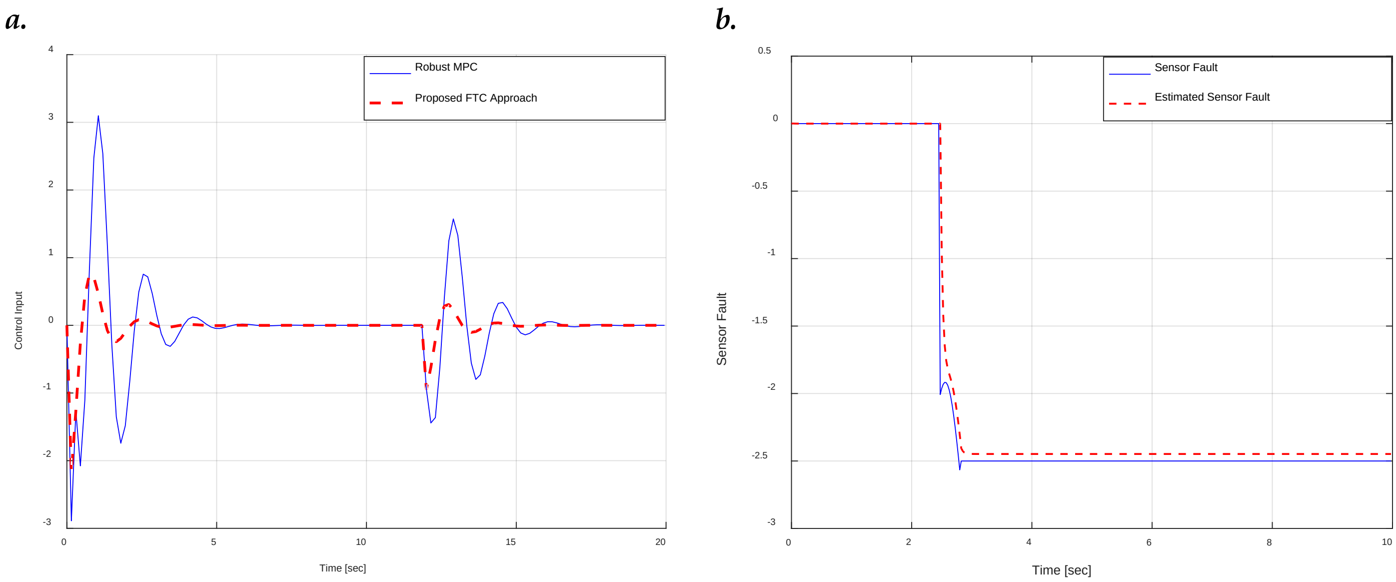


Fig. 4.4. Evolution of the CSS:

(a) Control Input. (b) Sensor Fault and its Estimation.

IV.4 Auxiliary Hybrid Systems with State-Dependent Switching based on T-S Fuzzy Framework: An Inverted Pendulum

Let us consider the privileged classical example that is considered as one among the common robotic paradigm in systems control theory, an inverted pendulum on a cart (Figure 4.5), where the dynamic equation is given below in (Teixeira,1999). Before validating the proposed approach in section 3.5.2 of chapter 3 "§3", for balancing the inverted pendulum to the desired range of vertical angle and copes with the time-delay and actuators' faults. We consider that the paradigm inspired from "§1" to describe the studied hybrid system is a combination of HS with state-dependent switching based on T-S fuzzy framework and auxiliary hybrid system, with pneumatic-electric hybrid actuator for the cart movements.

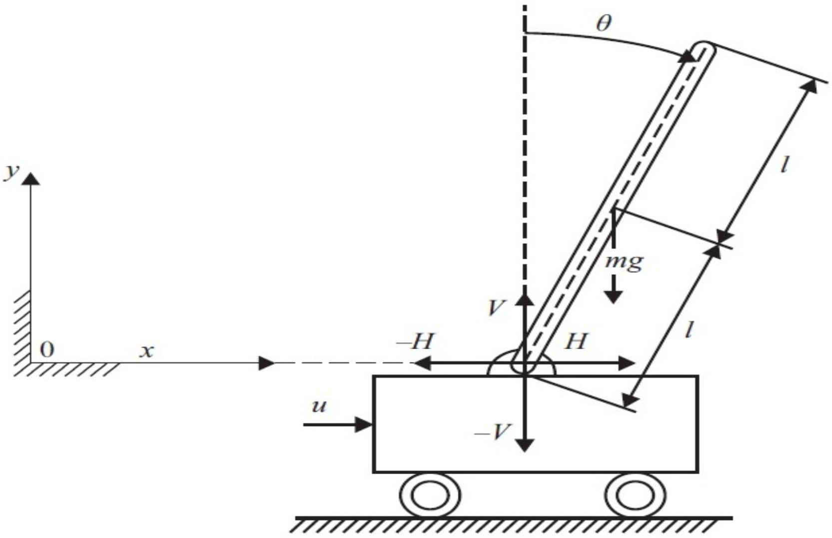


Fig. 4.5. An inverted pendulum on a cart (Ogata,1997).

The dynamic model is described as:

$$\begin{cases} \dot{x}_1(t) = & x_2(t) \\ \dot{x}_2(t) = & \dfrac{(g\,sin(x_1(t)) - aml\,x_2^2(t)\sin(2x_1(t))/2 + a\,cos(x_1(t))\,(f_c - u_r))}{(4l/3 - aml\,cos^2(x_1(t)))} \\ \dot{x}_3(t) = & x_4(t) \\ \dot{x}_4(t) = & \dfrac{-(amg\ \sin(2x_1(t))/2) + ((4aml\,x_2^2(t)\sin(x_1(t)))/3) + (4a(f_c - u_r)/3)}{(4/3 - am\,cos^2(x_1(t)))} \end{cases} \tag{4.1}$$

Where $x_1(t) = \theta$, $x_2(t) = \dot{\theta}$, $x_3(t) = x$ and $x_4(t) = \dot{x}$, are the pendulum angular position, pendulum angular velocity and the cart position and cart velocity, respectively, u_r and u denotes control inputs (external force) in presence of faults and external force F, respectively. In real application external forces have limited for robustness's control defined by $|F| \leq F_{max}$.

Compared with (Teixeira,1999) and (Boubaker,2017), the input control is related in this thesis to time-delay (coupled time-delay of the state and input) presented in (4.2), and occurring fault in the hybrid actuators, where the pneumatic actuator is a dominant actuator to generate joint velocity of the cart movements, with the assistance of the electric motor to achieve precise movements. Therefore, it's can be written as:

$$u_r(t) = u(t) + \rho\,\eta(t) \tag{4.2}$$

Where $\eta(t) = \epsilon\, x_4(t)\, cos(x_1(t))$, with $a = 1/m + M$ and $f_c = \rho\, sign\,\big(x_4(t)\big)$ represents the friction force between the cart and the track.

Then, using the control law (4.1) for FTC scheme, we rewrite (4.2) as:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{(g\, sin\big(x_1(t)\big) - aml\, x_2^2(t) \sin\big(2x_1(t)\big)/2 - a\rho\, \epsilon\, x_4(t)\, cos(x_1(t)) + a\, cos(x_1(t))\,(\,f_c - u))}{(4l/3 - aml\, cos^2(x_1(t)))} \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = \dfrac{-(amg\ \sin\big(2x_1(t)\big)/2) + ((4aml\, x_2^2(t) \sin\big(x_1(t)\big))/3) - a\,\rho\epsilon\, x_4(t) + (4a(\,f_c - u)/3)}{(4/3 - am\, cos^2(x_1(t)))} \end{cases} \tag{4.3}$$

Then the dynamic model is described as follows:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \underbrace{\begin{bmatrix} x_2(t) \\ \dfrac{(g\, sin\big(x_1(t)\big))}{(4l/3 - aml\, cos^2(x_1(t)))} \\ x_4(t) \\ \dfrac{-(amg\ \sin\big(2x_1(t)\big)/2)}{(4/3 - am\, cos^2(x_1(t)))} \end{bmatrix}}_{A(x(t))} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{-a\,\rho\,\epsilon\, x_4(t)\, cos(x_1(t))}{(4l/3 - aml\, cos^2(x_1(t)))} \\ 0 \\ \dfrac{-a\,\rho\,\epsilon\, x_4(t)}{(4/3 - am\, cos^2(x_1(t)))} \end{bmatrix}}_{A_d(x(t-t_d))}$$
$$+ \underbrace{\begin{bmatrix} 0 \\ \dfrac{a\, cos(x_1(t))}{(4l/3 - aml\, cos^2(x_1(t)))} \\ 0 \\ \dfrac{4a/3}{(4l/3 - aml\, cos^2(x_1(t)))} \end{bmatrix}}_{B(x(t))} (u(t)\ \underbrace{-f_c + aml\, x_2^2(t)\, sin(x_1(t))}_{\varphi(t)}\,) \tag{4.4}$$

Therefore, two fuzzy rules are used to describe the dynamic local model representation of the inverted-pendulum, following the steps in (Teixeira,1999) and (Boubaker,2017) the switched systems based on fuzzy paradigm are given as follows:

$$\begin{aligned} &Rule\ 1\!:\ if\ \ x_1(t)\ \ is\ near\ 0, \\ &\qquad then\ \ \dot{x}(t) = A_1 x(t) + A_{1d} x(t - t_d) + B_1(u(t) -\ \varphi(t)) \\ &Rule\ 2\!:\ if\ \ x_1(t)\ \ is\ \pm\pi/4, \\ &\qquad then\ \ \dot{x}(t) = A_2 x(t) + A_{2d} x(t - t_d) + B_2(u(t) -\ \varphi(t)) \end{aligned} \tag{4.5}$$

The state matrices A_i, A_{id} and B_i are:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{g}{(4l/3) - aml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{-a\,mg}{(4/3) - am} & 0 & 0 & 0 \end{bmatrix}, A_{1d} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{-a\,\rho\,\epsilon}{(4l/3) - aml} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{-a\,\rho\,\epsilon}{(4/3) - am} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{g\,2\sqrt{2}/\pi}{(4l/3) - (aml/2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{-a\,mg\,\frac{2}{\pi}}{(4/3) - (am/2)} & 0 & 0 & 0 \end{bmatrix}, A_{2d} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{-a\,\rho\,\epsilon\,\sqrt{2}/2}{(4l/3) - (aml/2)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{-a\,\rho\,\epsilon}{(4/3) - (am/2)} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ \dfrac{-a}{(4l/3) - aml} \\ 0 \\ \dfrac{4a/3}{(4/3) - am} \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ \dfrac{-a\,\sqrt{2}/2}{(4l/3) - (aml/2)} \\ 0 \\ \dfrac{(4a/3)}{(4/3) - (am/2)} \end{bmatrix},$$

The numerical values of the parameters are within table 4.1:

Where $a = 1/(m + M) = 0.769,\ b = \cos(88^{\circ}),$

TABLE 4.1. *INVERTED PENDULUM PARAMETERS*

Parameters	Description	Numerical value
l	Distance from the joint to the mass point m	0.5 m
m	Point Weight of the Pendulum	0.35 Kg
M	Weight of the Cart	0.95 Kg
g	Gravity	9.8 m/s^2
ρ	Cart friction coefficient	0.07 N s/rad
θ	Range Angle Joint	-π/5 ~ π/5
ϵ	Viscous friction of the joint	$\cos(88^{\circ})$ or 0.05 N s/rad
L	Total length of rail	3 m

With the substitution of the previous parameters we get:

$$A_1=\begin{bmatrix}0&1&0&0\\18.4210&0&0&0\\0&0&0&0\\-2.4787&0&0&0\end{bmatrix},A_{1d}=\begin{bmatrix}0&0&0&0\\0&0&0&-0.005056\\0&0&0&0\\0&0&0&-0.002527\end{bmatrix},\quad B_1=\begin{bmatrix}0\\-1.4454\\0\\0.9635\end{bmatrix},$$

$$A_2=\begin{bmatrix}0&1&0&0\\14.7296&0&0&0\\0&0&0&0\\-1.4015&0&0&0\end{bmatrix},A_{2d}=\begin{bmatrix}0&0&0&0\\0&0&0&-0.003170\\0&0&0&0\\0&0&0&-0.002244\end{bmatrix},\quad B_2=\begin{bmatrix}0\\-3.6292\\0\\0.8553\end{bmatrix},$$

To obtain the discrete switched system based on T-S fuzzy rules, we consider the sampling period be 0.03s; therefore, we obtain discrete switched fuzzy system with matrices states as follows:

$$A_1=\begin{bmatrix}1.0083&0.0301&0&0\\0.5542&1.0083&0&0\\0&0&1&0\\-0.0746&-0.0011&0&1\end{bmatrix},A_{1d}=\begin{bmatrix}0&0&0&0\\0&0&0&-0.005056\\0&0&0&0\\0&0&0&-0.002527\end{bmatrix},B_1=\begin{bmatrix}-0.0007\\-0.0435\\0\\0.0289\end{bmatrix},$$

$$A_2=\begin{bmatrix}0.9934&0.0299&0&0\\-0.4409&0&0&0\\0&0&1&0\\-0.0420&-0.0006&0&1\end{bmatrix},A_{2d}=\begin{bmatrix}0&0&0&0\\0&0&0&-0.003170\\0&0&0&0\\0&0&0&-0.002244\end{bmatrix},B_2=\begin{bmatrix}-0.0016\\-0.1086\\0\\0.0257\end{bmatrix},$$

Where : $C_1=[1 \quad 1 \quad 1 \quad 1],\quad C_2=[1 \quad 1 \quad 1 \quad 1]$

Along with defining some parameters for the optimization problem:

- The weighting matrices are $Q_0=\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix},\qquad R_0=1$
- The membership function for rule 1 and rule 2 are chosen as follows:

$$\begin{cases}M_1\big(x_1(k)\big)=1-\exp(-14(x_1-\pi/8)/1+\exp(-14(x_1+\pi/8)\\M_2\big(x_2(k)\big)=1-M_1\big(x_1(k)\big)\end{cases}$$

- Output and input Constraints are defined as:

$$\mathit{Input\ Constraints}\qquad\qquad -0.5<u(k)<0.5$$

$$\mathit{Output\ Constraints}\qquad\qquad -\pi/3<y(k)<\pi/3$$

- The initial conditions are : $x_1(0)=0.73,\ x_2(0)=0,\ u(0)=0.05,\ y(0)=0.05.$

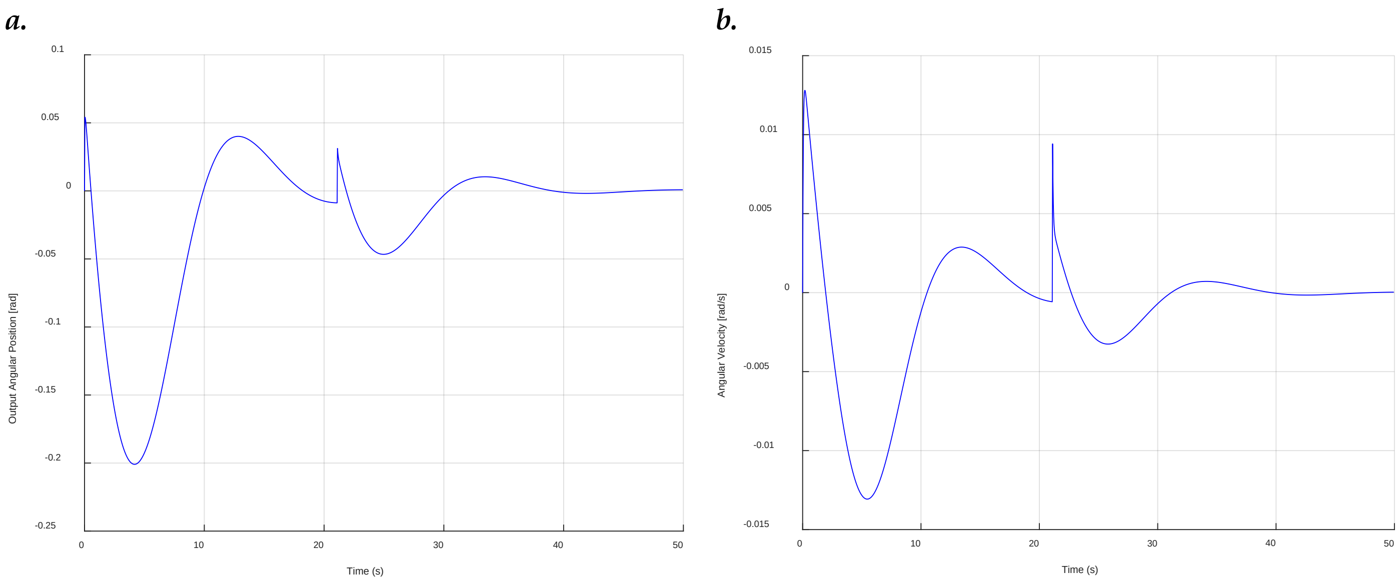


__Fig. 4.6.__ Evolution of Inverted Pendulum with actuators fault.

__(a)__ Angular Position. __(b)__ Angular Velocity

- Discussion

Figures 4.6.a and 4.6.b show up the angular position and angular velocity of the inverted pendulum on a cart, which converges to an original value (equilibrium point) smoothly and in a reasonable time. These results are obtained when a fault is introduced on the pneumatic actuator with loss of efficiency of 40% at time $t = 21\ [s]$. For better visibility of the impact of fault-tolerant predictive control (HFTPC) applied on the system, time-delay is taken into consideration. As a result, we notice at the start of the simulation that the amplitude increases when the fault occurs; whether the variation is in position or in speed as in figures 4.6.a and 4.6.b respectively.

