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# *Natural convection in rectangularenclosure with heating and coolingby sinusoidal temperature profiles on twoverticalsides*

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**Abstract**

We propose in this work, the numerical study of the natural convection laminar and permanent, in rectangular enclosures whose vertical walls are active and horizontal one are adiabatic. The active walls are heated and cooled by sinusoidal temperature profiles, three cases of heating and cooling have been considered. Steady state heat transfer by laminar natural convection has been studied numerically by solving equations of mass, momentum and energy. The enclosure is filled by a Newtonian and incompressible fluid. The number of Prandtl is fixed at 0.702 (case of the air), but the number of Rayleigh varies. By using the approximation of Boussinesq and the vorticity-stream function formulation, the flow is modelled by the differential equations with the derivative partial.

A computer code was developed, it uses finite volumes, for the discretization of the equations, and in order to show its reliability, we compare results resulting from the latter with other similar results existing in the literature, and we examine the effect of the Rayleigh number, and the parietal thermal conditions, on the results obtained, that it is qualitatively or quantitatively.

Keywords: Fluid Mechanic, Heat Transfer, Natural Convection, Closed enclosures,

## *1.0 Introduction*

Heat transfer by natural convection, in rectangular enclosures, was the subject of many theoretical and experimental studies because of their importance inmany engineering applications.

W. Zhang et al [1] investigated numerically, the unsteady conjugate conduction-natural convection in an enclosure, by a high accuracy multidomain temporal–spatial pseudospectral method. The enclosure is filled with Boussinesq fluid and is bounded by four finite thickness and conductive walls; one of the vertical sidewall is exposed to time-periodic temperature environment while the opposite sidewall holds constant temperature; the top and bottom walls are assumed to be adiabatic. Particular efforts are focused on the effects of three types of influential factors: the wall thermophysical properties, the time-periodic temperature patterns and the inclination, and the time-periodic flow patterns and heat transfer characteristics are presented. Numerical results reveal that within the present parameter range, the heat transfer rate increases almost linearly with the thermal conductivity ratio and thermal diffusivity ratio but decreases with the inclination angle.Moreover, the heat transfer could be enhanced or weakened by selecting different temperature pulsating period in the case of finite thickness wall, while it is always enhanced if the walls are zero thickness. The back heat transfer and heat transfer resonance phenomena are observed, and their relationships with thetime-periodic flow patterns and temperature distributions are analyzed. The findings are helpful to the understandings of the fluid flow and heat transfer mechanisms in the related enclosure configurations, and may be of engineering use in thermal design improvement. E. Bilgen and R.B. Yedder[2]presented a numerical study in rectangular enclosures, which have a vertical active wall with all the other walls insulated. The equallydivided active sidewall is heated and cooled with sinusoidal temperature profiles. Two cases have been considered: the first is the lower part is heated while the upper part is cooled and the second, the upper part is heated and lower part is cooled. Steady state heat transfer by laminar natural convection has been studied by numerically solving equations of mass, momentum and energy, to determine the thermal penetration in the enclosures and heat transfer as a function of Rayleigh number, the aspect ratio and the position of side heating with respect to side cooling. Rayleigh number was varied from $10^3$ to $10^6$ and the aspect ratio from 0.2 to 5, and the results are presented in the form of streamlines and isotherms, local and average Nusselt number, and heat penetration length.A. Dalal and M.K Das[3]studied the steady, laminar natural convection in a two-dimensional enclosure with three flat and one wavy wall. The top wall is heated with a sinusoidal temperature profile. The other three walls, including the wavy wall, are maintained at constant low temperature. Air is considered as the working fluid. They solved this problem numerically by

SIMPLE algorithm with deferred QUICK scheme in non-orthogonal curvilinear co-ordinates.

