

# STUDY OF TURBULENT NATURAL CONVECTION OF A NANOFLUID IN A SOLAR WATER HEATER ENCLOSURE

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## ABSTRACT

In this research we study the heat transfer by turbulent natural convection of a water-copper nanofluid in a solar water heater enclosure, the heating of enclosure is achieved by the solar collector wall at constant heat flux  $Q$ . The left vertical wall of the solar thermal storage tank is maintained at constant cold temperature  $T_c$ , the other parts of the enclosure are considered as adiabatic. The volume fractions of the nanoparticles are varied in the range of 0 to 0.20. The permanent forms of two-dimensional Reynolds-Averaged-Navier–Stokes equations and conservation equations of mass and energy are solved by the finite volume method with use of the SIMPLE algorithm for pressure-velocity coupling. The turbulence is modeled by the standard  $k$ - $\epsilon$  model. The Rayleigh number  $Ra$  is varied in the range of  $10^8$  to  $10^{12}$ . The streamlines and the isotherms and the variation of the average Nusselt number at the heated wall are shown for various combinations of the Rayleigh number  $Ra$  and different values of volume fraction of the nanoparticles.

**Keywords:** *Turbulent Flow, Solar Water Heater, Nanofluids.*

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## NOMENCLATURE

### Symbols :

$g$  acceleration of gravity,  $m\ s^{-2}$   
 $\bar{h}$  average heat transfer coefficient,  $Wm^{-2}K^{-1}$   
 $H$  the enclosure height,  $m$   
 $H_1$  solar collector output section height,  $m$   
 $L$  enclosure width,  $m$   
 $K$  thermal conductivity of the fluid,  $Wm^{-1}K^{-1}$   
 $p$  pressure,  $Pa$   
 $P$  dimensionless pressure  
 $u$  horizontal velocity,  $m\ s^{-1}$   
 $v$  vertical velocity,  $m\ s^{-1}$   
 $U$  dimensionless horizontal velocity  
 $V$  dimensionless vertical velocity  
 $Q$  heat flux,  $W\ m^{-2}$   
 $T_c$  temperature of cold surface,  $K$   
 $x, y$  coordinates,  $m$   
 $X, Y$  dimensionless coordinates

$\bar{Nu}$  average number of Nusselt,

$Ra$  Rayleigh number,  $g\ \beta\ H^4\ Q / K\ v\ \alpha$

$Pr$  Prandtl number,  $Pr = \nu / \alpha$

### Greek Letters:

$\alpha$  thermal diffusivity,  $m^2\ s^{-1}$

$\nu$  kinematic viscosity,  $m^2\ s^{-1}$

$\rho$  density,  $kg\ m^{-3}$

$\phi_v$  volume fraction of the nanoparticles

$\theta$  temperature dimensionless

$\Omega$  inclination angle of the solar collector

$\theta$  temperature dimensionless

$\beta$  thermal expansion coefficient at constant  
at constant pressure,  $K^{-1}$

### Indices / Exponents :

$f$  base fluid

$nf$  nanofluid

$s$  solid

## 1. INTRODUCTION

Today, with the increasing decline in the world's petroleum reserves and growing phenomenon of global warming resulting from the use of petroleum energy, It became necessary to find a replacement of this traditional energy. The solar energy is the best solution as it is a clean and a renewable source of energy. The research in this field led to the discovery of a lot of devices which can be used to convert the solar energy into thermal energy. The latter can be used in various everyday needs. Among such devices we find the solar water heater which is studied in the present work.