On the other hand, concerning the outputs (position and speed in figures 4.6.a, 4.6.b), though at time that outputs behavior is influenced by the fault occurred, a fast compensation is introduced to carry it back to the desired behavior and preserve the stability during the movement.

Figure 4.7.a shows up the control input using the estimated state provided from the coupled approach observer-MPC, to compensate the actuator faults. As a

result, we observe that the proposed control approach HFTPC raised in section 3.5.2 of chapter 3 adopted to compensate an occurred actuator fault.

Figure 4.7.b demonstrates low sensitivity at transitions moments between sub-systems, in addition to the stability of the HS in case of actuators faults.

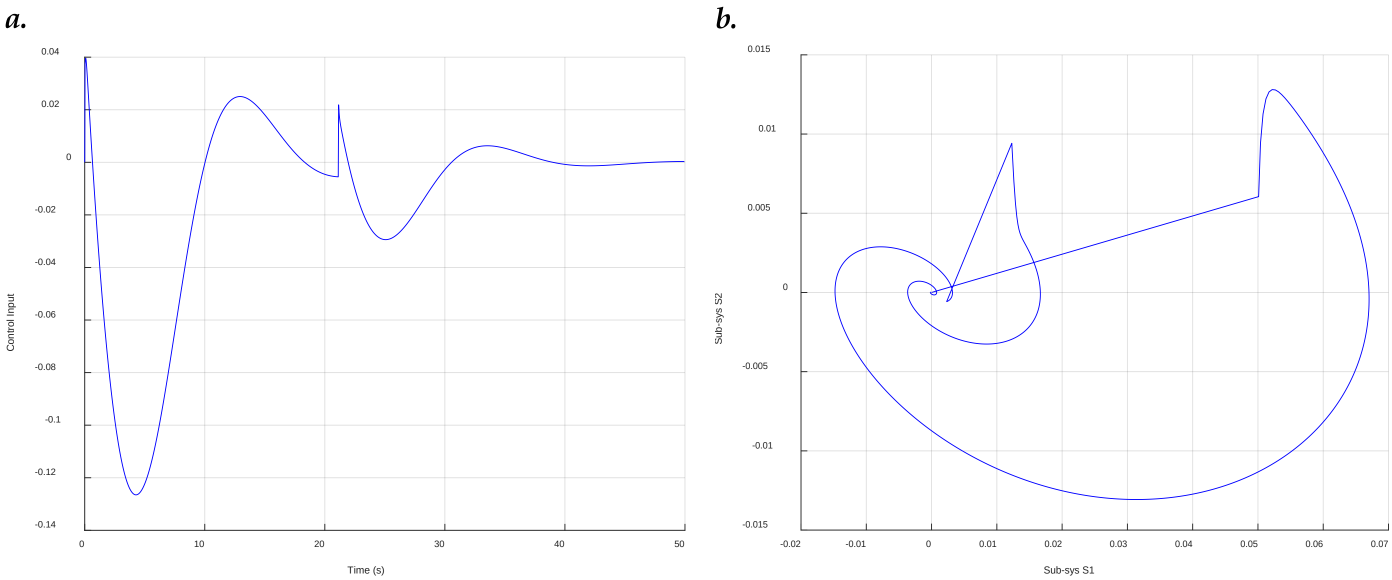


Fig. 4.7. *Evolution of Inverted Pendulum with actuators faults.*
(a) *Control Input.* ***(b)*** *Switching Signal on State Space*

IV.5 Embedded Hybrid Systems Framework: Industrial Robot Arm

In order to design the hybrid optimal control under hybrid actuation (electric and pneumatic actuators) for trajectory tracking of robot arm (Figure 4.15), it is essential to establish an appropriate mathematical model of the industrial robot arm to avoid joint stiffness (Zahaf,2020). Thus, several different representations of robot arm dynamics model are presented and compared, in order to determine the fastest and robust mode.

Taking into consideration the friction term, external disturbance and environment contact; a rigid dynamics robot muscle model with n links is described by the general equation given by (Singh,2013). On the other hand, the dynamic equation for a mobile manipulator arm is represented in (Sun,2016).

A combination of hybrid actuation by macro and mini actuators to operate joint arm dynamics is presented in (Shin,2010). While, a fast dynamic model based on a singularly perturbed system (Naidu,2002) for a hybrid actuation robot's arm is expressed by the motion equations in (Ishihara,2018).

In this section, the dynamics model is structured as hybrid actuation using the combination of a pneumatic and an electric motor actuator, that is considered as an effective actuator for each one.

The dynamic model of a forearm robot with the hybrid actuator is expressed as follows (Ishihara,2018; Noda,2014):

$$\begin{cases} I(\theta)\ddot{\theta} + M(\theta,\dot{\theta})\dot{\theta} \; + \; mgl\,cos(\theta) = \tau_{op}(v_p, v_m) \\ \tau_{op}(v_p, v_m) = \tau_p(P\,,\theta) + \tau_m(v_m\,,\theta) \\ \dot{P} = -F_{vp}P + v_p \end{cases} \tag{4.6}$$

Where, $I(\theta) \in \mathcal{R}^{n*n}$ is the Inertia Matrix, $M(\theta,\dot{\theta}) \in \mathcal{R}^{n*n}$ is external forces, friction forces and environment contact matrix, $m\ and\ \ l$ are the mass and length of the arm respectivily. $\theta\ and\ \dot{\theta}$ denote the joint angle and velocity. P is the pneumatic pressure, τ_{op} is the effective output torque generated by the electric motor τ_m and the pneumatic actuator τ_p. v_p and v_m denote the input voltages for the pneumatic pressure and electric motor.

In this work, to ensure the fastest and the most precise mode, we chose the conversion of pressure to a voltage scale, since the torque of an electric motor is more precise than the pneumatic pressure in the movement.

Thus, we note the transformation coefficient from the pneumatic pressure to electric torque F_{vp}, like in (Ishihara,2018).

In order to achieve this, unique state-space representation can be established to describe the hybrid actuator system.

State x is the state-space model which consists of the positions, the velocities and the Pneumatic Artificial Arm (PAM) pressure:

$$x = [\theta, \dot{\theta}, P]^T \tag{4.7}$$

The hybrid input $\boldsymbol{u}$ includes pneumatic and electric torques:

$$u = [\tau_p, \tau_m]^T \tag{4.8}$$

In real applications, a digital device is very often used. In addition, faults or failures can affect the actuators or sensors during the operation in real time. Then, the corresponding discrete-time faulty hybrid system model can be represented as:

$$\begin{cases} x_f(k+1) = \; Ax_f(k) + \; Bu_{fa}(k) \\ \qquad y_f(k) = Cx_f(k) \end{cases} \tag{4.9}$$

Where: $u_{fa}(k) = u(k) + f_a(k)$ and $x_f(k) = x(k) + f_s(k)$

***Remark 1**:*

- In this thesis, dry friction in the revolute joints may cause a movement delay, in fact that, the time delay in this thesis is subject to a dry friction and environment contact of Industrial Arm. In order that, to reinforce the system's functionality, at least one actuator remains efficient during any sample time "time step".
- In this thesis, the implemented time step size is similar to that used in (Ishihara,2018), $T_s = 20\;ms$.

The used hybrid system model is that described in (4.6), where only the actuators' faults are considered. The HFTPC design is applied to an Industrial Robot Arm with dynamic settings presented in Table 4.2, in three cases. In order to demonstrate the effectiveness and robustness of the proposed new approach, for trajectory tracking with and without actuators faults; and then compared with classical MPC strategy. In fact, there are several possibilities for faults; in our case, the pneumatic actuator can fail due to the loss of pressure, which results in a loss of effectiveness or accuracy.

For robot arm discrete state representation, we use an appropriate approximation technique. In order that, the faulty uncertain discrete-time hybrid system with time delay is described by the following system:

$$\begin{cases} x_f(k+1) = \; A_l x_f(k\,) + A_{ld} x_f(k-d_k) + \; B_l u_f(k\,) + F_a f_a(k\,) \\ \qquad y_f(k\,) = C_l x_f(k) \end{cases} \tag{4.10}$$

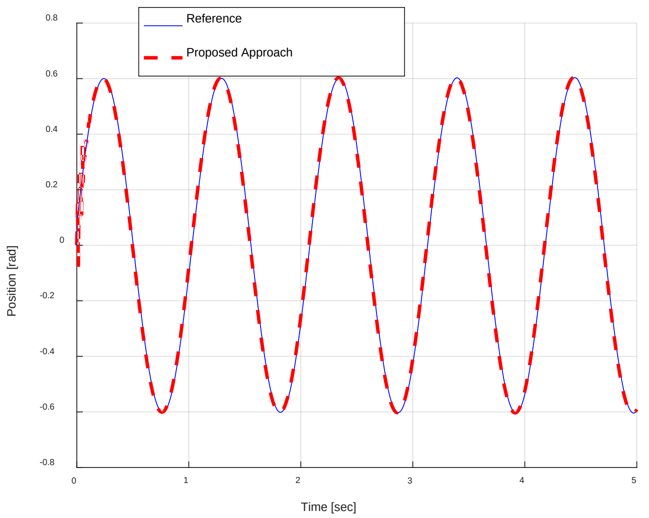
While the physical parameters of the industrial robot arm with hybrid actuators are given in Table 4.2. The weighting matrices are: $Q_0 = 3I$, $R_0 = 0.5$,

For the obtained results with two different modes:

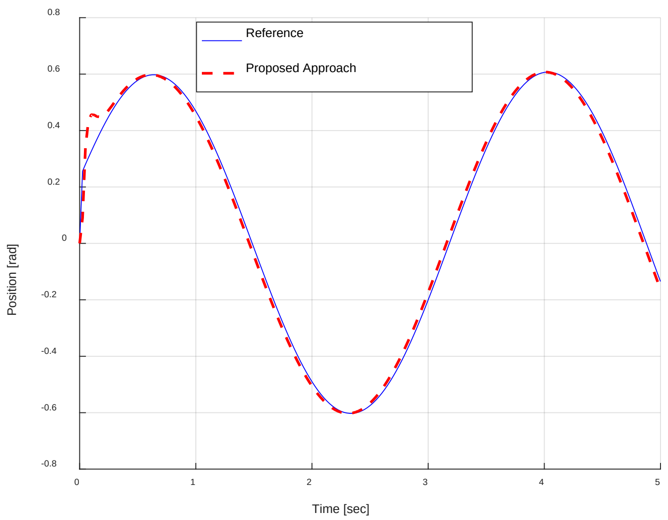
A- Results without faults

For Trajectory $f = 1.00\ [Hz]$: For Trajectory $f = 0.25\ [Hz]$:

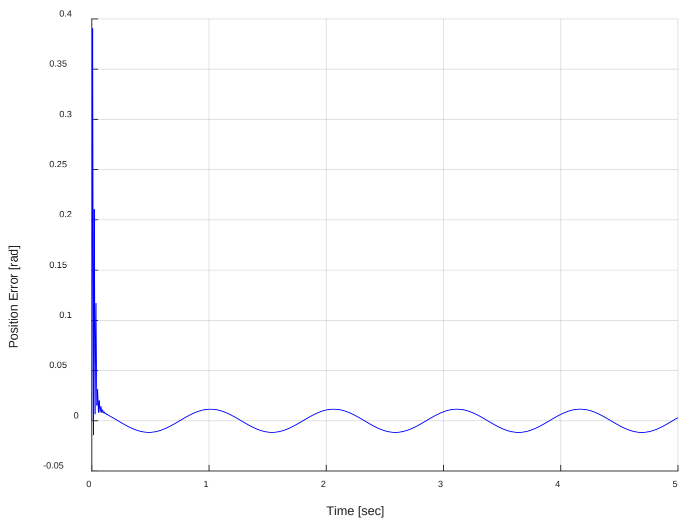
a.



b.



c.



d.

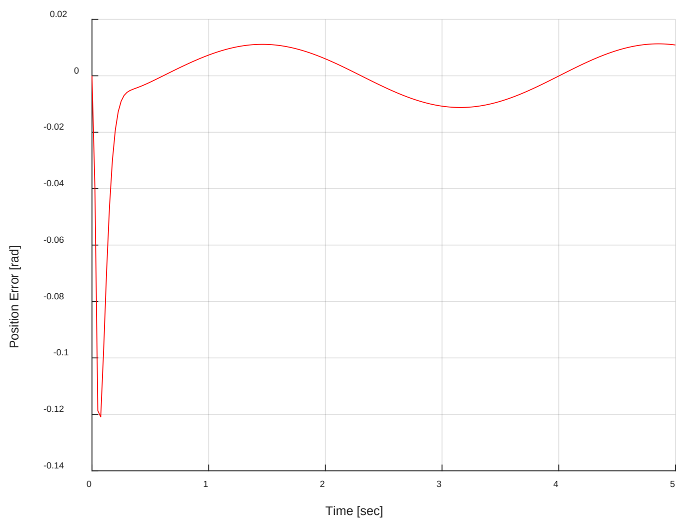


Fig. 4.8. *Evolution of Industrial Arm Position and Position Error.*

Figure 4.8.a and 4.8.b depict a good tracking performance for the joint angle, the proposed scheme HFTPC is used to track the trajectory reference of industrial arm without actuators faults. For that, two references are presented: a fast trajectory reference with $f = 1\ Hz$ and a slow one with $f = 0.25\ Hz$.

The robust MPC scheme can be extended to handle the uncertain system with time-delay, by introducing a variation of 10% on the parameters of the system including the mass and length of the arm.

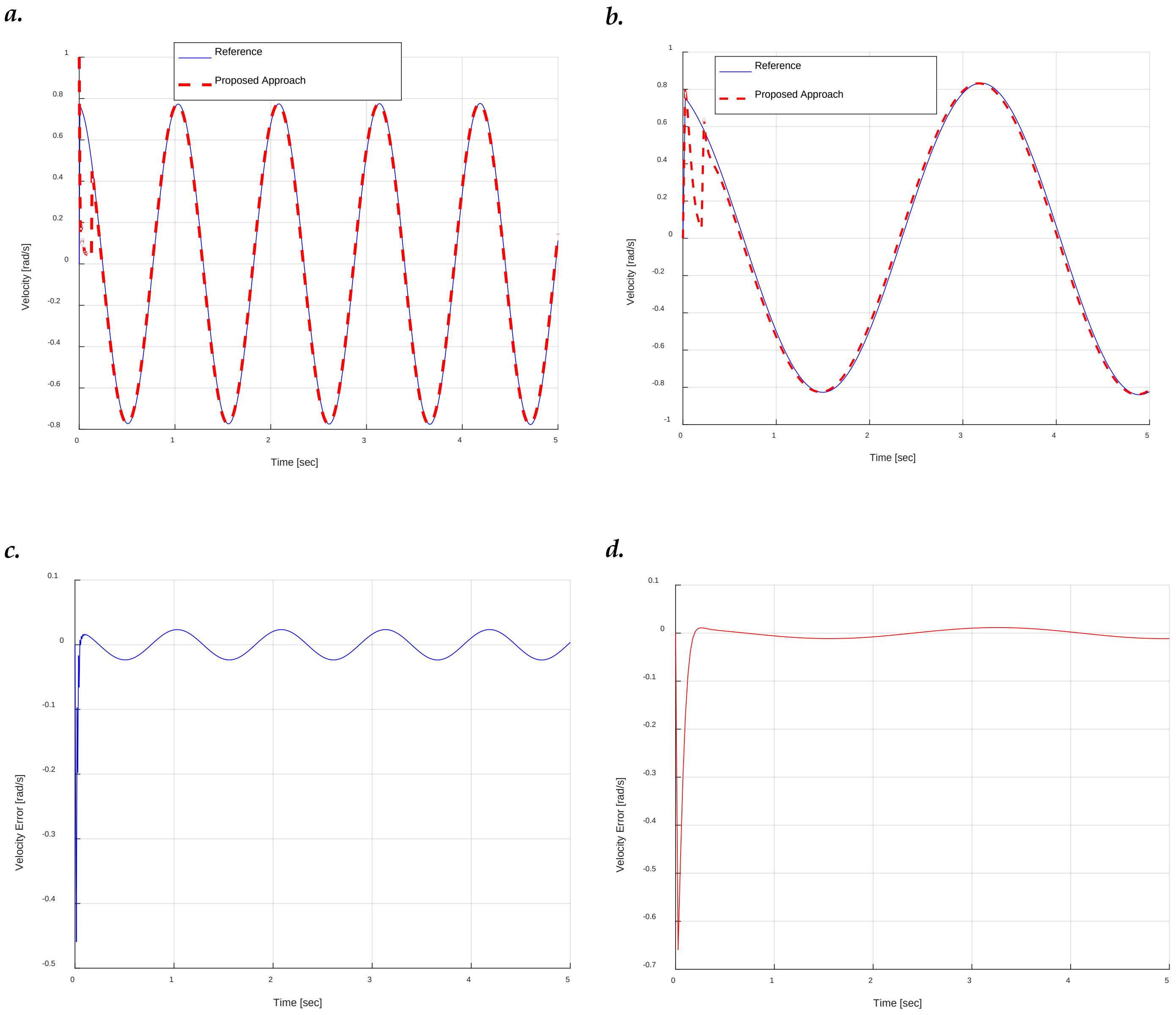


Fig. 4.9. *Evolution of Industrial Arm Velocity and Velocity Error.*

Figures 4.8.c, 4.8.d, 4.9.c and 4.9.d show up error dynamics for tracking position and velocity, which converges to a close value in both cases (slow and fast trajectory).