In this study we will analyze natural convection heat transfer in rectangular enclosures, having two side walls heated and cooled with sinusoidal temperature profiles, and the other walls are adiabatic. We see from the literature ere view, that the case of heating with sinusoidal temperature profile, on two side walls has not been addressed. This case has a certain importance in industrial processes.

## 2.0 Nomenclature

| | |
|---|---|
| $a$ | : Thermal diffusivity $(m^2.s^{-1})$ |
| $c_p$ | : Specific heat at constant pressure $(j.kg^{-1}.K^{-1})$ |
| $g$ | : Gravitational acceleration$(m.s^{-2})$ |
| $L$ | : Side of the rectangular enclosure $(m)$ |
| $Nu$ | ; Nusselt number |
| $P$ | : Pressure $(N.m^{-2})$ |
| $Pr$ | : Prandtl number |
| $Ra$ | : Rayleigh number |
| $S_\varphi$ | : Source |
| $t$ | : Time, $(s)$ |
| $T$ | : Fluid's temperature $(K)$ |
| $T_{ref}$ | : Reference temperature $(K)$ |
| $T(y)$ | :Sidewalltemperature, $=T_{ref}\pm\Delta T.sin(2\pi y/L)$ $(K)$ |
| $U$ | : Velocity component-coordinate x $(m.s^{-1})$ |
| $V$ | : Velocity component-coordinate y $(m.s^{-1})$ |
| $\vec{V}$ | : Velocity vector $(m.s^{-1})$ |
| $x,y,z$ | : Cartesian coordinates $(m)$ |
| *Greek letters* | |
| $\beta$ | : Thermal expansion coefficient $(K)$ |
| $\Gamma_\varphi$ | : Diffusion coefficient |
| $\Delta T$ | : Temperature difference, $=T(L/4)-T_{ref}(K)$ |
| $\lambda$ | : Thermal conductivity$(W.m^{-1}.K^{-1})$ |
| $\upsilon$ | : Kinematic viscosity $(m^2.s^{-1})$ |
| $\rho$ | : Density $(kg.m^{-3})$ |
| $\Pi$ | : Stress tensor |
| $\psi$ | : Stream function $(m^2.s^{-1})$ |
| $\omega$ | : Vorticity$(s^{-1})$ |
| *Superscripts* | |
| + | : Dimensionless |
| --- | : Average |
| *Subscripts* | |
| $AD$ | : Side AD |
| $BC$ | : Side BC |
| $L$ | : Local |
| $max$ | : Maximum |
| $mid$ | : Medium of the enclosure |
| $min$ | : Minimum |

## 3.0 Problem formulation and basic equations

We consider a rectangular enclosure, filled with an incompressible Newtonian fluid. Figure 1 represents a cross-section of the system. The vertical walls are heated and cooled by sinusoidal temperature profiles and those horizontal are adiabatic, the physical properties of the fluid are constant, except the density ρ whose variations are at theorigin of the natural convection. Viscous dissipation is neglected, just as the radiation (emissive properties of the two walls being neglected).We admit that the problem is bidimensionnal, permanent and laminar.

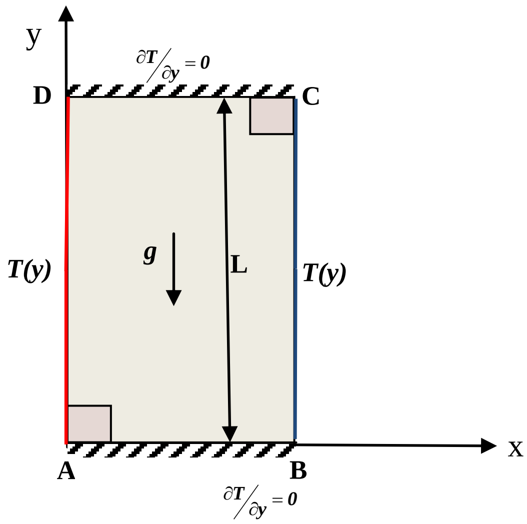


Figure 1: a cross-section of the system

- Continuity equation:

$$div\,\vec{V} = 0 \qquad (1)$$

- Momentum equation:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\overrightarrow{grad})\vec{V} = \frac{\rho}{\rho_0}\vec{g} + \frac{\nabla \Pi}{\rho_0} \qquad (2)$$