Due to the importance of this field we can find many published papers in the literature on numerical and experimental studies. Among these we can mention Renault and son [1] who studied experimentally the flow structure of turbulent natural convection in a rectangular enclosure. The results showed that the development of the boundary layer adjacent to the hot plate was similar to that observed in the case of a single plate. O. Aydin and W. Yang [2] investigated numerically the natural convection of air in a rectangular enclosure with localized heating from below and symmetrical cooling from sides, the Rayleigh number is varied from  $10^3$  to  $10^6$ , and the dimensionless heat source length is varied from 0.2 to 0.8, they concluded that for high Rayleigh number the heat transfer is dominated by convection and that the increase of the dimensionless heat source length improves the heat transfer, while for small Rayleigh numbers the heat transfer is dominated by conduction. Calcagni et al. [3] analysed numerically and experimentally the free convective heat transfer in a square enclosure constituted by a discrete heater located on the lower wall and cooled from sides, they found that for high Rayleigh numbers the increase of the heat source length produces a raise in heat transfer, and for Rayleigh numbers less than or equal to  $10^4$  the heat transfer is dominated by conduction, and by convection for Rayleigh number equal to  $10^5$ . Sharma et al. [4] carried out a numerical study of turbulent natural convection in a square enclosure with localized heating from below whose length varies from 0.2 to 0.8, and symmetrical cooling from the vertical side walls, the Rayleigh number was varied from  $10^8$  to  $10^{12}$ , they observed the formation of two counter clockwise rotating cells, and in the case of isothermal heating the Nusselt number increases with an increase in the heated width while the opposite is true for the isoflux heating. Kuznetsov and Sheremet [5] present a numerical study of turbulent natural convection in a rectangular enclosure having finite thickness walls with local heating at the bottom of the cavity, the Grashof number was varie from  $10^8$  to  $10^{10}$ , they obtained average Nusselt number correlations as a function of Grashof number.

There are many methods to improve the heat transfer by natural convection, among such methods we find the use of nanofluids which attracted the attention of many researchers. Many studies have been carried out on this subject, among them: Khanafer et al. [6] who investigated the heat transfer enhancement in a two-dimensional enclosure utilizing a water-copper nanofluid, for a range of Grashof numbers  $10^3$  to  $10^5$  and volume fractions 0 to 0.25, the results illustrate that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. Abu-Nada et al. [7] studied the heat transfer enhancement in horizontal annuli using different nanofluids, the results showed that for high Rayleigh numbers and high L/D ratios, nanoparticles with high thermal conductivity result in significant enhancement in heat transfer. Oztop and Abu-Nada [8] investigated the heat transfer by natural convection in a partially heated enclosure using nanofluids, the results showed that both increasing the value of Rayleigh number and heater size -while keeping the remaining problem parameters fixed -enhance the heat transfer and flow strength.

Although there is a large number of studies carried out on the heat transfer of nanofluids by natural convection within the enclosures, the majority of these studies have been completed for the laminar regime and only a very small number of these studies concentrated on the turbulent natural convection, for this reason we achieved this study in order to provide the maximum amount of information about the heat transfer by turbulent natural

convection in a solar water heater enclosure filled with a copper-water nanofluid with volume fraction varied between 0 and 0.20 and for a Rayleigh number varied between  $10^8$  and  $10^{12}$ .

## 2. MATHEMATICAL FORMULATION

### 2-1. Physical Model

Figure 1 shows the physical model of the problem, which is a solar water heater enclosure divided into two parts, the first part is called the solar collector and is heated at a constant heat flux  $Q$ , while the second section is a thermal storage tank in the form of a triangle with a cooled left vertical wall which is maintained at a constant temperature  $T_c$ , the remaining enclosure parts are taken as thermally insulated. In order to simplify the problem we consider that this enclosure is infinite in length in the Z direction. The heat is transferred within the solar water heater enclosure by turbulent natural convection by means of copper-water nanofluid with volume fraction of copper nanoparticles varied from 0 to 0.20. The Rayleigh number is varied in the range of  $10^8$  to  $10^{12}$ .

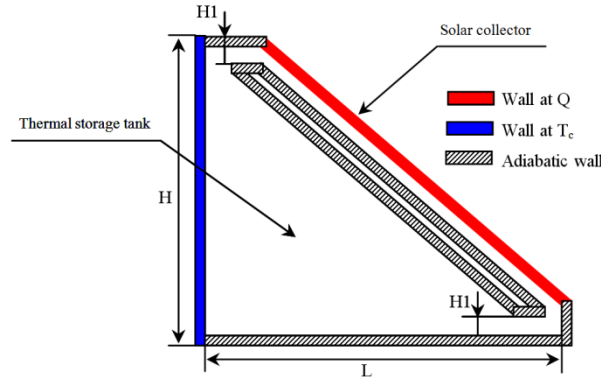


FIGURE 1. Schematic of physical model

### 2-2. Governing Equations

For a simple formulation of the problem, we considered that the fluid is Newtonian and incompressible, the flow is stationary and two-dimensional, and the density in the buoyancy term obeys the Boussinesq approximation while the physical properties of the fluid are constant. The mathematical model of the problem is summarized in the following equations:

$$\frac{\partial(\rho_{nf} u)}{\partial x} + \frac{\partial(\rho_{nf} v)}{\partial y} = 0 \quad (1)$$

$$\frac{\partial}{\partial x}(\rho_{nf} uu) + \frac{\partial}{\partial y}(\rho_{nf} vu) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( 2(\mu_{eff})_{nf} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left[ (\mu_{eff})_{nf} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] \quad (2)$$

$$\frac{\partial}{\partial x}(\rho_{nf} uv) + \frac{\partial}{\partial y}(\rho_{nf} vv) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left[ (\mu_{eff})_{nf} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[ 2(\mu_{eff})_{nf} \frac{\partial v}{\partial y} \right] + \rho_{nf} \beta_{nf} g (T - T_c) \quad (3)$$

$$\frac{\partial}{\partial x}(\rho_{nf} u C_{p,nf} T) + \frac{\partial}{\partial y}(\rho_{nf} v C_{p,nf} T) = \frac{\partial}{\partial x} \left( (K_{eff})_{nf} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( (K_{eff})_{nf} \frac{\partial T}{\partial y} \right) \quad (4)$$

$$\frac{\partial}{\partial x}(\rho_{nf} uk) + \frac{\partial}{\partial y}(\rho_{nf} vk) = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right)_{nf} \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left( \mu + \frac{\mu_t}{\sigma_k} \right)_{nf} \frac{\partial k}{\partial y} + P_k + G_k - \rho_{nf} \varepsilon \quad (5)$$

$$\frac{\partial}{\partial x}(\rho_{nf} u \varepsilon) + \frac{\partial}{\partial y}(\rho_{nf} v \varepsilon) = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right)_{nf} \frac{\partial \varepsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right)_{nf} \frac{\partial \varepsilon}{\partial y} + [C_{\varepsilon 1}(P_k + C_{\varepsilon 3} G_k) - C_{\varepsilon 2} \varepsilon] \frac{\varepsilon}{k} \quad (6)$$

Where

$$P_k = (\mu_t)_{nf} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]; G_k = -\frac{(\mu_t)_{nf}}{\sigma_T} g \beta_{nf} \frac{\partial T}{\partial y}; (\mu_{eff})_{nf} = (\mu + \mu_t)_{nf}; (\mu_t)_{nf} = C_\mu \frac{\rho_{nf} k^2}{\varepsilon}; (K_{eff})_{nf} = K_{nf} + \frac{\mu_t C_{p,nf}}{\sigma_T}$$

For k-ε turbulence model, the constants used are:  $C_\mu = 0.09$ ;  $C_{\epsilon 1} = 1.44$ ;  $C_{\epsilon 2} = 1.92$ ;  $\sigma_T = 1.0$ ;  $\sigma_k = 1.0$ ;  $\sigma_\epsilon = 1.3$

The density and specific heat of the nanofluid are calculated according to [9] from:

$$\rho_{nf} = (1 - \varphi_v)\rho_f + \varphi_v\rho_s ; \quad (\rho C_p)_{nf} = (1 - \varphi_v)(\rho C_p)_f + \varphi_v(\rho C_p)_s \tag{7}$$

The thermal expansion coefficient and the viscosity of the nanofluid is obtained by [9]

$$(\rho\beta)_{nf} = (1 - \varphi_v)(\rho\beta)_f + \varphi_v(\rho\beta)_s ; \quad (\mu_{eff})_{nf} = \frac{\mu_f}{(1 - \varphi_v)^{2.5}} \tag{8}$$

The effective thermal conductivity of the nanofluid is determined using the Maxwell model [9]

$$\frac{(k_{eff})_{nf}}{k_f} = \frac{k_s + 2k_f + 2\varphi_v(k_s - k_f)}{k_s + 2k_f - \varphi_v(k_s - k_f)} \tag{9}$$