***Remark* 2:**

The tracking position is extensively studied in (Ishihara,2018; Tuvayanond,2017; Sun,2016; Potocnik,2008). However, tracking the joint angle in this work for the fast mode is more stable than the slow mode. In addition, the trajectory tracking in the fast mode is scarce fluctuated, and the range of error dynamics is less than in (Ishihara,2018). This show an improvement of the trajectory tracking performances, in addition of the stability of the closed loop during the processing based on the proposed approach in theorem 3.5.

TABLE 4.2. *INDUSTRIAL ARM PHYSICAL PARAMETERS*

Parameters	**Description**	**Numerical value**
L	Length of the Arm	0.5 m
M	Weight of the Arm	0.45 Kg
g	Gravity	9.8 m/s^2
θ	Range Angle Joint	$-\pi/5 \sim \pi/5$

B- Results with faults

The proposed strategy must keep up the tracking performances of the desired trajectories and preserve the conditions of stability in the presence of faults. An air leaks or another fault appears. The main objective is preserve high performances and guarantee the stability of the HS, by providing an efficient tracking of reference paths while faults is tolerated and compensate. In our case, we consider only an actuator fault not an actuator failure. In the following, the simulation results of the HFTMPC strategy raised in section 3.5.2 of chapter 3 for the hybrid robot arm are presented.

To prove the ability of our proposed strategy to handle the faulty hybrid systems, Figures 4.10. 4.11 and 4.12 depict how the HFTPC approach performs to compensate failures of hybrid actuators' faults in two cases. The results portray a good tracking of the desired trajectory, in addition of an effective compensation of actuators' faults to keep the robot arm in movement.

Figures 4.10 and 4.12 show the evolution of the outputs dynamic (position, position error and velocity error) with an actuator fault in u. The applied control input undergoes a power loss of 35% due to a fault at system start-up, followed by a loss of efficiency of 25% at time $t = 1.6\ [s]$, pursued by another loss of efficiency about 45% at time $t = 3.8\ [s]$, which corresponds to a successive faults on the actuator. These faults are compensated and controlled “Figure 4.11” by the control input u_1, at the moment that the control input u_2 is affected by faults f_i. To note, that the instant of the fault has occurred is assumed to be unknown during the simulation, while the estimation and compensation of the faulty behavior is done over 1 [s], synchronously after its occurrence. So, the control law is computed in order to reduce the effect of the actuator fault on the system based on the estimation task and the proposed control law.

However, there is a perpetuation of the desired trajectory tracking of the robot arm, with good performances even in the presence of the actuator fault. Moreover, the proposed HFTPC guarantees the stability of the robot arm while it is in the movement process. Consequently, the HFTPC strategy makes it possible to preserve the good performances of the trajectories tracking in closed-loop, as close as possible to the desired path, and ensuring the stability of the HS in the presence of faults. The reliable and good performances that obtained confirm the effectiveness of the control strategy introduced in this study. The occurrence of faults did not prompt on infeasibility or instability; on the other hand, the imposed constraints are satisfied.

In order to achieve the required performances of the robot arm with hybrid actuators. We are proposing a slow and fast tracking paths $f = 0.25\ [Hz]$ and $f = 1\ [Hz]$ respectively, to test the effectiveness of the HFTPC approach, through features of the consumed time to compensate the faults effect and the convergence of chattering interval to stabilize the system.

For Trajectory $f = 1.00\ [Hz]$: For Trajectory $f = 0.25\ [Hz]$:

a. *b.*

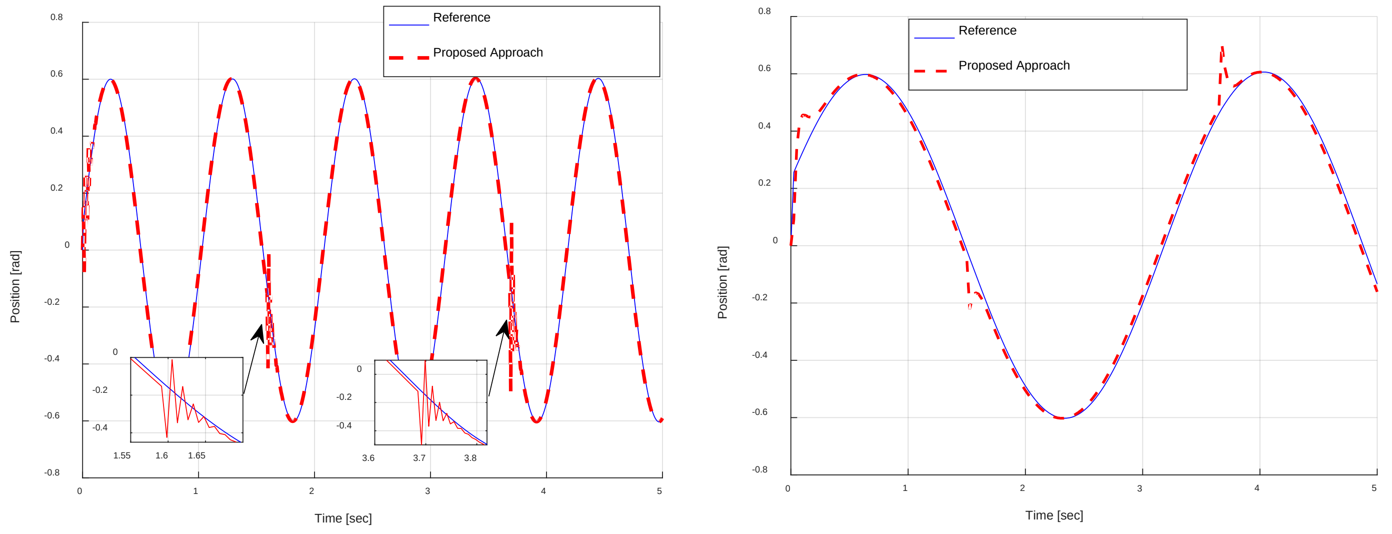


Fig. 4.10. *Evolution of Industrial Arm Position.*

a. *b.*

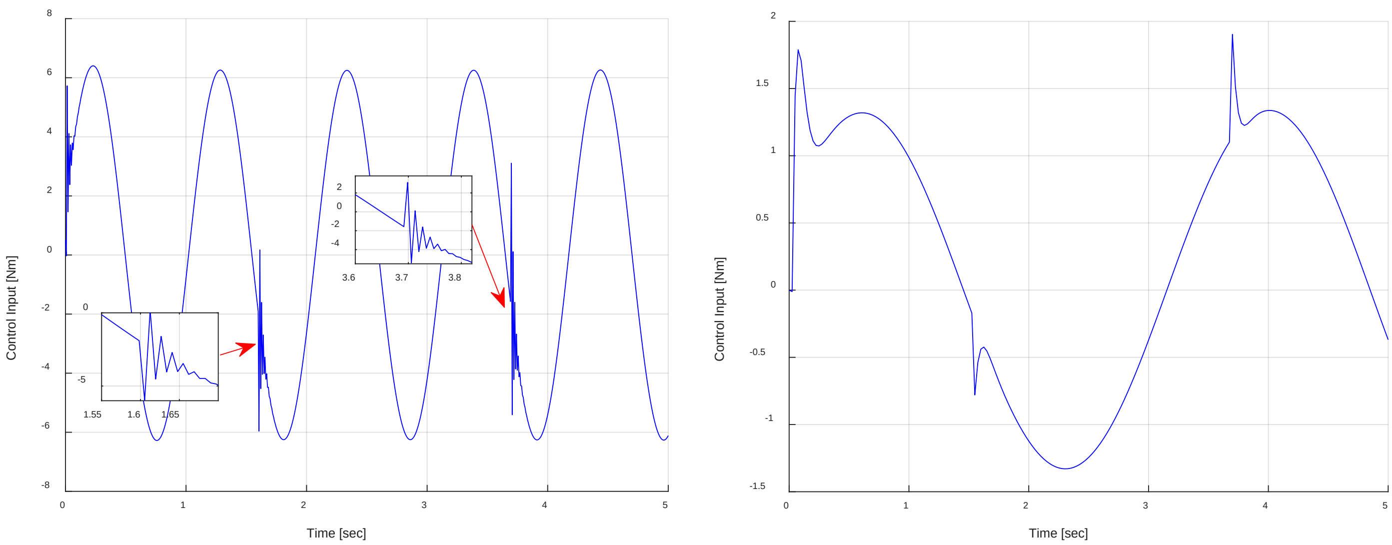


Fig. 4.11. *Evolution of Control Input.*

To cope with the failures in the slow mode, the HFTPC approach lasted over 0.2 s to compensate the effect of faults. In order to avoid total breakdown of the system, the same convergence time is used for the fast mode with more fluctuations in the compensation of defects.

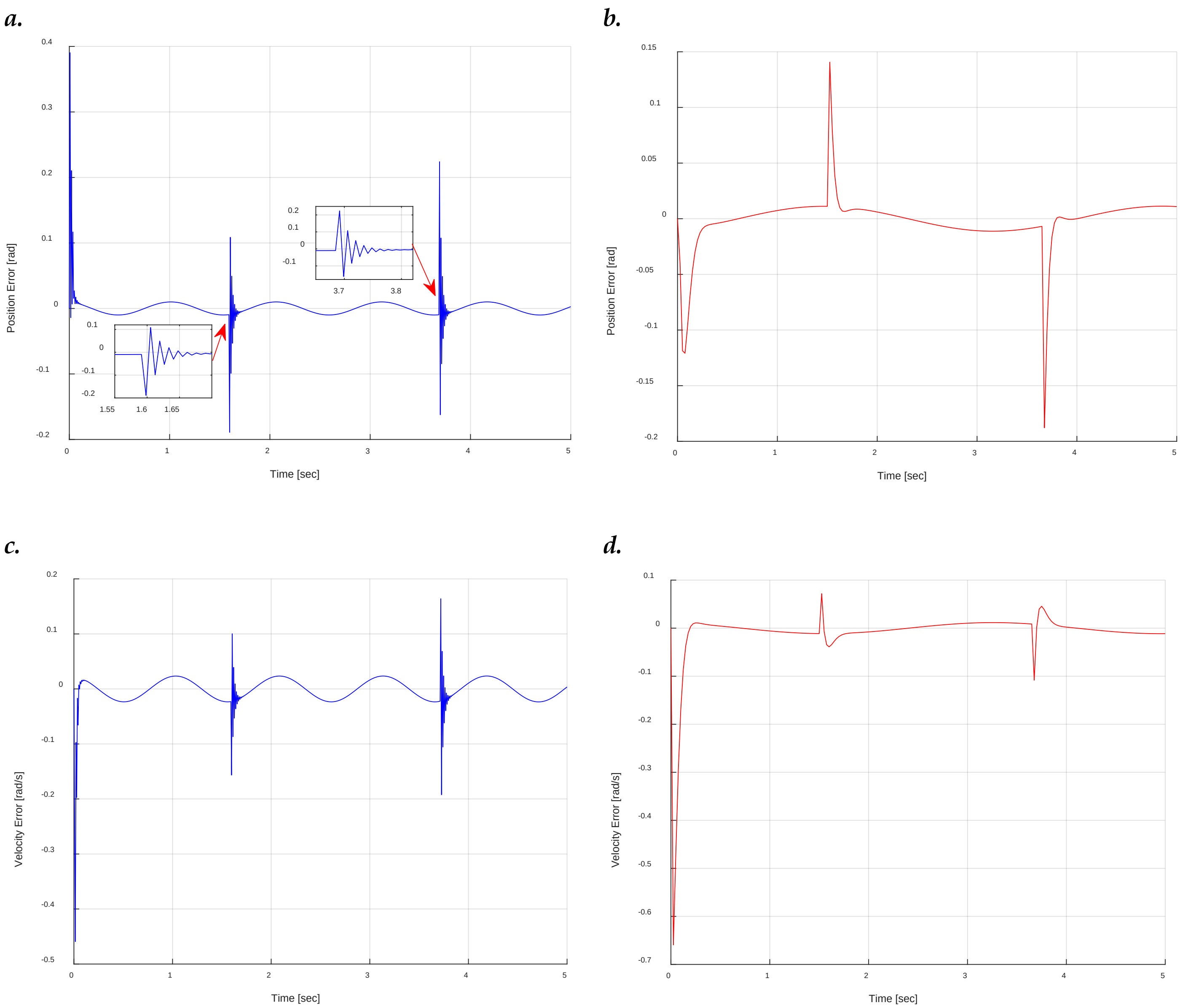


Fig. 4.12. *Evolution of Industrial Arm Position Error and Velocity Error.*

For further comparison of existing results in different works with the proposed approach in (Zahaf,2020), Table 4.3. Shows different tasks with related works in the literature.

TABLE 4.3. ***PERFORMANCES COMPARISON***

Comparison Tasks	Ref (Sun,2016)	Ref (Ishihara, 2018)	Ref (Wu,2017)	Ref (Noda, 2014)	Our Control Approach HFTPC	Observations
Used Techniques	Adaptive	MPC	Fuzzy Sliding Mode	No	**MPC + FTC**	The tracking desired position is smoother
Hybrid Controller	No	No	Yes	No	**Yes**	Our approach is based on MPC and HFTPC to ensure stability and robustness
Hybrid Actuators Control	No	Yes	No	Yes	**Yes**	/
Actuator Faults	No	No	No	No	**Yes**	We consider all kinds of faults
Sensor Faults	No	No	No	No	**No**	/
Time delay	No	No	No	No	**Yes**	/
Simulation	Arm of Robot	Arm of Robot	Arm of Robot	Arm of Robot	**Industrial Arm of Robot**	/
Trajectory (Position)	Joint Angle (rad): [-0.6 ~ 0.6]	Joint Angle (rad): [0.8 ~ 1.8]	Displace ment (mm): [0 ~ 15]	Joint Angle (rad): [-0.4 ~ 0.7]	**Joint Angle (rad): [-0.6 ~ 0.6]**	The trajectory tracking is in two directions as in (T.Noda, 2014) and (W. Sun,2016)which is more challenging than (Wu, 2017) and (Ishihara, 2018)

C- Comparison with MPC Approach

In this part, we compared this study's strategy with the classical MPC approach that used the QP method.

In this comparative study, we have preserved the same characteristics of the studied system, but with an increase in the strength of the actuator fault. We only consider an actuator fault not failure; i.e., we illustrate the case of partial loss of the actuator affected by an actuator fault on u. Thus, we test again the applied control input with 45% power loss at system start-up due to an actuator fault, followed by a 35% efficiency loss at time $t = 1.6$ $[s]$, followed by another loss of efficiency of 55% at time $t = 3.8$ $[s]$. The obtained results are presented in Figures 4.13.

We notice that faults f_1 and f_2 are well estimated, in the case of the actuator fault occurrence. Therefore, in Figure 4.14, an estimation of different scenarios of faults (loss of pressure and overpressure of the pneumatic actuator) are presented based on the proposed estimation mechanism, where, the observer estimates the faults f_i at reasonable time. We note that if a fault appears on the first actuator "input" or on the second actuator "input", the fault is well estimated. So, using this observer, the actuator faults are detected and located, even if they appear simultaneously on both outputs behavior. As a result, we remark that the used observer provides a satisfactory estimate of faults, even if the states are unknown. Therefore, the state estimation error is given in the Figure 4.14.d.

If we compare the results of the tracking performances obtained by our proposed control design with those of predictive control based on QP method of the robot arm (Figures 4.13.b and 4.13.b), it is obvious that the desired objective has been improved using the HFTPC strategy, compared with the MPC scheme based on QP. We notice in the mentioned figures that the obtained control input by QP method mostly has a less efficiency control performances for the position and velocity tracking. The latter can be converted to an inefficiency of the controller to compensate the effect of the actuator faults at reasonable time, if the faults amplitude is increasing and become too considerable to manage robustly.