- Heat equation:

$$\frac{\partial T}{\partial t} + (\vec{V}.\overrightarrow{grad})T = \frac{\lambda}{\rho\, c_p}\nabla^2 T \qquad (3)$$

We pass directly to the writing dimensionless equations, by posing the following dimensionless quantities:

$x^+ = \frac{x}{L}$, $y^+ = \frac{y}{L}$, $U^+ = \frac{U}{a/L}$, $V^+ = \frac{V}{a/L}$, $\omega^+ = \frac{\omega}{a/L^2}$, $T^+ = \frac{T - T_{ref}}{\Delta T}$, $\psi^+ = \frac{\psi}{a}$ and $t^+ = \frac{t}{L^2/a}$.

With:

Characteristic length: $L$

Characteristic velocity: $a/L$

Equation (1) becomes:

$$\frac{\partial U^+}{\partial x^+} + \frac{\partial V^+}{\partial y^+} = 0 \qquad (4)$$

Projections of Equation (2), following x axis and y axis, after some long manipulations give:

$$\frac{\partial \omega^+}{\partial t^+} + U^+\frac{\partial \omega^+}{\partial x^+} + V^+\frac{\partial \omega^+}{\partial y^+} = \qquad (5)$$

$$Ra.Pr\left(\frac{\partial T^+}{\partial x^+}cos(\alpha) - \frac{\partial T^+}{\partial y^+}sin(\alpha)\right) + Pr\left(\frac{\partial^2 \omega^+}{\partial x^{+2}} + \frac{\partial^2 \omega^+}{\partial y^{+2}}\right)$$

And Equation (3) becomes:

$$\frac{\partial T^+}{\partial t^+} + U^+\frac{\partial T^+}{\partial x^+} + V^+\frac{\partial T^+}{\partial y^+} = \frac{\partial T^{+2}}{\partial x^{+2}} + \frac{\partial T^{+2}}{\partial y^{+2}} \qquad (6)$$

This, after the introduction of the dimensionless vorticity defined by:

$$\omega^+ = -\left[\frac{\partial^2 \psi^+}{\partial x^{+2}} + \frac{\partial^2 \psi^+}{\partial y^{+2}}\right] \qquad (7)$$

With: $U^+ = \frac{\partial \psi^+}{\partial y^+}$ and $V^+ = -\frac{\partial \psi^+}{\partial x^+}$

And Pr and Ra are the following dimensionless numbers:

$$Pr = \frac{\upsilon}{a} \text{ and } Ra = \frac{g\beta\beta(_H - T_C)L^3}{a\upsilon}$$

The initial and the boundary conditions are the following ones:
For $t^+=0$, we have:

$$T^+=0,\ U^+=V^+=\frac{\partial\psi^+}{\partial x^+}=\frac{\partial\psi^+}{\partial y^+}=0 \text{ and } \omega^+=0$$

For $t^+>0$, for all the sides we have:

$$U^+=V^+=\frac{\partial\psi^+}{\partial x^+}=\frac{\partial\psi^+}{\partial y^+}=0 \quad \text{and} \quad \omega^+=-\left(\frac{\partial^2\psi^+}{\partial x^{+2}}+\frac{\partial^2\psi^+}{\partial y^{+2}}\right)$$

And for the first, the second and the third condition of heating, we have respectively:

On the sides AB and BC:

[$\frac{\partial T^+}{\partial y^+}=0$, $T^+=sin(2\pi y^+)$], [$\frac{\partial T^+}{\partial y^+}=0$, $T^+=sin(2\pi y^+)$] and [$\frac{\partial T^+}{\partial y^+}=0$, $T^+=-sin(2\pi y^+)$]