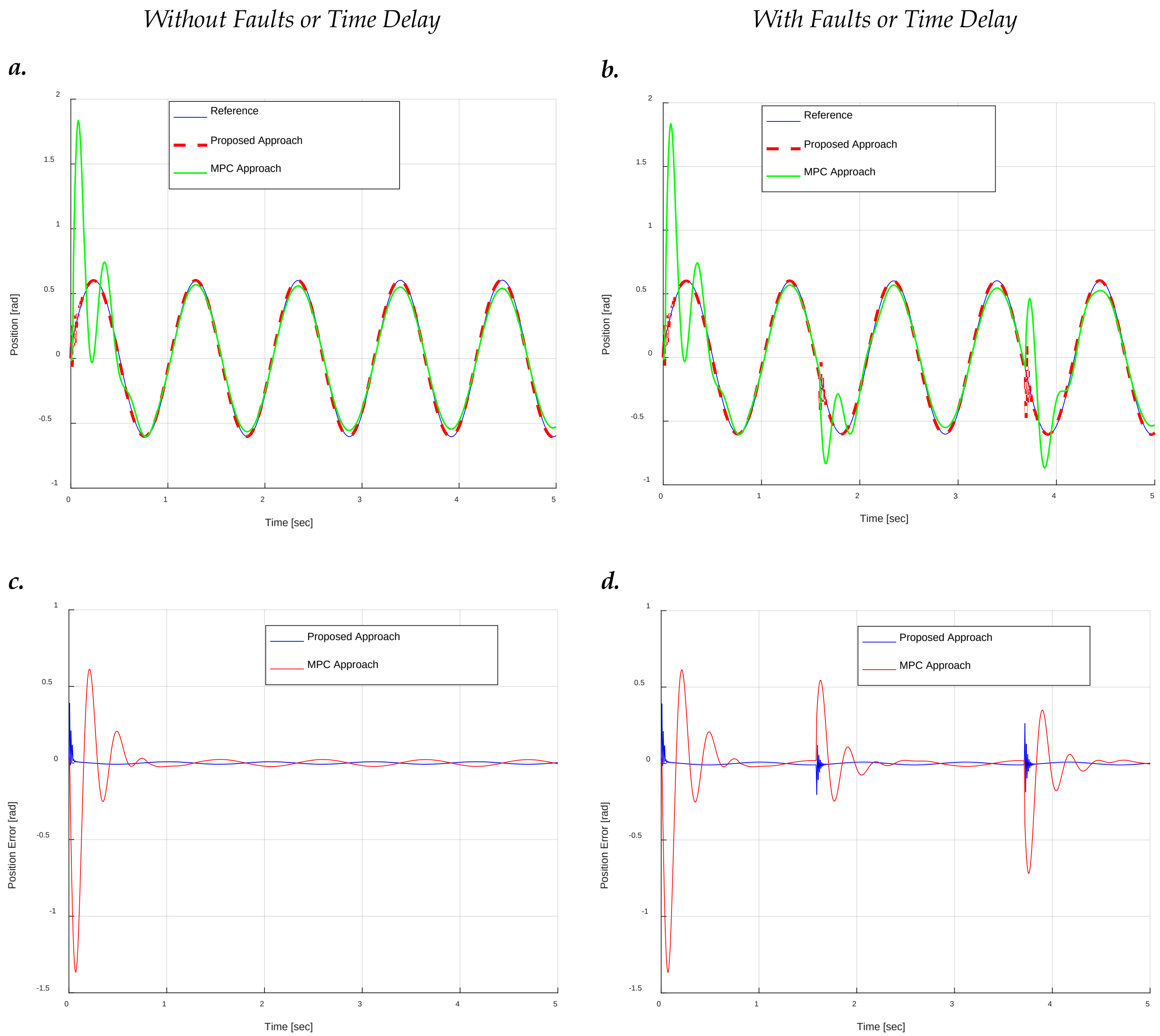


Fig. 4.13. Comparison between Proposed Approach (HFTPC) and MPC Approach
***(a)** and **(b)** represent Position. **(c)** and **(d)** represent Position Error.*

Therefore, we consider that the fault amplitude becomes too significant at the instant t = 3.8 [s]. In this case, the intrinsic robustness of the MPC scheme using QP is no longer sufficient to ensure the stability of the faulty system. In fact, this fault was not taken into account in computation of the control law at the moment it occurred. The second drawback of this approach (MPC scheme using QP) is that the model of the system, as well as the delay, must be perfectly known to be able to write the different matrices necessary for the synthesis of the system.

The results of using the proposed control design show that the effect of the fault is treated and compensated at a reasonable time. In addition, the robustness in terms of stability for the closed loop system is ensured, in the presence of an additive-actuator faults and time-varying delay. As a result, the proposed approach HFTPC, through results in Figures: 4.13.a, 4.13.b, 4.13.c and 4.13.d outperforms the classical MPC scheme in coping with faulty hybrid actuators in the presence of time delay.

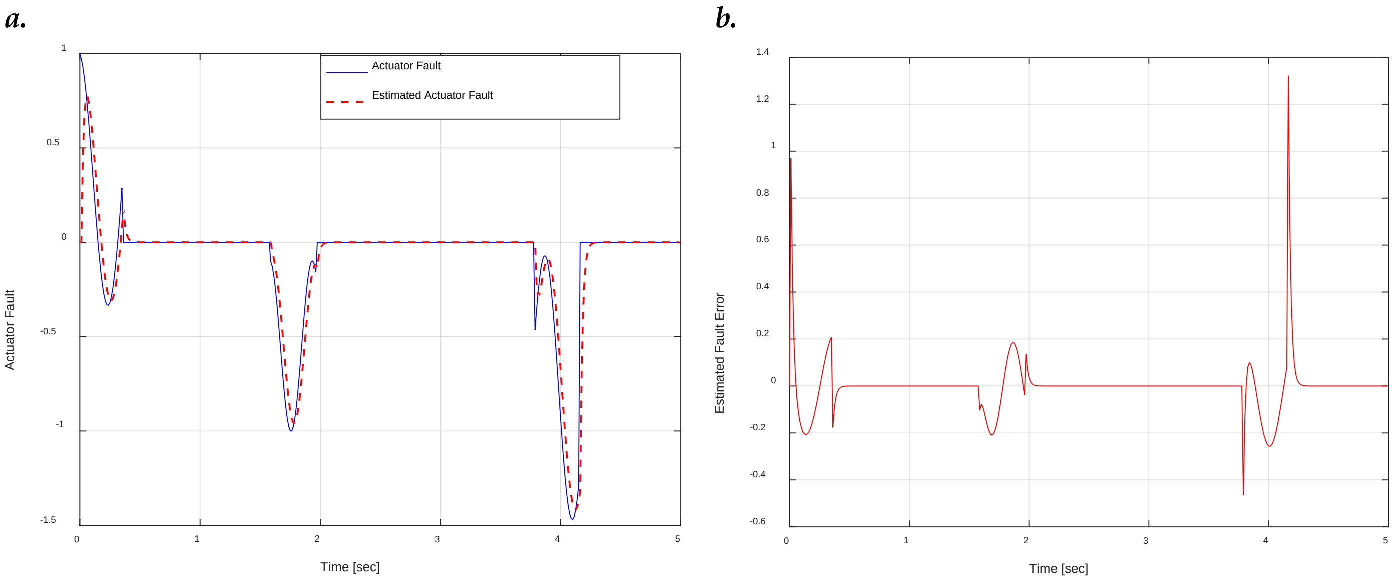


Fig. 4.14. *Intermittent pneumatic actuator fault $f_a(k)$ and its estimations using Proposed Approach (HFTPC)* ***(a)*** *Pneumatic Actuator Fault and its Estimation.* ***(b)*** *Error of Estimated Actuator Fault.*

Table 4.4 presents a comparison of results of: the proposed HFTPC approach and the classical MPC strategy.

The results show a longer convergence time, in addition of more fluctuations to compensate failures of hybrid actuators in the classical MPC approach compared to the proposed strategy in section 3.5 from chapter 3.

TABLE 4.4. *COMPARISON WITH MPC APPROACH*

Comparison Results	*MPC Approach*	*The Approach HFTPC*
Chattering interval of Position (rad)	-0.1 ~ 1. 8	0 ~ 0.6
Convergence Time (sec)	0.8	0.2
Chattering interval of Faulty Case (rad)	-0.9 ~ 1. 8	0 ~ 0.6
Convergence Time of Faulty Case (sec)	0.7 ~ 1. 0	0.2
Error Dynamics (rad)	-1.5 ~ 0. 6	-0.2 ~ 0. 35

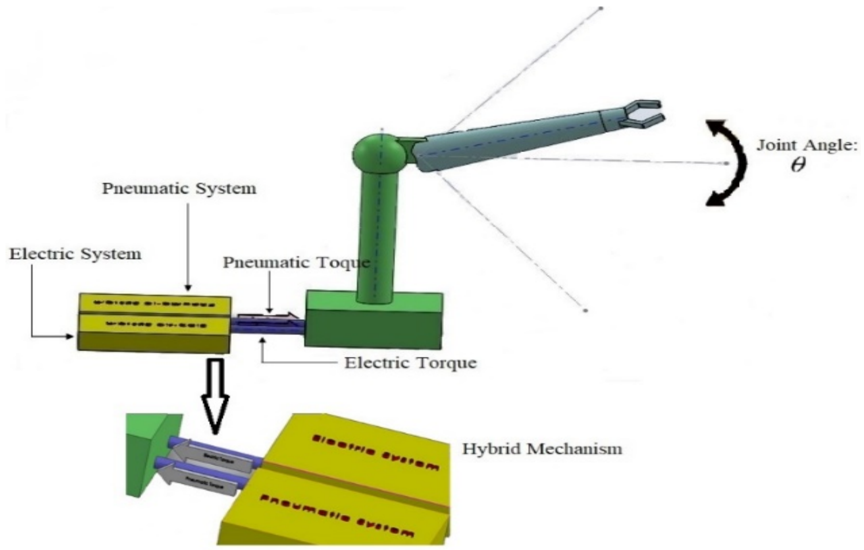


Fig. 4.15. *The 3D Model of Industrial Robot Arm with Hybrid Actuators System (Electric and Pneumatic)*

IV.6 Conclusion

In this chapter, we have examined and treated four examples of hybrid systems of different nature: a hybrid system with State-Dependent Switching, the Continuous Switched System, the Auxiliary HS with State-Dependent Switching using Fuzzy rules and the last is an Embedded HS with two pneumatic and electric actuators.

In the first case, we have presented a strategy to synthesize a controller to ensure the stability of the hybrid system. Our focus was on an accurate estimation of the system states, using an observer in the synthesis of the robust MPC control.

Then, we performed that the system is occurring fault on the sensor which led to failure, for a continuous switched system. We have proposed a robust MPC approach, by adopting an adequate mechanism of estimation to cope with the sensor failures to keep up the desired performances.

In the third case, we have assumed that the system is experiencing a state delay and an actuator fault in the third example. We have proposed a predictive control for auxiliary HS with state-dependent switching based on T-S models. which makes it possible to define for a specific case the fault and time-delay, to meet specific performances, namely the quality of convergence as well as the precision and rapidity of the response and undesirable behavior compensation. Various simulations and estimated errors support these results. In order to improve these performances, the last example of this chapter presents an enhancement of the previous control design.

In the last part of this chapter, we dealt with the issues of faults and time-varying delay for hybrid actuator system, the pneumatic and electric actuators must be controlled to achieve the desired paths. The control must guarantee acceptable performances from the point of view of consumption and avoiding pollutant emissions, given that the hybrid system is highly non-linear. It is necessary that the control system compensates the power loss at the occurring fault moment, otherwise a low or excess pressure decreases or increases the control signal, thus promoting the undesirable behavior.

In this regard, a new strategy of FTC called HFTPC has been proposed for the hybrid system; at which this control scheme is designed based on the observers, aiming to estimate the faults and states of the hybrid system simultaneously. The convergences of the observer and the controller are obtained using the asymptotic stability of Lyapunov. These convergences are formulated in terms of LMIs to obtain the gains of the controller and the observers. The use of the nonlinear sector approach has reduced the conservatism related to the number of LMIs to be solved. We have shown by simulation results the outperforming performances of the proposed control approach (HFTPC), compared to the existing control strategies as that based on using the classical optimization methods as QP. The control approach (HFTPC) gives us a significant improvement for the tracking of the desired trajectory and effective compensation of different kinds of faults, to keep the desired behavior of some classes of hybrid systems.

Conclusion and Further Research

Motivated by further analysis and synthesis requirements for an efficient control approaches of hybrid systems. Fault tolerant control for HS is an interesting topic for both academic and industrial communities, due to specific structures and properties of HS. Thus, several non-hybrid system FTC strategies are not always valid for HS. From this point, it is necessary to propose more efficient mechanisms to maintain high performances of HS in the presence of undesirable associated inputs.

Since the optimality concept is considered as the main objective of many control strategies, the developed HFTPC achieves the optimal solution for some classes of HS studied in this dissertation. In addition, it ensures the HS stability based on less conservative conditions. The achieved goal of optimality and stability is related to new proposed control law, that is composed by estimated state and dynamic error, where the reconfiguration mechanism is basically founded on the concepts of observability and estimation that are raised in chapter 2 and chapter 3.

Generally, the cost to consider for using HS is complexity and difficult synthesis of reliable control design. So, the present results have opened a window to the control aspect of HS, starting by a unified description of hybrid systems. Meanwhile, there are still many pertaining problems to be investigated, especially in the aspect of reconfiguration mechanism for the faulty systems; where a perfect knowledge of outputs behavior allows to design a robust and reliable control scheme.

This dissertation presents a robust constructive FTC strategy as a new control approach, based on Predictive Control theory for faulty hybrid systems with time-delay. Using *min-max* optimization method, necessary and sufficient conditions are derived in term of LMIs to compute the optimal control. These less conservative conditions reduce the computation burden, in addition to satisfy the imposed constraints. Moreover, a new description of HS is presented throughout this thesis and the existing published works. Besides, a robust hybrid fault-tolerant predictive control (HFTPC) approach with time-delay is proposed in chapter 3 for some frameworks of HS as the main contribution. Wherein, the hybrid control design is based on a robust MPC to cope with time delay as continuous dynamic and robust fault tolerant predictive control as discrete dynamic. Therefore, the objective of this study is to design an optimal fault tolerant predictive control for trajectory tracking, applied to a class of nonlinear hybrid actuator systems subject to actuator faults and time delay. In fact, the introduction of time-delay and actuator faults into a hybrid system model results in a dynamic system converted to a strict feedback model. To improve the dynamic performances and decrease the conservatism, a dynamic estimator is implemented to estimate the faults of the actuators, in order to compute an optimal solution.

However, an inspiring analysis is provided to improve the dynamics of a manipulator arm with hybrid actuators, which can be extended for some classes of hybrid systems. The state-space model has been extended by introducing the output tracking errors, in order to increase the hybrid controller degrees of freedom. Then, an optimal control is designed to operate the industrial robot arm to the desired position, and compensate the loss of efficiency or failure of an actuator affected by faults and/or in the presence of time-delays "depending on delay-range". Using Lyapunov-Krasovskii function, combined with an optimized cost function and observer error, we have established a necessary framework to obtain a stable and less conservative conditions in terms of LMIs.

We can summarize the contributions of this work in the following points:

- The proposed hybrid optimal fault-tolerant predictive control scheme is made up of two elements; the first is a robust MPC control to cope with time varying delay, and the second is the robust hybrid fault-tolerant predictive control to deal with actuators faults and external disturbances; in order to perform a robust trajectory tracking.
- The proposed control approach is actively reacting to faults and time-delays, by the reconfiguration of the controller to keep-up the stability

and the desired performances of the whole system, regardless of the detection and isolation loop.

- The proposed control law is based on two dynamics: an estimated state and an error dynamic. The objective is to provide a reliable estimation, that makes it possible to supplement or replace the information sent by the actuator in the case of faults or time-delay. Then, the estimated actuator faults are used to compensate the undesirable behavior and allow us to reconstruct the robust optimal control.
- In fact, the valid and the efficient testing of the contribution that is raised in this study is performed through: classical, academic and industrial examples in chapter 4 using the proposed control schemes. Therefore, we have proposed two FTC approaches based on the predictive control theory to compute the optimal control. In the first approach we have used the QP method (A classical optimization method); while the second approach consists of computing the robust optimal control using *min-max* optimization criterion (HFTPC), to derive necessary conditions in term of LMIs, wherein the imposed constraints, time-delay and faults are underlying on the cost function. The proposed hybrid control approach (HFTPC) outperforms the classical MPC using QP method, by reducing the computation burden and deriving less conservative conditions in terms of LMI.

The present work reveals some future perspectives for further investigation and development. The direction to be developed and formalized concerns the evaluation and improvement of the observer performances for hybrid systems, namely, precision and speed. It would also be interesting to reuse the proposed observers in fault-tolerant control for systems with unknown varying-delay and hybrid systems in the presence of faults and bounded disturbances.

APPENDICES

APPENDIX A

Structures and Representation of Nonlinear Systems

Generally, the studied systems in control theory are not often linear and convex problem. Therefore, in this section, based on an academic physical example "Pendulum Inverted converted to an industrial example (Zahaf,2020)", we present the T-S and the LFT (Fractional Linear Transformation) representations as formalisms to convert the uncertain non-linear problem to an uncertain linear problem, that is well adapted to the control of uncertain nonlinear systems. Then, we apply the LMIs formulation to convert the studied system into convex problem. These two formalisms can be considered as an interesting alternative to represent a large classes of HS.

$$\begin{cases} I(\theta)\ddot{\theta} + M(\theta,\dot{\theta})\dot{\theta} \ + \ mgl\,cos(\theta) = \tau_{op}(v_p, v_m) \\ \tau_{op}(v_p, v_m) = \tau_p(P\,,\theta) + \tau_m(v_m\,,\theta) \\ \dot{P} = -F_{vp}P + v_p \end{cases} \qquad (A.1)$$

State x is the state-space model which consists of the position, the velocity and the Pneumatic Artificial Arm (PAM) pressure:

$$x = [\theta, \dot{\theta}, P]^T \qquad (A.2)$$

The hybrid input u includes pneumatic and electric torque

$$u = [\tau_p, \tau_m]^T \qquad (A.3)$$

A.1 System Representation: the system (A.1) can written partial as follow:

$$I\ddot{\theta} + M(\theta,\dot{\theta})\dot{\theta} \ + \ mglcos(\theta) = \tau_{op}(v_p, v_m) \qquad (A.4)$$

$$I\ddot{\theta} + c(t)\dot{\theta} + q(t)\theta = \ F(t) \qquad (A.5)$$

In order to provide an appropriate representation of the uncertain NL system, the next uncertainties are considered in our example:

$$c(t) = \bar{c}\left(1 + 0.3\delta_c(t)\right), q(t) = \bar{q}\ \left(1 + 0.2\delta_q(t)\right), P(t) = \bar{P}\left(1 + 0.2\delta_P(t)\right) \quad (A.6)$$

$$|\delta_c(t)| < 1, \qquad \left|\delta_q(t)\right| < 1, \qquad |\delta_P(t)| < 1 \quad (A.7)$$

$$c(t) \in [0.8\bar{c}, 1.2\bar{c}], \quad q(t) \in [0.9\bar{q}, 1.1\bar{q}], \quad P(t) \in [0.9\bar{P}, 1.9\bar{P}] \quad (A.8)$$

Where $\bar{c}, \bar{q}$ and $\bar{P}$ are the nominal values of $c(t), q(t)$ and $P(t)$, respectively; and the bounded uncertainties are $\delta_c(t), \delta_q(t)$ and $\delta_P(t)$.