And On the sides AD and CD:

[$T^+=-sin(2\pi y^+)$, $\frac{\partial T^+}{\partial y^+}=0$], [$T^+=sin(2\pi y^+)$, $\frac{\partial T^+}{\partial y^+}=0$] and [$T^+=-sin(2\pi y^+)$, $\frac{\partial T^+}{\partial y^+}=0$]

## 4.0 NumericalMethod

To solve the Equations (5) and (6) with the associated boundary conditions, we consider a numerical solution by the finite volumes method, exposed by S.V. Patankar[4]. For Equation (7), we consider a numerical solution by the centred differences method.

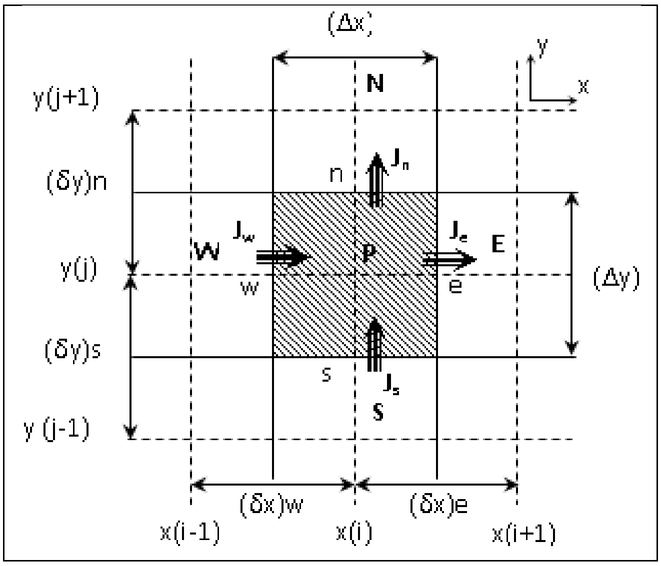


Figure 2: A typical control volume and its neighbours in a computational domain

The enclosure is cut out according to directions x and y, in several elementary volumes or "control volumes" each one is equal to «Δx.Δy.1». The problem is bidimensionnal; the thickness in z direction is taken equal to unity.

The center of a typical control volume is a point P and the center of its side faces "east", "west", "north" and "south", are the points e, w, n and s, respectively. Four other control volumes surround each interior control volume. The centres of these volumes are points E, W, N and S.

### 4.1 Discretization equation transfer of a variable φ

The general differential equation is:

$$\frac{\partial\varphi}{\partial t}+\frac{\partial}{\partial x^+}(U^+\varphi-\Gamma_\varphi\frac{\partial\varphi}{\partial x})+\frac{\partial}{\partial y^+}(V^+\varphi-\Gamma_\varphi\frac{\partial\varphi}{\partial y})=S_\varphi \quad (8)$$

We illustrate sources and diffusion coefficients in Table 1.

**Table 1: Sources and diffusion coefficients**

| $\varphi$ | $\Gamma_\varphi$ | $S_\varphi$ |
|---|---|---|
| $T^+$ | 1 | 0 |
| $\omega^+$ | $Pr$ | $Ra.Pr\left(cos(\alpha)\dfrac{\partial T^+}{\partial x^+} - sin(\alpha)\dfrac{\partial T^+}{\partial y^+}\right)$ |

The discretization equation is obtained by integrating the conservation equation over the control volume shown in Figure 2, after some manipulations we have the final discretization equation:

$$a_P\,\varphi_P = a_N\,\varphi_N + a_S\,\varphi_S + a_E\,\varphi_E + a_W\,\varphi_W + b \qquad (9)$$

The equation coefficients are well defined by S.V. Patankar[4].The power law scheme is used to discretize the convective terms in the governing equations.

The discretization equations of the boundary conditions are written in the same form of equation (9).

After the discretization, we obtain a system of linear equations which we solve by the iterative method of "under relaxation".

Once the temperature distribution is available, the local Nusselt number in the physical domain is defined as:

$$Nu_{L\ AD,BC} = -\left.\frac{\partial T^+}{\partial x^+}\right|_{x=0,1}$$

The average Nusselt number is obtained by integrating the local Nusselt numbers along each side:

$$\overline{NU}_{AD} = \int_0^1 \left(\frac{\partial T^+}{\partial x^+}\right)_0 dy \text{ and } \overline{NU}_{BC} = \int_0^1 \left(\frac{\partial T^+}{\partial x^+}\right)_1 dy$$

## 5.0 *Results and discussion*

TherectangularcavitieswerestudiedatRayleighnumbersvaryingfrom$10^3$to$10^6$.Threecasesof heating and cooling wereconsideredbyasinusoidaltemperatureprofile.Prandtl numberwas0.702.

### 5.1 Grid study

In this study several grids were used arbitrarily, to see their effect on the results. Table 1 shows us the variation of the maximum of the stream function value according to the number of nodes for each grid. We choose the grid (51x51)from which we notice that this value does not vary significantly any more.

**Table 1: Variation of the stream function value according the number of nodes**

| | Ra=$10^3$ | | Ra=$10^4$ | | Ra=$10^5$ | | Ra=$10^6$ | |
|---|---|---|---|---|---|---|---|---|
| nixnn | $\psi_{max}$ | \|Er\| % | $\psi_{max}$ | \|Er\| % | $\psi_{max}$ | \|Er\| % | $\psi_{max}$ | \|Er\| % |
| 21x21 | | - | 2.78 | - | 11.02 | - | 14.73 | - |
| 31x31 | 0.28 | 0.00 | 2.83 | 1.76 | 11.03 | 0.12 | 14.64 | 0.57 |
| 41x41 | 0.28 | 0.00 | 2.83 | 0.00 | 11.11 | 0.68 | 14.74 | 0.67 |
| 51x51 | 0.28 | 0.00 | 2.83 | 0.00 | 11.15 | 0.37 | 14.85 | 0.73 |
| 61x61 | 0.28 | 0.00 | 2.83 | 0.00 | 11.19 | 0.34 | 14.93 | 0.50 |
| 71x71 | 0.28 | 0.00 | 2.83 | 0.00 | 11.22 | 0.25 | 14.98 | 0.35 |
| 81x81 | 0.28 | 0.00 | 2.83 | 0.00 | 11.23 | 0.14 | 15.02 | 0.29 |
| 91x91 | 0.28 | 0.00 | 2.83 | 0.00 | 11.25 | 0.12 | 15.05 | 0.20 |

### 5.2 Numerical code validation

D.V.G. Davis [5] considers a rectangular enclosure; the two vertical walls are heated differently whereas the horizontal walls are insulated. We applied our computer code to this case and we compared our results with theirs, we notice that they are in concord; the maximum relative variation in all the cases does not reach 1%. Table 2 illustrates this comparison well.

**Table 2: Comparison between our results and those of Davis [5]**

| | | Davis [5] | Our results | \| Relative variation \| (%) |
|---|---|---|---|---|
| $Ra=10^3$ (51x51) | $Nu_{AB}$ | 1.117 | 1.119 | 0.18 |
| | $\Psi_{mid}$ | -1.174 | -1.173 | 0.09 |
| $Ra=10^4$ (51x51) | $Nu_{AB}$ | 2.238 | 2.252 | 0.63 |
| | $\Psi_{mid}$ | -5.071 | -5.059 | 0.24 |
| $Ra=10^5$ (61x61) | $Nu_{AB}$ | 4.509 | 4.531 | 0.49 |
| | $\Psi_{mid}$ | -9.111 | -9.112 | 0.01 |
| | $\Psi_{min}$ | -9.612 | -9.611 | 0.01 |

## 5.3 Isotherms and streamlines

### 5.3.1 Influence of the Rayleigh number

Figure 3, Figure 4, Figure 5 and Figure 6 represent the isotherms and the streamlines,in the first case of heating, fordifferentvalues of the Rayleigh number.