A.2 T-S Model Representation for Uncertain Nonlinear System

In this part, we try to represent the system (A.1 and A4) in the form of a T-S model, under the conditions (A.7 and A8) that considered as constraints. Therefore, the sector nonlinearity approach is used to (A.6) and (A.8), where the parameters $c\ (t)$ and $k\ (t)$ represented as follow:

$$\begin{aligned} c(t) &= F_{c,1}\big(c(t)\big)\, c_M + F_{c,2}\big(c(t)\big)\, c_m \\ q(t) &= F_{q,1}(q(t))\, q_M + F_{q,2}(q(t))\, q_m \\ P(t) &= F_{P,1}(P(t))\, P_M + F_{P,2}(P(t))\, P_m \end{aligned} \quad (A.9)$$

With

$$\begin{aligned} c_M &= max\{c(t)\} = 1.2\bar{c}, & c_m &= min\{c(t)\} = 0.8\bar{c} \\ q_M &= max\{q(t)\} = 1.1\bar{q}, & q_m &= min\{q(t)\} = 0.9\bar{q} \\ P_M &= max\{P(t)\} = 1.1\bar{P}, & P_m &= min\{P(t)\} = 0.9\bar{P} \end{aligned} \quad (A.10)$$

$$\begin{aligned} F_{c,1}\big(c(t)\big) &= \frac{c(t) - c_m}{c_M - c_m}, & F_{c,2}\big(c(t)\big) &= \frac{c_M - c(t)}{c_M - c_m} \\ F_{q,1}\big(q(t)\big) &= \frac{q(t) - q_m}{q_M - q_m}, & F_{q,2}\big(q(t)\big) &= \frac{q_M - q(t)}{q_M - q_m} \\ F_{P,1}\big(P(t)\big) &= \frac{P(t) - P_m}{P_M - P_m}, & F_{P,2}\big(P(t)\big) &= \frac{P_M - P(t)}{P_M - P_m} \end{aligned} \quad (A.11)$$

The uncertain nonlinear system (A.1), (A.6) is presented in the T-S model as follow:

$$\dot{X}(t) = \begin{pmatrix} 0 & 1 & 0 \\ -q(t)/I & -c(t)/I & 0 \\ 0 & 0 & -P(t) \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 1/I \\ P(t) \end{pmatrix} u(t) \quad (A.12)$$

$$\dot{X}(t) = \sum_{i=1}^{4} \mu_i(t)\,(A_i X(t) + B_i u(t)) \qquad (A.13)$$

$$\begin{aligned}
\dot{X}(t) = &\begin{pmatrix} 0 & 1 & 0 \\ -F_{q,1}\big(q(t)\big)q_M/I & -F_{c,1}\big(c(t)\big)c_M/I & 0 \\ 0 & 0 & -F_{P,1}\big(P(t)\big)P_M \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,1}\big(q(t)\big)q_M/I & -F_{c,1}\big(c(t)\big)c_M/I & 0 \\ 0 & 0 & -F_{P,1}\big(P(t)\big)P_m \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,1}\big(q(t)\big)q_M/I & -F_{c,2}\big(c(t)\big)c_m/I & 0 \\ 0 & 0 & -F_{P,1}\big(P(t)\big)P_M \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,1}\big(q(t)\big)q_M/I & -F_{c,2}\big(c(t)\big)c_m/I & 0 \\ 0 & 0 & -F_{P,1}\big(P(t)\big)P_m \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,2}\big(q(t)\big)q_m/I & -F_{c,1}\big(c(t)\big)c_M/I & 0 \\ 0 & 0 & -F_{P,2}\big(P(t)\big)P_M \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,2}\big(q(t)\big)q_m/I & -F_{c,1}\big(c(t)\big)c_M/I & 0 \\ 0 & 0 & -F_{P,2}\big(P(t)\big)P_m \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,2}\big(q(t)\big)q_m/I & -F_{c,2}\big(c(t)\big)c_m/I & 0 \\ 0 & 0 & -F_{P,2}\big(P(t)\big)P_M \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 & 1 & 0 \\ -F_{q,2}\big(q(t)\big)q_m/I & -F_{c,2}\big(c(t)\big)c_m/I & 0 \\ 0 & 0 & -F_{P,2}\big(P(t)\big)P_m \end{pmatrix} X(t) \\
&+ \begin{pmatrix} 0 \\ 1/I \\ P_M \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ 1/I \\ P_m \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ 1/I \\ P_M \end{pmatrix} u(t) \\
&+ \begin{pmatrix} 0 \\ 1/I \\ P_m \end{pmatrix} u(t) \qquad (A.14)
\end{aligned}$$

$\dot{X}(t) = (\ \theta(t) \quad \dot{\theta}(t) \quad P)^T$ is state vector and $u(t) = F(t)$ is the input control, where the membership functions $\mu_i(t)$ are described as follows:

$$\begin{array}{ll} \mu_1(t) = F_{q,1}(t)F_{c,1}(t)F_{P,1}(t), & \mu_2(t) = F_{q,1}(t)F_{c,2}(t)F_{P,1}(t), \\ \mu_3(t) = F_{q,1}(t)F_{c,1}(t)F_{P,2}(t), & \mu_4(t) = F_{q,1}(t)F_{c,2}(t)F_{P,2}(t), \\ \mu_5(t) = F_{q,2}(t)F_{c,1}(t)F_{P,1}(t), & \mu_6(t) = F_{q,2}(t)F_{c,1}(t)F_{P,1}(t), \\ \mu_7(t) = F_{q,2}(t)F_{c,1}(t)F_{P,2}(t), & \mu_8(t) = F_{q,2}(t)F_{c,1}(t)F_{P,2}(t), \end{array} \quad (A.15)$$

Then, the state matrices A_i and B_i are given by:

$$\begin{array}{ll} A_1 = \begin{pmatrix} 0 & 1 & 0 \\ -q_M/I & -c_M/I & 0 \\ 0 & 0 & -P_M \end{pmatrix}, & A_2 = \begin{pmatrix} 0 & 1 & 0 \\ -q_M/I & -c_M/I & 0 \\ 0 & 0 & -P_m \end{pmatrix}, \\ A_3 = \begin{pmatrix} 0 & 1 & 0 \\ -q_M/I & -c_m/I & 0 \\ 0 & 0 & -P_M \end{pmatrix}, & A_4 = \begin{pmatrix} 0 & 1 & 0 \\ -q_M/I & -c_m/I & 0 \\ 0 & 0 & -P_m \end{pmatrix}, \\ A_5 = \begin{pmatrix} 0 & 1 & 0 \\ -q_m/I & -c_M/I & 0 \\ 0 & 0 & -P_M \end{pmatrix}, & A_6 = \begin{pmatrix} 0 & 1 & 0 \\ -q_m/I & -c_M/I & 0 \\ 0 & 0 & -P_m \end{pmatrix}, \\ A_7 = \begin{pmatrix} 0 & 1 & 0 \\ -q_m/I & -c_m/I & 0 \\ 0 & 0 & -P_M \end{pmatrix}, & A_8 = \begin{pmatrix} 0 & 1 & 0 \\ -q_m/I & -c_m/I & 0 \\ 0 & 0 & -P_m \end{pmatrix}, \end{array} \quad (A.16)$$

$$B_1 = B_3 = \begin{pmatrix} 0 \\ P_M/I \end{pmatrix}, \qquad B_2 = B_4 = \begin{pmatrix} 0 \\ P_m/I \end{pmatrix}, \quad (A.17)$$

A.3 LFT Representation for Uncertain Nonlinear System

A second approach to represent the uncertain system (A.1) is introduced as an alternative approach. We following the steps in (Bezzaoucha,20014), where the principal of this approach based on the separation of the certain and uncertain states in model. Therefore, we get:

$$\begin{cases} \begin{pmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \\ \dot{P}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -\bar{q}/I & -\bar{c}/I & 0 \\ 0 & 0 & \bar{P}/I \end{pmatrix} \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \\ P(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1/I \\ \bar{P}/I \end{pmatrix} F(t) + \begin{pmatrix} 0 & 1 & 0 \\ -0.2/I & -0.1/I & 0 \\ 0 & 0 & 0.1/I \end{pmatrix} \begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{pmatrix} \\ y(t) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \\ P(t) \end{pmatrix} \end{cases} \quad (A.18)$$

$$\begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{pmatrix} = \begin{pmatrix} \delta_c(t) & 0 & 0 \\ 0 & \delta_q(t) & 0 \\ 0 & 0 & \delta_P(t) \end{pmatrix} \begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix}, \begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = \begin{pmatrix} 0 & \bar{c} & 0 \\ \bar{q} & 0 & 0 \\ 0 & 0 & \bar{P} \end{pmatrix} \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \\ P(t) \end{pmatrix} \quad (A.19)$$

APPENDIX B

Predictor and Predictions Computation Based on State Space Representation

This section is devoted to the modelling model predictive control based on state space model derived from nonlinear systems. Meanwhile, a classical synthesis is shown to obtain the control law (Richalet,1978; Clake,1987). Thus, the discrete-time state space representation hybrid model is represented as:

$$\begin{cases} x(k+1,j) = Ax(k,j) + Bu(k,j) \\ y(k,j) = Cx(k,j) \end{cases} \tag{B.1}$$

Where $x(k,j) \in R^n$ is the states vector belonging to a state space $M \subset R^n$, $u(k,j) \in R^m$ input control and $y(k,j) \in R^l$ is the output vector of system, with constraints in inputs control $u(k,j)$ and outputs $y(k,j)$. Over a prediction horizon N_h:

$$\begin{matrix} u_{min,j} \le u(k+i,j) \le u_{max,j} \\ y_{min,j} \le y(k+i,j) \le y_{max,j} \end{matrix} \quad for\ i = 1 \dots N \tag{B.2}$$

Note: for simplicity, we write e.g., $u_{min,j} = u_{min}$, $u(k,j) = u(k)$ for all variables.

At first, we compute the n predictions based on mathematical manipulation of states space model:

$$x(k+1) = Ax(k) + Bu(k) = Ax(k) + Bu(k-1) + B\Delta u(k)$$

$$x(k+2) = Ax(k+1) + Bu(k+1)$$

$$= A^2x(k) + (AB+B)u(k-1) + (AB+B)\Delta u(k) + B\Delta u(k+1)$$

$$\vdots$$

$$x(k+n) = A^n x(k) + (A^{n-1}B + \cdots + B)u(k-1) + (A^{n-1}B + \cdots + B)\Delta u(k) + \cdots + B\Delta u(k+n-1) \tag{B.3}$$

Hence, the predictor can be formulated in the next matrix form as:

$$\hat{X} = \begin{bmatrix} \hat{x}(k) \\ \hat{x}(k+1) \\ \vdots \\ \hat{x}(k+n-1) \end{bmatrix} = \Psi_0 x(k) + \Gamma_0 u(k-1) + \Lambda \Delta u \qquad (B.4)$$

$$\text{With}: \quad \Delta U = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+n-1) \end{bmatrix}, \quad \Psi_0 = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^n \end{bmatrix}, \quad \Gamma_0 = \begin{bmatrix} B \\ AB+B \\ \vdots \\ A^{n-1}B + \cdots + B \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \mathrm{B} & 0 & \dots & 0 \\ AB+B & \mathrm{B} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{n-1}B + \cdots + B & A^{n-2}B + \cdots + B & \dots & \mathrm{B} \end{bmatrix}$$

Also, (B.4) can be written as follows:

$$\hat{y}(k+n|k) = CA^n x(k) + \sum_{i=0}^{n-1} CA^{n-i-1} Bu(k-1) + \sum_{i=0}^{n-1} CA^{n-i-1} \mathrm{B}\Delta U \qquad (B.5)$$

With: $\Psi_h = C\Psi_0$, $\Gamma_h = C\Gamma$, $\Lambda_h = C\Lambda$

$$\hat{y} = C\hat{x} = \Psi_h x(k) + \Gamma_h u(k-1) + \Lambda_h \Delta U \qquad (B.6)$$

In predictive control, only the first computed input control (more specifically, only the current computed input control) is considered and applied for next iteration, thus minimize the cost function using the current input control $u(k-1)$ is not relevant, the output predictor (B.6) can be written as follow:

$$\hat{y} = C\hat{x} = \Psi_h x(k) + \Lambda_h \Delta U \qquad (B.7)$$

$$\hat{y} = C\hat{x} = \Psi_h x(k) + \Lambda_h u(k) \qquad (B.8)$$

APPENDIX C

LMIs Tools for Analysis, Synthesis and Transformations of Matrices

Generally, the studied problems in control theory are not necessary linear and convex. Therefore, it is necessary to have some mathematical transformations to convert the non-linear convex problems to LMIs problems to obtain relaxed conditions based on optimization criterion.

C.1 Schur Complement

Lemma C.1 (El Ghaoui,1997): *Let $R(x) \in \Re^{m*m}$ is definite positive matrix and $R(x) = R(x)^T$, $Q(x) \in \Re^{m*n}$ is full rank matrix $Q(x) = Q(x)^T$ and $S(x) \in \Re^{n*n}$ any matrix. The Three matrices are dependently affine of variable $\boldsymbol{x}$. The following inequalities are equivalent:*

$$1. \; Q(x) - S(x)R(x)^{-1}S(x)^T > 0, \qquad R(x)0 \tag{C.1}$$

$$2. \begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0 \tag{C.2}$$

- Then, by using the property in (Salvador,2005), the generalized Schur complement becomes:

Let the next matrices $U(x)$ and $M(x)$ coordinate with the above matrices, with $R(x) > 0$ and $M(x) > 0$. The following two inequalities are equivalent:

$$1. \; Q(x) - S(x)R(x)^{-1}S(x)^T - U(x)M(x)^{-1}U(x)^T > 0, \tag{C.3}$$

$$2. \begin{bmatrix} Q(x) & S(x) & U(x) \\ S(x)^T & R(x) & 0 \\ U(x)^T & 0 & M(x) \end{bmatrix} > 0 \tag{C.4}$$

C.2 Congruence Property

For particular classes of optimization problems, the obtained linear convex inequalities require some supplementary variables to relax the computation of optimal control. These proprieties are developed based on the next inequalities:

- $A^T PA - Q < 0,$
- $A^T P + PA + Q < 0,$

For the first inequality we have the next results:

Lemma C.2 : *let* A, G, L, P *and* Q *are matrices with appropriate dimensions. The next inequalities are equivalents:*

1. $A^T PA - Q < 0, \qquad P > 0 \qquad (C.5)$

2. $\begin{bmatrix} -Q & A^T P \\ PA & -P \end{bmatrix} < 0 \qquad (C.6)$

3. $\exists G \begin{bmatrix} -Q & -A^T G^T \\ -GA & -G - G^T + P \end{bmatrix} < 0, \qquad P > 0 \qquad (C.7)$

4. $\exists G, L \begin{bmatrix} -Q + A^T L^T + LA & -L + A^T G^T \\ -L^T + GA & -G - G^T + P \end{bmatrix} < 0, \quad P > 0 \qquad (C.8)$

Remak:

- Inequality (C.7) is obtained in (De Oliveira,1999).
- Inequality (C.8) is obtained in (Peaucelle,2000).

For the transformation of the rest inequalities we have:

$(C.5) \Leftrightarrow (C.6)$: *by Shur Complement.*

$(C.6) \Longrightarrow (C.7)$ *and* $(C.8)$: *is just to put* $G = P$ *et* $L = 0$

$(C.7)$ *and* $(C.8) \Longrightarrow (C.5)$: *by congreuence property, multiplying left and right by* $[I \; A^T]$

Follow the steps for the first inequality, then results for the second inequality are:

<u>Lemma C.3</u> (Peaucelle,2000): *let* A, G, L, P *and* Q *are matrices with appropriate dimensions. The next inequalities are equivalents:*

$$1. A^T P + PA + Q < 0, \tag{C.9}$$

$$2. \exists G, L \begin{bmatrix} A^T L^T + LA + Q & P - L + A^T G \\ P - L^T + G^T A & -G - G^T \end{bmatrix} < 0, \tag{C.10}$$

Also, for the transaction between the inqualities, we have :

$(C.10) \Longrightarrow (C.9)$: $by\ congreuence\ property, multiplying\ left\ and\ right\ by$ $[I\ A^T]$

$(C.9) \Longrightarrow (C.10)$: $if\ (B.9)\ is\ virified, \exists \varepsilon > 0\ such\ as\ A^T P + PA + Q + \frac{\varepsilon}{2} A^T A < 0$, *by using Schur complement, and posed that* $L = P$ *et* $G = \varepsilon I$ *in (C.10), we get*:

$$\begin{bmatrix} A^T P^T + PA + Q & \varepsilon A^T \\ \varepsilon A & -2\varepsilon I \end{bmatrix} < 0.$$

<u>Note</u>: *the necessary conditions are verified and ensure the boundedness of the following inequalities*

- Lemma 2 : $(C.6) \Longrightarrow (C.7)$ *and* $(C.8)$,

- Lemma 3 : $(C.9) \Longrightarrow (C.10)$

if L, G *are not constrained matrices.*

B.3 Variables or Elements Projection

The next lemma is mostly used in LMIs formulations in several constraints control problems by transforming some variables to decrease the dimension of the feasibility problems. For that, we recall two versions of Finsler's Lemma.