When the Rayleigh number is weak, lower or equal to $10^4$, the heat transfer is essentially conductive, so the isotherms of Figure 3 are almost parallel. Nevertheless there is a movement of the fluid: the particles, which warm up on the hot half walls, tend to rise along them, then to go down again along cold half walls. Thus the flow is organized in six principal cells which turn very slowly in opposite directions. The laminar convection is weak.

For $Ra=10^5$ and $Ra=10^6$ the isotherms of the Figures 5 and Figure 6 change appreciably to follow the direction of the flow, values of streamlines mentioned on the same Figure, increase also appreciably, which translates a transformation of the conductive transfer to the convective transfer.

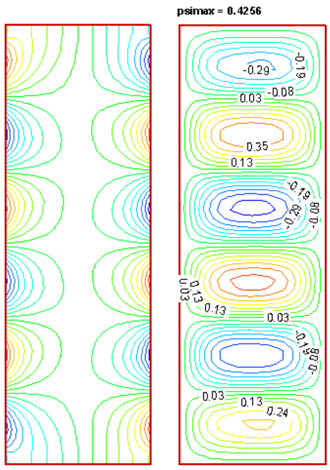


**Figure 3: Isotherms and Streamlines in the 1st case for Ra=$10^3$**

**Figure 4: Isotherms and Streamlines in the 1st case for Ra=$10^4$**

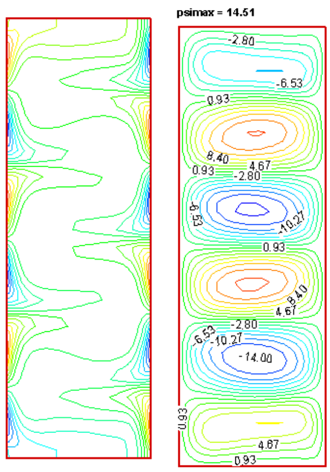


**Figure 5: Isotherms and Streamlines in the 1st case for Ra=$10^5$**

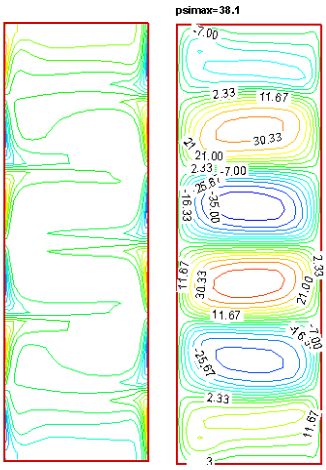


**Figure 6: Isotherms and Streamlines in the 1st case for Ra=$10^6$**

### 5.3.2 Influence of the parietal thermal conditions

Figure 6, Figure8 and Figure10 illustrate respectively the isotherms and the streamlines**,** the firstcaseof heating for Ra=$10^6$, the second case of heating for Ra=$10^4$ and the third case for Ra=$10^5$.

In the first case of heating, the flow is organized in six cells, whose centres move towards the right side wall, which is heated and cooled in different parts; we can say that the flow is moreun stable in the enclosure. In these cond cases and the third case of heating the structure of the flow is multi cellular, in these cond cases of heating, the flow is more intense. For the third case of heating the centres of the cells move towards the active side walls of the enclosure heaving a thermal stratification in the middle of this one, like illustrate it well the isotherms of Figure 9 which are parallel in this region.

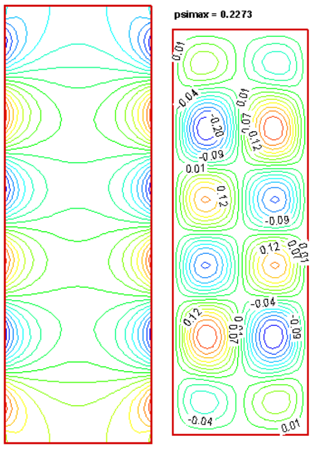


**Figure 7: Isotherms and Streamlines in the 2st case for Ra=$10^3$**

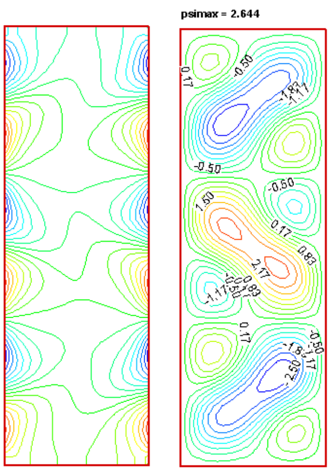


**Figure 8: Isotherms and Streamlines in the 2st case for Ra=$10^4$**

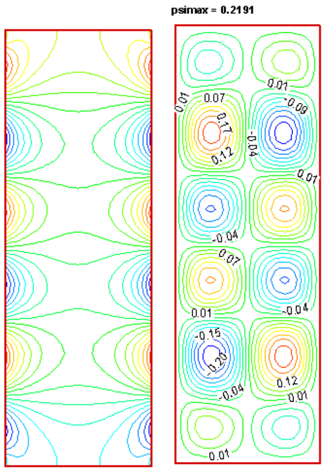


**Figure 9: Isotherms and Streamlines in the 3st case for Ra=$10^3$**

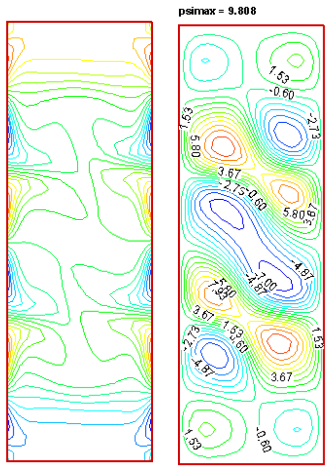


**Figure 10: Isotherms and Streamlines in the 3st case for Ra=$10^5$**

## 5.4 Local Nusselt number

Variation of the Nusselt local number along a wall subjected to as in usoidal profile of temperature is surely not similar to that of an isothermal wall. Figure11, which respectively illustrate the variations of Nusselt local numbers, along the sides AD and BC, in the first case of heating, show that the value of this Nusselt local number increases with the Rayleigh number, and we also notice that variations of these Nusselt local numbers are negatives and positives according to whether we heat or cold parts of the sidewall.

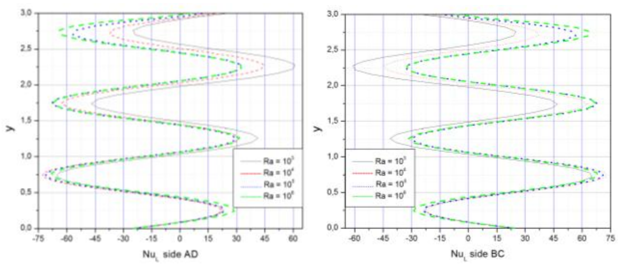


**Figure 11: Local Nusselt number in the 1st case**

## *6.0 Conclusions*

The suggested calculation code, which uses the finite volumes method, with the vorticity-stream function formulation, makes it possible, to find with a good agreement, the literature results, which solves problems similar, to that studied. Thus we theoretically studied, the bidimensionnal thermal natural convection, in laminar flow and permanent, in a rectangular enclosure. We examined, in particular, the influence, of the Rayleigh number and the parietal thermal conditions. The results underline that the thermal type of excitation imposed on the walls influences much the mode of flow within the enclosure considered and for low Rayleigh number values, the coefficient of heat transfer is dominated by the mechanism of the conduction.

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