<u>Lemma C.4</u> (Finsler) (De Oliveira,2001) :

Let the vector $x(t) = [x_1(t), \dots, x_m(t)]^T \in \Re^n$, and $Q = Q^T \in \Re^{n*n}$, $M \in \Re^{m*n}$ and $N \in \Re^{m*n}$ are matrices such as $rang(M) < n$ and $rang(N) < n$. The following four statements are equivalents:

$$1. x^T Q^T x < 0\ \ \forall x \neq 0\ \ such\ as\ x = 0, Nx = 0 \tag{C.11}$$

2. The orthogonal complements $M_{\perp} and\ N_{\perp}$ and $M\ \ and\ N$, respectively, verified the next conditions: $M_{\perp}^{T} Q M_{\perp} < 0 \quad and \quad N_{\perp}^{T} Q N_{\perp} < 0$ $(C.12)$

3. There exists a scalar real $\sigma \in \Re$ such as:

$$Q - \sigma M^T M < 0 \quad and \quad Q - \sigma N^T N < 0 \tag{C.13}$$

4. There exists a real matrix $X \in \Re$ such as:

$$Q + N^T M + M^T X^T N < 0 \tag{C.14}$$

C.4 The Quadratic Matrices and its Derivatives

The advantages of next Lemmas are the ability to decrease the conservatism by formulating inequalities to a quadratic and non-quadratic form and vice versa, as that let us define the definite positive matrix $Q > 0$:

Lemma C.5*:* (De Oliveira,2001)

Let $Q \in S^n$, and $X \in \mathcal{R}^{m \times n}$ such that $rank(R) < n$; the following expressions are equivalent:

1. $X^T Q X < 0,$ $(C.15)$
2. $\exists X \in \mathcal{R}^{n \times m} : \ Q + XR + R^T X^T < 0.$ $(C.16)$

Lemma C.6 (Zhou,1988): *let X and Y two matrices with appropriate dimensions.*

- let γ a positive constant:

$$X^T Y + Y^T X \le \gamma X^T X + \gamma^{-1} Y^T Y \tag{C.17}$$

Lemma C.7 (Wang,1992) ***:*** *let X and Y two matrices with appropriate dimensions. The next inequality is always verified for any $Q = Q^T > 0$:*

$$XY^T + YX^T \le XQX^T + YQ^{-1}Y^T \tag{C.18}$$

Proof (Zhou,1988) *:*

$$Q > 0 \Rightarrow (QX - Y)^T Q^{-1} (QX - Y) \ge 0 \Leftrightarrow XY^T + YX^T \le XQX^T + YQ^{-1}Y^T \tag{C.19}$$

C.5 Time-Delay Transformation

Lemma C.8: (Bououden,2016)

For any given integers v_1 and v_2 satisfying $v_1 < v_2$, and any matrix W, the following statement holds:

$$\sum_{s=k-v_2}^{k-v_1-1}\left(x^T(s+1)Wx(s+1) - x^T(s)Wx(s)\right)$$
$$= x^T(s-v_1)Wx(s-v_1) - x^T(s-v_2)Wx(s-v_2) \quad (C.20)$$

BIBLIOGRAPHY

Bibliography

Aihara, K., and Suzuki, H., (2010). "Theory of hybrid dynamical systems and its applications to biological and medical systems". Philos. Trans. R. Soc. A, 368, 4893–4914.

Ait Ladel, A., Benzaouia, A., Outbib, R., Ouladsine M., and El Adel, E., (2021). "Robust Fault Tolerant Control of Continuous-Time Switching Systems: An LMI Approach". Nonlinear Analysis: Hybrid Systems, 39, 100950.

Altin, B., Ojaghi, P., and Sanfelice, B.G. (2018). "A Model Predictive Control Framework for Hybrid Systems". IFAC-Papers OnLine, 51(20), 128–133.

Amarasinghe, D., Mann, G. K. I., and Gosine, R. G., (2007). "Vision-Based Hybrid Control Scheme for Autonomous Parking of a Mobile Robot". Advanced Robotics, 21(8), 905–930.

Aminsafaee, M., and Shafiei, M. H., (2019). "Stabilization of Uncertain Nonlinear Discrete-Time Switched Systems with State Delays: A Constrained Robust Model Predictive Control Approach". Journal of Vibration and Control, 25(14), 2079–2090.

Andry, A. N., Shapiro, E. Y., and Chung, J. C., (1983). "Eigenstructure Assignment for Linear Systems". IEEE Trans. Aero. and Electron. Syst. AES, 19(5), 711-729.

Anstaklis, P. J., Kohn, W., Nerode, A., and Sastry, S., (1995). editors. "Hybrid Systems II. Lecture Notes in Computer Science". Springer-Verlag, New York, USA.

Antsaklis, P. J., (2000). "A Brief Introduction to the Theory and Applications of Hybrid Systems". Proceedings of the IEEE, Special Issue on Hybrid Systems: Theory and Applications, 88(7), 879–886.

Arbib, C., and De Santis, E., (2020). "Almost Always Observable Hybrid Systems". Nonlinear Analysis: Hybrid Systems, 36, 100838.

Aubrun, C., Sauter, D., Noura H., and Robert, M., (1993). "Fault Diagnosis and Reconfiguration of Systems Using Fuzzy Logic: Application to a Thermal Plant". International Journal of Systems Science, 24(10), 1945-1954.

Bader, K., Lussier, B., and Schön, W., (2017). "A Fault Tolerant Architecture for Data Fusion: A Real Application of Kalman Filters for Mobile Robot Localization". Robotics and Autonomous Systems, 88, 11–23.

Bakiotis, C., Raymond, J., and Rault, A., (1979). "Parameter and Discriminant Analysis for Jet Engine Mechanical State Diagnosis". Proc. of The 18th IEEE Conf. on Decision and Control Including the Symposium on Adaptive Processes, December 12-14, 1979, Fort Lauderdale, USA.

Barelli, L., Bidini, G., Ciupăgeanu, D. A., Pianese, C., Polverino, P., and Sorrentino, M., (2020). "Stochastic Power Management Approach for a Hybrid Solid Oxide Fuel Cell/Battery Auxiliary Power Unit for Heavy Duty Vehicle Applications". Energy Conversion and Management, 221, 113197.

Belarbi, K., and Megri, F., (2007). "Stable Model-Based Fuzzy Predictive Control on Fuzzy Dynamic Programming". IEEE Transactions on Fuzzy Systems, 15(4), 746–754.

Bemporad, A., and Morari, M., (1999). "Control of Systems Integrating Logic, Dynamics, and Constraints". Automatica, 35(3), 407–427.

Bemporad, A., Heemels W.P.M.H., and De Schutter, B., (2000). "On Hybrid Systems and Closed-Loop MPC Systems". IEEE Transactions on Automatic Control, 47(5), 863–869.

Bemporad, A., Ferrari-Trecate, G., and Morari, M., (2000). "Observability and Controllability of Piecewise Affine and Hybrid Systems". IEEE Transactions on Automatic Control, 45(10), 1864–1876.

Benveniste, A., and Guernic, P. L., (1990). "Hybrid Dynamical Systems Theory and the Signal Language". IEEE Transactions on Automatic Control, 35(5), 535-546.

Bezzaoucha, D., (2014). Commande Tolérante aux Défauts des Systèmes Non-Linéaires Représentés par des Modèles Takagi-Sugeno. PhD Thesis, Université de Lorraine, Nancy, France.

Blanke, M., Kinnaert, M., Lunze, J., and Staroswiecki, M., (2006). "Diagnosis and Fault-Tolerant Control", 2nd Ed. Springer-Verlag, Berlin, Germany.

Bobal, V., Kubalcik, M., Dostal, P., and Matejicek, J., (2013). "Adaptive Predictive of Time-Delay Systems". Computers and Mathematics with Applications, 66, 165–176.

Borrelli, F., Baotic, M., Bemporad A., and Morari, M., (2005). "Dynamic Programming for Constrained Optimal Control of Discrete-Time Linear Hybrid Systems". Automatica, 41(10), 1709–1721.

Bošković, J.D., and Mehra R.K., (2003). "Failure Detection, Identification and Reconfiguration in Flight Control". In: Caccavale F., Villani L. (eds) Fault Diagnosis and Fault Tolerance for Mechatronic Systems: Recent Advances. Springer Tracts in Advanced Robotics, Springer, 129 – 167, Berlin, Germany.

Boskovic, J. D., and Mehra, R. K., (1998). "A Multiple Model-Based Reconfigurable Flight Control System Design". In Proceedings of the 37th IEEE Conference on Decision and Control, December 16-18, 1998, Tampa, Florida, USA.

Boubaker, O., and Iriarte, R., (2017). "The Inverted Pendulum in Control Theory and Robotics: From Theory to New Innovations". IET Control, Robotics and Sensors Series 111, London, UK.

Bounemeur, A., Chemachema, M., and Essounbouli, N., (2018). "Indirect Adaptive Fuzzy Fault Tolerant Tracking Control for MIMO Nonlinear Systems with Actuator and Sensor Failures". ISA Transactions, 79, 45–61.

Bououden, S. Chadli, M., Zhang, L., and Yang, T., (2016). "Constrained Model Predictive Control for Time-Varying Delays Systems: Application to an Active Car Suspension". International Journal of Control, Automation and Systems, 14(1), 51-58.

Boyd, S., El Ghaoul, L., Feron, E., and Balakrishnan, V., (1994). "Linear Matrix Inequalities in System and Control Theory". vol. 15. Society for Industrial Mathematics, Philadelphia, USA.

Branicky, M.S., (1995). "Studies in Hybrid Systems: Modeling, Analysis, and Control". PhD Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

Branicky, M. S., Borkar, V. S., and Mitter, S. K., (1998). "A Unified Framework for Hybrid Control: Model and Optimal Control Theory". IEEE Transactions on Automatic Control, 43(1), 31–45.

Branicky, M.S., (1998). "Multiple Lyapunov Functions and Other Analysis Tools for Switched and Hybrid Systems". IEEE Transactions on Automatic Control, 43(4), 475–482.

Camacho, E. F., and Bordons, C., (2004). "Model predictive control". Springer-Verlag, London, UK.

Camacho, E.F., Ramirez, D.R., Limon, D., Munoz de la Pena, D., and Alamo, T., (2010). "Model Predictive Control Techniques for Hybrid Systems". Annual Reviews in Control, 34(1), 21–31.

Camacho, E. F., Alamo, T., and Munoz de la Pena, D., (2010). "Fault-Tolerant Model Predictive Control". IEEE 15th Conference on Emerging Technologies and Factory Automation, September 13-16, 2010, Bilbao, Spain.

Cavanini, L., and Ippoliti, G., (2018). "Fault Tolerant Model Predictive Control for an Over-Actuated Vessel". Ocean Engineering, 160, 1–9.

Chaib, S., Boutat, D., Benali, A., and Barbot, J.P., (2005). "Observability of the Discrete State for Dynamical Piecewise Hybrid Systems". Nonlinear Analysis: Theory, Methods & Applications. 63(3), 423–438.

Chen, B. S., and Lee, C, H., and Chang, Y. C., (1996). "H/sup/spl infin//Tracking Design of Uncertain Nonlinear SISO Systems: Adaptive Fuzzy Approach". IEEE Transactions on Fuzzy Systems, 4(1), 32-43.

Chen, J., Xu, C., Wu, C., and Xu, W., (2018). "Adaptive Fuzzy Logic Control of Fuel-Cell-Battery Hybrid Systems for Electric Vehicles". IEEE Transactions on Industrial Informatics, 14(1), 292-300.

Chen J., and Patton, R.J., (1999). "Robust Model-Based Fault Diagnosis for Dynamic Systems". Kluwer Academic Publishers, Boston, USA.

Chen, J., and Zhang, H. Y., (1990). "Parity Vector Approach for Detecting Failures in Dynamic Systems", International Journal of Systems Science, 21(4), 765-770.

Ciubotaru, B., Staroswiecki, M., and Christophe, C., (2006). "Fault Tolerant Control of the Boeing 747 Short-Period Mode using the Admissible Model Matching Technique". In IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes 06, August 30 – September 1, 2006, Beijing, China.

Clark, D.W., Mohtadi, C., and Tuffs, P.S., (1987). “Generalized Predictive Control: Part I: The Basic Algorithm”. Part II: Extensions and Interpretation. Automatica, 23(2), 137-160.

Dashkovskiy, S., and Feketa, P., (2017). “Prolongation and Stability of Zeno Solutions to Hybrid Dynamical Systems”. in: Proc. of the 2017 IFAC World Congress, July 9-14, 2017, 3484-3489, Toulouse, France.

De Almeida, F. A., and Liebling, D., (2010). “Fault-Tolerant Model Predictive Control with Flight-Test Results”. International Journal of Adaptive Control and Signal Processing, 23(8), 363-375.

De Carlo, R., Branicky, M.S., Pettersson, S., and Lennartson, B., (2000). “Perspectives and Results on the Stability and Stabilizability of Hybrid Systems”. Proc. IEEE, 88(7), 1069–1082.

De Jager, B., Van Keulen T., and Kessels, J., (2013). “Optimal Control of Hybrid Vehicles. Advanced in Industrial Control”. Springer, London, UK.

De La Sen, M., and Luo, N., (2000). “Design of Linear Observers for a Class of Linear Hybrid Systems”. International Journal of Systems Science, 31(9), 1077–1090.

De Oliveira, M.C., Bernussou, J., and Geromel, J.C., (1999). “A New Discrete-Time Robust Stability Condition”. Systems & Control Letters, 37(4), 261-265.

De Oliveira, M., and Skelton, R., (2001). “Stability Tests for Constrained Linear Systems”. In Perspectives in Robust Control, Lecture Notes in Control and Information Sciences 268 edited by S. O. Reza Moheimani, Springer, 241–257, London, UK.

De Souza Júnior, A. B., De Castro Diniz, E., De Araújo Honório, D., Barreto, L. H. S. C. and Nogueira dos Reis, L. L., (2014). “Hybrid Control Robust Using Logic Fuzzy Applied to the Position Loop for Vector Control to Induction Motors”. Electric Power Components and Systems, 42(6), 533-543.

Di Benedetto, M. D., Di Gennaro, S., and D’Innocenzo, A., (2009). “Discrete State Observability of Hybrid Systems”. International Journal of Robust Nonlinear Control, 19(14), 1564–1580.

Ding, B., and Huang, B., (2007). “Constrained Robust Model Predictive Control for time-delay Systems with Polytopic Description”. International Journal of Control, 80(4), 509–522.

Ding, L., He, Y., Wu, M., and Wang, Q., (2018). “New Augmented Lyapunov-Krasovskii Functional for Stability Analysis of Systems with Additive Time-Varying Delays”. Asian Journal of Control, 20(5), 1–8.

Ding, S., Jeinsch, P., and Ding, E., (2000). “A Unified Approach to the Optimization of Fault Detection Systems”. International Journal of Adaptive Control and Signal Processing, 14(7), 725–745.

El Ghaoui, L., Oustry, F., and Ait Rami, M.A., (1997). “Cone Complementary Linearization Algorithm for Static Output-Feedback and Related Problems”. IEEE Transaction and Automatic Control, 42(8), 1171-1176.

Elzaghir, W., Zhang, Y., Natrajan, N., Massey F., and Mi, C. C., (2018). "Model Reference Adaptive Control for Hybrid Electric Vehicle with Dual Clutch Transmission Configurations". IEEE Trans on Vehicular Technology, 67(2), 991-999.

English, J. D., and Maciejewski, A. A., (1998). "Fault Tolerance for Kinematically Redundant Manipulators: Anticipating Free-Swinging Joint Failures". IEEE Transactions on Robotics and Automation, 14(4), 566-575.

Essounbouli, N., and Hamzaoui, A., (2006). "Direct and Indirect Robust Adaptive Fuzzy Controllers for a Class of Nonlinear Systems". International Journal of Control, Automation, and Systems, 4(2), 146-154.

Ferranti, L., Wan, Y., and Keviczky, T., (2019). "Fault-Tolerant Reference Generation for Model Predictive Control with Active Diagnosis of Elevator Jamming Faults". International Journal of Robust and Nonlinear Control, 29(16), 5412-5428.

Flila, S., Dufour, P., and Hammouri, H., (2008). "Optimal Input Design for Online Identification: A Coupled Observer-MPC Approach". IFAC-Proceedings Volumes, 41(2), 11457–11462.

Frank, P.M., (1990). "Fault Diagnosis in Dynamic Systems using Analytical and Knowledge-Based Redundancy: A Survey and Some New Results". Automatica, 26(3), 459–474.

Gahinet, P., Nemirovski, A., Laub, A. J., and Chilali, M., (1995). "LMI Control Toolbox". MathWorks, Natick, Massachusetts, USA.

Gao, L., Zhang, M., and Yao, X., (2019). "Stochastic Input-to-State Stability for Impulsive Switched Stochastic Nonlinear Systems with Multiple Jumps". International Journal of Systems Science, 50(9), 1860-1871.

Gao, L., Cao, Z., & Wang, G., (2019). "Almost Sure Stability of Discrete-Time Nonlinear Markovian Jump Delayed Systems with Impulsive Signals". Nonlinear Analysis: Hybrid Systems, 34, 248–263.

Gao, Z., and Antsaklis, P. J., (1991). "Stability of the Pseudo-Inverse Method for Reconfigurable Control Systems". International Journal of Control, 53(3), 717–729.

Gao, Z., and Antsaklis, P. J., (1992). "Reconfigurable Control System Design via Perfect Model-Following". International Journal of Control, 56(4), 783–798.

Gertler, J., and Monajemy, R., (1995). "Generating Directional Residuals with Dynamic Parity Relations". Automatica, 31(4), 627-635.

Gertler, J., (1997). "Fault Detection and Isolation Using Parity Relations". Control Engineering Practice, 5(5), 653-661.

Gertler, J., (1998). "Fault Detection and Diagnosis in Engineering Systems". Marcel Dekker, New York, USA.

Ghaemi, S., Sabahi, K., and Badamchizadeh, M. A., (2019). "Lyapunov–Krasovskii Stable T2FNN Controller for a Class of Nonlinear Time-Delay Systems". Soft Computing, 23(4), 1407–1419.

Goebel, R., Sanfelice R. G., and Teel, A. R., (2012). “Hybrid Dynamical Systems: Modeling, Stability and Robustness”. Princeton University Press, New Jersey, USA.

Goncalves, J.M.M.S., (2000). “Constructive Global Analysis of Hybrid Systems”. PhD Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

Gueguen, H., and Zaytoon, J., (2004). “On the Formal Verification of Hybrid Systems”. Control Engineering Practice, 12(10), 1253–1267.

Hauroigne, P., Riedinger P., and Iung, C., (2011). “Switched Affine Systems using Sampled-Data Controllers: Robust and Guaranteed Stabilization”. IEEE Transactions on Automatic Control, 56(12), 2929–2935.

He, Z., and Xie, W., (2016). “Control of Non-Linear Switched Systems with Average Dwell Time: Interval Observer-Based Framework”. IET Control Theory & Applications, 10(1), 10–16.

Heemels, W.P.M.H., De Schutter, B., and Bemporad, A., (2001). “Equivalence of Hybrid Systems Models”. Automatica, 37(7), 1085–1091.

Henry, D., and Zolghadri, A., (2006). “Norm Based Design of Robust FDI Schemes for Uncertain Systems Under Feedback Control: Comparison of Two Approaches”. Control Engineering Practice, 14(9), 1081–1097.

Hespanha, J.P., and Morse, A.S., (1999). “Stability of Switched Systems with Average Dwell-Time”. In: Proceedings of the 38th IEEE Conference on Decision and Control, December 7-10, 1999, Phoenix, Arizona, USA.

Hetel, L. Daafouz, J., and Iung, C., (2006). “Stabilization of Arbitrary Switched Linear Systems with Unknown Time Varying Delays”. IEEE Transactions on Automatic Control, 51(10), 1668–1674.

Hetel, L., (2007). “Robust Stability and Control of Switched Linear Systems”. PhD thesis, Université de Nancy, Nancy, France.

Hosseini-Pishrobat, M., and Keighobadi, J., (2019). “Extended State Observer-Based Robust Non-Linear Integral Dynamic Surface Control for Triaxial MEMS Gyroscope”. Robotica, 37(3), 481–501.

Hu, L., Huang, B., and Cao, Y., (2004). “Robust Digital Model Predictive Control for Linear Uncertain Systems with Saturations”. IEEE Transactions on Automatic Control, 49(5), 792–796.

Hu, X.B., and Chen, W.H., (2004). “Model Predictive Control for Constrained Systems with Uncertain State-Delays”. International Journal of Robust and Nonlinear Control, 14(17), 1421–1432.

Hui, Y., Michel, A.N., and Hou, L, (1998). “Stability Theory for Hybrid Dynamical Systems”. IEEE Transactions on Automatic Control, 43(4), 461–474.

Ichalal, D., (2009). « Estimation et Diagnostic de Systemes Non-Lineaires d’Ecrits par un Model Takagi-Sugeno and Control”. PhD thesis, Université de Nancy, Nancy, France.

Isermann, R., (1984). "Process Fault Detection Based on Modelling and Estimation Methods: A Survey". Automatica, 20(4), 387-404.

Isermann, R., and Balle, P., (1997). "Trends in the Application of Model-Based Fault Detection and Diagnosis of Technical Processes". Control Engineering Practice, 5(5), 709-719.

Isermann, R., (2005). "Model-Based Fault Detection and Diagnosis - Status and Applications". Annual Reviews in Control, 29(1), 71–85.

Isermann, R., (2006). "Fault-Diagnosis Systems: An Introduction from Fault Detection to Fault Tolerance". Springer, Berlin, Germany.

Ishihara, K., and Morimoto, J., (2018). "An Optimal Control Strategy for Hybrid Actuator Systems: Application to an Artificial Muscle with Electric Motor Assist". Neural Networks, 99, 92–100.

Jaimoukha, I., Li, Z., and Papakos, V., (2006). "A Matrix Factorization Solution to the $h-/h\infty$ Fault Detection Problem". Automatica, 42(11), 1907–1912.

Jasso-Fuentes, H., Menaldi, J. L., Prieto-Rumeau, T., and Robin, M., (2018). "Discrete-Time Hybrid Control in Borel Spaces: Average Cost Optimality Criterion". Journal of Mathematical Analysis and Applications, 462(2), 1695–1713.

Ji, Z., Wang, L., and Guo, X., (2008). "On Controllability of Switched Linear Systems". IEEE Transactions on Automatic Control, 53(3), 796–801.

Jiang, J., and Yu, X., (2012). "Fault Tolerant Control Systems: A Comparative Study between Active and Passive Approaches". Annual Reviews in Control, 36(1), 60–72.

Johansson, K. H., Egerstedt, M., Lygeros, J., and Sastry, S., (1999). "On the Regularization of Zeno Hybrid Automata". Systems & Control Letters. 38(3),141-150.

Kalman, R., (1960). "A New Approach to Linear Filtering and Prediction Problems". Transactions of the ASME - Journal of Basic Engineering, 82(1), 35–45.

Karabacak, O., Kivilcim, A., and Wisniewski, R., (2020). "Almost Global Stability of Nonlinear Switched Systems with Time-Dependent Switching". IEEE Transactions on Automatic Control, 65(7), 2969–2978.

Karmakar, N., (1984). "A New Polynomial Time Algorithm for Linear Programming". Combinatorica. 4(4), 373-395.

Kerrigan, E. C., and Maciejowski, J. M., (1999). "Fault-Tolerant Control of a Ship Propulsion System Using Model Predictive Control". European Control Conference (ECC), 4602-4607, August 31 – September 3, 1999, Karlsruhe, Germany.

Khairy, M., Elshafei A. L., and Emara, H. M., (2010). "LMI Based Design of Constrained Fuzzy Predictive Control". Fuzzy Sets and Systems, 161(6), 893-918.

Khan, O., Mustafa, G., Khan, A. Q., and Abid, M., (2019). "Robust Observer-Based Model Predictive Control of Non-Uniformly Sampled Systems". ISA Transactions. 98, 37-46.

Klancar, G., and Skrjanc, I., (2007). "Tracking-Error Model-Based Predictive Control for Mobile Robots in Real Time". Robotics and Autonomous Systems, 55(6), 460–469.

Kolathaya, S., and Ames, A.D., (2017). "Parameter to State Stability of Control Lyapunov Functions for Hybrid System Models of Robots". Nonlinear Analysis: Hybrid Systems, 25, 174–191.

Konstantopoulos, L., and Antsaklis, P., (1996). "An Eigenstructure Assignment Approach to Control Reconfiguration". In Proceedings of 4th IEEE Mediterranean Symposium on Control and Automation, June 10 – 14, 1996, Crete, Greece.

Kothare, M. V., Balakrishnan, V., and Morari, M., (1996). "Robust Constrained Model Predictive Control Using Linear Matrix Inequalities". Automatica, 32(10), 1361–1379.

Kundu, A., and Chatterjee, D., (2017). "On Stability of Discrete-Time Switched Systems". Nonlinear Analysis: Hybrid Systems, 23, 191–210.

Labiod. S., Boucherit, M. S., and Guerra, T. M., (2005). "Adaptive Fuzzy Control of a Class of MIMO Nonlinear Systems". Fuzzy sets and systems, 151(1), 59-77.

Labiod, S., and Boucherit, M. S., (2006). "Indirect Fuzzy Adaptive Control of a Class of SISO Nonlinear Systems". Arabian Journal for science and engineering, 31(1), 61-74.

Labiod, S., and Guerra, T.M., (2016). "Direct Adaptive Fuzzy Control for a Class of Nonlinear Systems with Unknown Control Gain Sign". In 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 380-385, July 24 – 29, 2016, Vancouver, BC, Canada.

Lazar, M., Heemels, W.P.M.H., Weiland, S. and Bemporad, A., (2006). "Stabilizing Model Predictive Control of Hybrid Systems". IEEE Transactions on Automatic Control, 51(11), 1813–1818.

Leth, J., Rasmussen, J. G., Schioler, H., and Wisniewski, R., (2015). "Parameter Estimation for A Class of Stochastic Hybrid Systems with State-Dependent Switching Noise". European Control Conference (ECC), July 15-17, 2015, Linza, Austria.

Li, L., Chadli, M., Ding, S. X., Qiu, J., and Yang, Y., (2018). "Diagnostic Observer Design for T-S Fuzzy Systems: Application to Real-Time-Weighted Fault-Detection Approach". IEEE Transactions on Fuzzy Systems, 26(2), 805-816.

Li, Q.K., Zhao J., Liu X.J., and Dimirovski, G.M., (2011). "Observer-Based Tracking Control for Switched Linear Systems with Time-Varying Delay". Int. J. Robust. Nonlinear Control, 21(3), 309–327.

Li, Q.K., Zhao, J., Dimirovski, G.M., and Liu, X.J., (2009). "Tracking Control for Switched Linear Systems with Time-Delay: A State-Dependent Switching Method". Asian Journal of Control, 11(5), 517–526.

Li, S., Ahn, C. K., and Xiang, Z., (2020). "Command Filter Based Adaptive Fuzzy Finite-Time Control for Switched Nonlinear Systems Using State-Dependent Switching Method". IEEE Transactions on Fuzzy Systems, 29(4), 833-845.

Lien, C.H, Yu, K.W., and Chang, H.C., (2020). "Robust Mixed Performance of Continuous Switched Systems with Time-Delay". Asian Journal of Control, 22(1), 1–11.

Lin, F., Wang L.Y., Chen, W., and Polis, M. P., (2020). “On Controllability of Hybrid Systems”. IEEE Transactions on Automatic Control, doi:10.1109/TAC.2020.3015665.

Lin, Y., and al., (2018). “Output Feedback Thruster Fault-Tolerant Control for Dynamic Positioning of Vessels Under Input Saturation”. IEEE Access, 6, 76271-76281.

Lincoln, P. and Tiwari, A., (2004). “Symbolic Systems Biology: Hybrid Modeling and Analysis of Biological Networks”. Lecture Notes in Computation and Control, vol. 2993, pp. 660–672, Springer, Berlin, Germany.

Liu, B., and Marquez, H., (2008). “Controllability and Observability for A Class of Controlled Switching Impulsive Systems”. IEEE Transactions on Automatic Control, 53(10), 2360–2366.

Luenberger, D., (1971). “An Introduction to Observers”. IEEE Transactions on Automatic Control, 16(6), 596–602.

Lygeros, J., Tomlin, C, and Sastry, S., (1999). “Controllers for Reachability Specifications for Hybrid Systems”. Automatica, Special Issue on Hybrid Systems; 35(3), 349–370.

Lygeros, J., Johansson, H. K., Sim´c, S.N., Zhang, J. and Sastry, S.S., (2003). “Dynamical Properties of Hybrid Automata”. IEEE Transactions on Automatic Control, 48(1), 2–17.

Lygeros, J., Tomlin, C., and Sastry, S., (2008). “Hybrid Systems: Modelling, Analysis and Control”. Princeton Univ. Press. New Jersey, USA.

Maciejowski, J. M., (2000). “Fault-Tolerant Aspects of MPC”. In Proceedings of the IEEE Seminar on Practical Experiences with Predictive Control, February 16, 2000, Middlesbrough, UK.

Maciejowski, J. M., (2002). “Predictive Control with Constraints”. Prentice-Hall, Harlow, UK.

Maciejowski, J. M., and Jones, C., (2003). “MPC Fault-Tolerant Flight Control Case Study: Flight 1862”. IFAC Proceedings Volumes, 36(5), 119–124.

Marcucci, T., and Tedrake, R., (2020). “Warm Start of Mixed Integer Programs for Model Predictive Control of Hybrid Systems”. IEEE Transactions on Automatic Control, 66(6), 2433-2448.

Mayne, D. Q., Rawlings, J. B., Rao, C. V., and Scokaert, P. O. M., (2000). “Constrained Model Predictive Control: Stability and Optimality”. Automatica, 36(6), 789–814.

Medina, E.A., and Lawrence, D.A., (2008). “Reachability and Observability of Linear Impulsive Systems”. Automatica, 44(5), 1304–1309.

Mignone, D., (2002). “Control and Estimation of Hybrid Systems with Mathematical Optimization”. PhD thesis, Swiss Federal Institute of Technology, Zurich, Switzerland.

Minh, V.T., (2013). “Stability for Switched Dynamic Hybrid Systems”. Mathematical and Computer Modelling, 57(1), 78–83.

Mironovski, L. A., (1980). “Functional Diagnosis of Dynamic System - A Survey”. Automation and Remote Control, 41, 1122-1143.

Morari, M., (1994). "Advances in Model Based Predictive Control". Oxford University Press, Oxford, UK.

Naghshtabrizi, P., Teel A., and Hespanha, J.P., (2008). "Exponential Stability of Impulsive Systems with Application to Uncertain Sampled-Data Systems". Systems Control Letters, 57(5), 378–385.

Naidu, D., (2002). "Singular Perturbations and Time Scales in Control Theory and Applications: An Overview". Dynamics of Continuous Discrete and Impulsive Systems Series B., 9, 233–278.

Nesterov, Y., and Nemirovski, A., (1994). "Interior point polynomial methods in convex programming: theory and applications". SIAM, Philadelphia, PA, USA.

Noda, T., Teramae, T., Ugurlu, B., and Morimoto, J., (2014). "Development of an Upper Limb Exoskeleton Powered via Pneumatic Electric Hybrid Actuators with Bowden Cable". IEEE/RSJ International Conference on Intelligent Robots and Systems, 3573-3578, September 14 – 18, 2014, Chicago, IL, USA.

Nodozi, I., and Rahmani, M., (2020). "LMI-Based Robust Mixed Integer Model Predictive Control for Hybrid Systems". Int. J. Control, 93(10), 1357–1376.

Oberdieck, R., and Pistikopoulos, E. N., (2015). "Explicit Hybrid Model-Predictive Control: The Exact Solution". Automatica, 58, 152–159.

Ocampo-Martinez, C., and Puig, V., (2009). "Fault-Tolerant Model Predictive Control within the Hybrid Systems Framework: Application to Sewer Networks". Journal of Guidance, Control and Dynamics, 33(2), 757-787.

Ogata, K., (1997). "Modern Control Engineering". 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, New Jersey, USA.

Oishi, Y., and Fujioka, H., (2010). "Stability and Stabilization of Aperiodic Sampled-Data Control Systems using Robust Linear Matrix Inequalities". Automatica, 46(8), 1327–1333.

Or, Y., and Ames, A. D., (2011). "Stability and Completion of Zeno Equilibria in Lagrangian Hybrid Systems". IEEE Transactions on Automatic Control, 56(3),1322-1336.

Orani, N., Pisano, A., Franceschelli, M., Giua, A., and Usai, E., (2011). "Robust Reconstruction of the Discrete State for A Class of Nonlinear Uncertain Switched Systems". Nonlinear Analysis: Hybrid Systems, 5(2), 220–232.

Patton, R., Frank, P., and Clark, R., (2000). "Issues of Fault Diagnosis for Dynamic Systems". Springer-Verlag, London, UK.

Patton, R., Frank, P., and Clark, R., (1989). "Fault Diagnosis in Dynamic Systems: Theory and Application". Prentice Hall international, New Jersey, USA.

Patino, D., Riedinger P., and Iung, C., (2009). "Practical Optimal State Feedback Control Law for Continuous Time Switched Affine Systems with Cyclic Steady State". International Journal of Control, 82(7), 1357–1376.

Peaucelle, D., Arzelier, D., Bachelier, O., and Bernussou, J., (2000). "A New Robust D-Stability Condition for Real Convex Polytopic Uncertainty". Systems & Control Letters, 40(1), 21-30.

Petreczky, M., and Van Schuppen, J. H., (2010). “Span-Reachability and Observability of Bilinear Hybrid Systems”. Automatica, 4(3), 501–509.

Pettersson, S., Lennartson, B., (2002). “Hybrid System Stability and Robustness Verification Using Linear Matrix Inequalities”. International Journal of Control, 75(16/17), 1335–1355.

Pettersson, S., (2006). “Designing Switched Observers for Switched Systems using Multiple Lyapunov Functions and Dwell-Time Switching”. In: IFAC Proceedings Volumes, 39(5), 18–23.

Phat, V.N., (2010). “Switched Controller Design for Stabilization of Nonlinear Hybrid Systems with Time-Varying Delays in State and Control”. Journal of the Franklin Institute, 347(1), 195–207.

Philippe, M., Millerioux, G., and Jungers, R.M., (2017). “Deciding the Boundedness and Dead-Beat Stability of Constrained Switching Systems”. Nonlinear Analysis: Hybrid Systems, 23, 287–299.

Praveen Kumar Reddy, M., and Rajasekhara Babu, M., (2019). “A Hybrid Cluster Head Selection Model for Internet of Things”. Cluster Computing, 22(6), 13095–13107.

Potocnik, B., Bemporad, A., Torrisi, F.D., Music, G., and Zupancic, B., (2004). “Hybrid Modelling and Optimal Control of Multiproduct Batch Plant”. Control Engineering Practice, 12(9), 1127–1137.

Potocnik, B., Mušic, G., Škrjanc, I., and Zupancic, B., (2008). “Model-based Predictive Control of Hybrid Systems: A Probabilistic Neural-Network Approach to Real-time Control”. Journal of Intelligent and Robotic Systems, 51(1), 45–63.

Qian, J., Dufour, P., and Nadri, M., (2013). “Observer and Model Predictive Control for On-line Parameter Identification in Nonlinear Systems”. IFAC-Proceedings Volumes, 46(23), 571–576.

Raimondo, D. M., Marseglia, G. R., Braatz, R. D., and Scott, J. K., (2013). “Fault-Tolerant Model Predictive Control with Active Fault Isolation”. Conference on Control and Fault-Tolerant System, October 9-11, 2013, Nice, France.

Rebel, M., Esfanjani, R.M., Nikravesh, S.K.Y., and Allgower, F., (2011). “Model Predictive Control of Constrained Non-Linear Time Delay Systems”. IMA Journal of Mathematical Control and Information, 28(2), 183–201.

Ren, W., and Xiong, J., (2019). “Stability Analysis of Impulsive Switched Time-Delay Systems with State-dependent Impulses”. IEEE Transactions on Automatic Control, 64(9), 3928–3935.

Richalet, J., Rault, A., Testud, J.L., and Papon, J., (1978). “Model Predictive Heuristic Control: Application to Industrial Processes”. Automatica, 14(5), 413-428.

Rodrigues, M., Theilliol, D., and Sauter, D., (2006). “Fault Tolerant Control Design for Switched Systems”. IPV-IFAC-Proceeding Volumes, 39(5), 223–228.

Salvador, C. H., (2005). "Stratégie de Commande Intégrée Intelligente de Procèdes de Traitement des Eaux Usées par la Digestion Anaérobie". Thèse de doctorat, Institut National Polytechnique de Grenoble, Grenoble, France.

Scholte, E. and Campbell, M. E., (2008). "Robust Nonlinear Model Predictive Control with Partial State Information". IEEE Transactions on Control Systems Technology., 16(4), 636–651.

Scokaert, P. O. M., Mayne, D. Q., and Rawlings, J. B., (1999). "Suboptimal Model Predictive Control (Feasibility Implies Stability)". IEEE Transactions on Automatic Control, 44(3), 648–654.

Shahid Shaikh, M., (2004). "Optimal Control of Hybrid Systems: Theory and Algorithms". PhD thesis, Department of Electrical and Computer Engineering, McGill University, Montreal, Canada.

Shim, H., and Tanwani, A., (2011). "On a Sufficient Condition for Observability of Switched Nonlinear Systems and Observer Design Strategy". Proc. American Control Conference, 1206–1211, June 29 – July 1, 2011, San Francisco, CA, USA.

Shin, D., Seitz, F., Khatib, O., Cutkosky, M., (2010). "Analysis of Torque Capacities in Hybrid Actuation for Human-Friendly Robot Design". IEEE International Conference on Robotics and Automation, 799-804, May 3 -7, 2010, Anchorage, Alaska, USA.

Singh, H. P., and Sukavanam, N., (2013). "Stability Analysis of Robust Adaptive Hybrid Position/Force Controller for Robot Manipulators using Neural Network with Uncertainties". Neural Computing and Applications, 22(7), 1745-1755.

Siroupour, S., and Shahdi, A., (2006). "Model Predictive Control for Transparent Teleoperation Under Communication Time Delay". IEEE Transactions on Robotics, 22(6), 1131–1145.

Staroswiecki, M. (2005a). "Fault Tolerant Control: The Pseudo-Inverse Method Revisited". IFAC-Proceeding Volumes, 38(1), 418–423.

Stroustrup, J., and Niemann, H., (2002). "Fault Estimation- A Standard Problem Approach". International Journal of Robust and Nonlinear Control, 12(8), 649–673.

Sun, W., and Wu, Y., (2016). "Adaptive Motion/Force Tracking Control for a Class of Mobile Manipulators". Asian Journal of Control, 18(2), 1–8.

Taghieh, A., and Shafiei, M.H., (2020). "Observer-Based Model Predictive Control of Switched Nonlinear Systems with Time Delay and Parametric Uncertainties". Journal of Vibration and Control, doi.:10.1177/1077546320950523.

Tao, T., Zhao, W., Du, Y., Cheng, Y., and Zhu, J., (2020). "Simplified Fault-Tolerant Model Predictive Control for a Five-Phase Permanent-Magnet Motor with Reduced Computation Burden". IEEE Transactions on Power Electronics, 35(4), 3850-3858.

Tanwani, A., (2014). "Hybrid-Type Observer Design Based on A Sufficient Condition for Observability in Switched Nonlinear Systems". International Journal of Robust Nonlinear Control, 24(6), 1064–1089.

Taringoo, F., (2012). "Control and Optimization of Hybrid Systems on Riemannian Manifolds". PhD thesis, Department of Electrical and Computer Engineering, McGill University, Montreal, Canada.

Teixeira, M.C.M., and Zak, S.H., (1999). "Stabilizing Controller Design for Uncertain Nonlinear Systems using Fuzzy Models". IEEE Transaction on Fuzzy Systems, 7(2), 133–142.

Thau, F., (1973). "Observing the State of Non-Linear Dynamic Systems". International Journal of Control, 17(3), 471–479.

Theilliol, D., Noura, H., and Ponsart, J. C., (2002). "Fault Diagnosis and Accommodation of a Three-Tank System Based on Analytical Redundancy". ISA Transactions, 41(3), 365-382.

Theilliol, D., Sauter, D., and Ponsart, J. C., (2003). "A Multiple Model Based Approach for Fault Tolerant Control in Non-Linear Systems". IFAC Proceedings Volumes, 36(5), 149-154.

Tohidi, S. S., Sedigh, A. K., and Buzorgnia, D., (2016). "Fault Tolerant Control Design using Adaptive Control Allocation Based on the Pseudo-Inverse Along the Null Space". International Journal of Robust and Nonlinear Control, 26(16), 3541–3557.

Tsai, C., Cheng, C., M. B., and Lin, S. C., (2007). "Robust Tracking Control for a Wheeled Mobile Manipulator with Dual-Arms using Hybrid Sliding-Mode Neural Network". Asian Journal of Control, 9(4), 377–389.

Tsui, C., (1999). "A Design Example with Eigenstructure Assignment Control whose Loop Transfer is Fully Realized". Journal of Franklin Institute, 336(7), 1049–1053.

Tuvayanond, W., and Parnichkun, M., (2017). "Position Control of a Pneumatic Surgical Robot Using PSO Based 2-DOF H∞ Loop Shaping Structured Controller". Mechatronics, 43, 40–55.

Usman, A., (2016). "Optimal Control of Constrained Hybrid Dynamical Systems: Theory, Computation and Applications". PhD thesis, Georgia Institute of Technology, Atlanta, USA.

Vesely, V., and Rosinova, D. A., (2009). "Robust Output Model Predictive Control Design: BMI Approach". International Journal of Innovative Computing, Information and Control, 5(4), 1115–1123.

Wada, N., Saito, K., and Saeki, M., (2006). "Model Predictive Control for Linear Parameter Varying Systems Using Parameter Dependent Lyapunov Function". IEEE Trans. Circuits Syst. II, Exp. Briefs, 53(12), 1446–1450.

Wang, A. P., and Lin, S., (1999). "The Parametric Solutions of Eigenstructure Assignment for Controllable and Uncontrollable Singular Systems". Journal of Mathematical Analysis and Applications, 248(2), 549–571.

Wang, L., (2009). "Model Predictive Control System Design and Implementation using Matlab". Springer, London, UK.

Wang, L., and al., (2017). "Iterative Learning Fault-Tolerant Control for Injection Molding Processes Against Actuator Faults". Journal of Process Control, 59, 59–72.

Wang, L. X., (1993). "Stable Adaptive Fuzzy Control of Nonlinear Systems". IEEE Transactions on fuzzy systems, 1(2), 146-155.

Wang, L.X., and Mendel, J.M., (1992). "Fuzzy Basis Functions, Universal Approximation and Irthogonal Least-Squares". IEEE Transactions on Neural Networks, 3(5), 807-814.

Wang, T., Gao, H., and Qiu, J., (2016). "A Combined Fault-Tolerant and Predictive Control for Network-Based Industrial Processes". IEEE Transactions on Industrial Electronics, 63(4), 2529-2536.

Wang, Z., Chen, G., and Ba, H., (2019). "Stability Analysis of Nonlinear Switched Systems with Sampled-Data Controllers". Applied Mathematics and Computation, 357(15), 297–309.

Wang, Z., Gao, L., and Liu, H., (2021). "Stability and Stabilization of Impulsive Switched Systems with Inappropriate Impulsive Switching Signals under Asynchronous Switching". Nonlinear Analysis: Hybrid Systems, 39, 100976.

Wu, C., Teo K. L., and Li, R., and Zhao, Y., (2006). "Optimal Control of Switched with Time-Delays". Applied Mathematics Letters, 19(10), 1062–1067.

Wu, Q. and al., (2017). "Development and Hybrid Force/Position Control of a Compliant Rescue Manipulator". Mechatronics, 46, 143–153.

Wu, X., Shi, P., Tang, Y., and Zhang, W., (2016). "Input-to-State Stability of Nonlinear Stochastic Time-Varying Systems with Impulsive Effects". International Journal of Robust and Nonlinear Control, 27(10), 1792–1809.

Xia, Y., Liu, G. P., Shi, P., Chen, J., and Rees, D., (2008). "Robust Constrained Model Predictive Control Based on Parameter-Dependent Lyapunov Functions". Circuits Syst. Signal Process., 27(4), 429–446.

Xia, Y., Yang, H., Shi, P., and Fu, M., (2010). "Constrained Infinite-Horizon model predictive control for Fuzzy-Discrete Time Systems". IEEE Transactions on Fuzzy Systems, 19(3), 429–436.

Xie, G., and Wang, L., (2004). "Necessary and Sufficient Conditions for Controllability and Observability of Switched Impulsive Control Systems". IEEE Transactions on Automatic Control, 49(6), 960–966.

Xu, H.L., Liu, X.Z., and Teo, K.L., (2008). "A LMI Approach to Stability Analysis and Synthesis of Impulsive Switched Systems with Time Delays". Nonlinear Analysis: Hybrid Systems, 2(1), 38-50.

Yang, D., Li, X., and Song, S., (2020). "Design of State-Dependent Switching Laws for Stability of Switched Stochastic Neural Networks with Time-Delays". IEEE Transactions on Neural Networks and Learning Systems, 31(6), 1808–1819.

Yang, D., Li, X., and Qiu, J., (2019). "Output Tracking Control of Delayed Switched Systems via State-Dependent Switching and Dynamic Output Feedback". Nonlinear Analysis: Hybrid Systems, 32, 294–305.

Yang, H., (2009). "Fault Tolerant Control Design for Hybrid Systems". PhD Thesis, Lille 1 University and Nanjing University.

Yang, H., Jiang, B., and Cocquempot, V., (2010). "Hybrid Systems with Time-Dependent Switching". Fault Tolerant Control Design for Hybrid Systems. Lecture Notes in Control and Information Sciences, vol. 397, pp. 11–58. Springer, Berlin, Germany.

Yang, X., and Maciejowski, J. M., (2015). "Fault-Tolerant Control using Gaussian Processes and Model Predictive Control". International Journal of Applied Mathematics and Computer Science, 25(1), 133–148.

Yang, X., and Maciejowski, J. M., (2012). "Fault-Tolerant Model Predictive Control of a Wind Turbine Benchmark". IFAC-Proceedings Volumes, 45(20), 337–342.

Youssef, T., Chadli, M., Karimi, H. R., and Wang, R., (2017). "Actuator and Sensor Faults Estimation Based on Proportional Integral Observer for T-S Fuzzy Model". Journal of the Franklin Institute, 354(6), 2524-2542.

Yu, L., Barbot, J.P., Boutat, D., Benmerzouk, D., (2011). "Observability Forms for Switched Systems with Zeno Phenomenon or High Switching Frequency". IEEE Transactions on Automatic Control, 56(2), 436–441.

Zahaf, A. Bououden, S., Chadli, M., Zelinka, I., and Boulkaibet, I., (2019). "Observer Based Model Predictive Control of Hybrid Systems". Advanced Control Engineering Methods in Electrical Engineering Systems. ICEECA'2017. Lecture Notes in Electrical Engineering, vol. 522, pp. 198 – 207, Springer, Cham, Switzerland.

Zahaf, A., Chemachema, M., Bououden, S., and Boulkaibet, I., (2020). "Fault Tolerant Predictive Control for Constraints Hybrid Systems with Sensors Failures". Proceedings of the 4th International Electrical Engineering and Control Applications. ICEECA'2019. Lecture Notes in Electrical Engineering, vol. 682, pp. 973 - 984, Springer, Singapore, Singapore.

Zahaf, A., Beunemeur, A., Bououden, S., and Boulkaibet, I., (2020). "Fault Diagnosis of Uncertain Hybrid Actuators Based Model Predictive Control". Proceedings of the 4th International Electrical Engineering and Control Applications. ICEECA'2019. Lecture Notes in Electrical Engineering, vol. 682, pp. 961 – 971, Springer, Singapore, Singapore.

Zahaf, A., Bououden, S., Chadli, M., and Chemachema, M., (2020). "Robust Fault Tolerant Optimal Predictive Control of Hybrid Actuators with Time-Varying Delay for Industrial Robot Arm". Asian J Control. https://doi.org/10.1002/asjc.2444

Zhai, D., Lu. A.Y., Li, J.H., and Zhang, Q.L., (2016). "Simultaneous Fault Detection and Control for Switched Linear Systems with Mode-Dependent Average Dwell-Time". Applied Mathematics and Computation, 273, 767-792.

Zhai, M., Long, Z., and Li, X., (2019). "Fault-Tolerant Control of Magnetic Levitation System Based on State Observer in High Speed Maglev Train". IEEE Access, 7, 31624-31633.

Zhang. H., (2007). "Robust Optimal Control of Hybrid Systems". PhD Thesis, The Australian National University, Canberra, Australia.

Zhang, S., and Nie, H., (2020). "Asynchronous Feedback Passification for Discrete-Time Switched Systems Under State-Dependent Switching with Dwell Time Constraint". International Journal of Adaptive Control and Signal Processing. doi:10.1002/acs.3090

Zhang, X., (2020). "Robust Integral Sliding Mode Control for Uncertain Switched Systems under Arbitrary Switching Rules". Nonlinear Analysis: Hybrid Systems, 37, 100900

Zhang, Y., and Jiang, J., (2000). "Design of Proportional Integral Reconfigurable Control Systems via Eigenstructure Assignment". Proceedings of the 2000 American Control Conference, ACC, June 28-30, 2000, Chicago, Il, USA.

Zhang, Y., and Jiang, J., (2001). "Integrated Active Fault-Tolerant Control Using IMM Approach". IEEE Transactions on Aerospace and Electronic systems, 37(4), 1221-1235.

Zhao, F., Koutsoukos, X., Haussecker, H., Reich, J., and Cheung, P., (2005). "Monitoring and Fault Diagnosis of Hybrid Systems". IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 99(14), 1225–1240.

Zhao, X., Zhang, L., Shi, P., and Liu, M., (2012). "Stability and Stabilization of Switched Linear Systems with Mode-Dependent Average Dwell Time". IEEE Transactions on Automatic Control, 57(7), 1809–1815.

Zhendong Sun, S., Ge, S., and Lee, T.H., (2002). "Controllability and Reachability Criteria for Switched Linear Systems". Automatica, 38(5), 115 – 786.

Zheng, Q., Ling, Y., Wei, L., and Zhang, H., (2018). "Mixed H∞ and Passive Control for Linear Switched Systems Via Hybrid Control Approach". International Journal of Systems Science, 49(9), 1–14.

Zhou, K., and Khargonedkar, P., (1988). "Robust Stabilization of Linear Systems with Norm-Bounded Time-Varying Uncertainty". Systems & Control Letters, 10, 17-20.

Zhu, F., and Antasklis, P.J., (2015). "Optimal Control of Hybrid Switched Systems: A Brief Survey". Discrete Event Dynamic Systems, 25(3), 345–364.

Zolghadri, A., (1996). "An Algorithm for Real-Time Failure Detection in Kalman Filter". IEEE Transactions on Automatic Control, 41(10), 1537–1539.

Zong, G., and Zhao, H., (2018). "Input-to-State Stability of Switched Nonlinear Delay Systems Based on a Novel Lyapunov-Krasovskii Functional Method". Journal of Systems Science and Complexity, 31(4), 875–888.