The heat transfer is characterized by the average Nusselt number which is calculated using the formula:

$$\overline{Nu} = \frac{\bar{h}\sqrt{(H - H_1)^2 + (L - L_1)^2}}{K_f} \tag{10}$$

### 2-3. Boundary conditions

The different boundary conditions of the problem are expressed as follows

Hot wall	Cod wall	Adiabatic wall
$Q = -k \frac{\partial T}{\partial y}; k = u = v = 0; \epsilon = \infty$	$T = T_c; u = v = 0; \epsilon = \infty$	$\frac{\partial T}{\partial y} = 0; k = u = v = 0; \epsilon = \infty$

## 3. NUMERICAL PROCEDURE

The equations of mathematical model are solved numerically using the finite volume method [10]. All numerical simulations of this study are performed using the commercial software Fluent. The SIMPLE algorithm is used for pressure-velocity coupling. The discretization of the convective terms in the conservation equations is made with the "QUICK" scheme. The convergence of all equations is reached when the sum of normalized residuals at each node of the computational domain -and for each algebraic equation obtained after discretization- becomes less than  $10^{-2}$ .

## 4. GRID INDEPENDENCE STUDY AND VALIDATION

The influence of the number of nodes on the accuracy of the results has been studied for the case of  $Ra = 10^{12}$  and  $\varphi_v = 0$ . From the grid which has 15028 nodes the average Nusselt number becomes constant, see Figure 2. Therefore, the simulation using an unstructured mesh of 15028 nodes with triangular cell types (see Figure 3) was used in all calculations.

In order to verify the accuracy of the numerical results obtained in this work, we compared them with those obtained by Aminossadati and Ghasemi [11]. The comparison curves for the average Nusselt number are shown in Figure 4. They show an excellent agreement, which confirms the accuracy of the present simulation.

## 5. RESULTS

In this study, we investigated the effects of using a copper-water nanofluid with volume fraction varied from 0 to 0.20 on fluid flow and heat transfer by turbulent natural convection inside a solar water heater enclosure in the range of the Rayleigh number from  $10^8$  to  $10^{12}$ .

### 5-1. Thermal fields

This field is shown by the temperature contours in Figure 5 for a Rayleigh number which varies in the range of  $10^8$  to  $10^{12}$ , and volume fraction  $\varphi_v$  varied from 0 to 0.20.

The recovered heat through the hot wall is transported by turbulent convection to the upper portion of the solar thermal storage tank due to molecules of fluid being in the solar collector. The heat is then discharged through

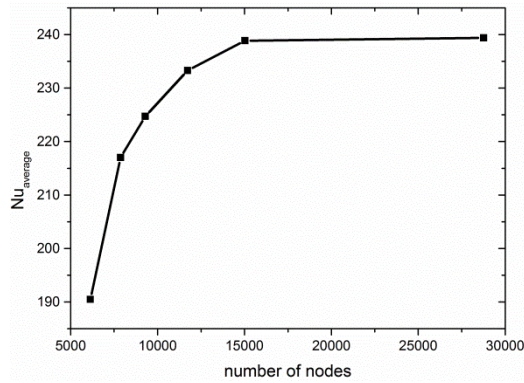


FIGURE 2. Average Nusselt number along the heated wall for  $\phi_v = 0$  and  $Ra = 10^{12}$

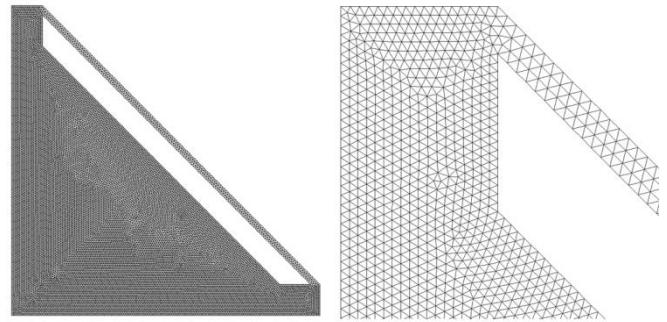


FIGURE 3. The final mesh (a) and detail (b)

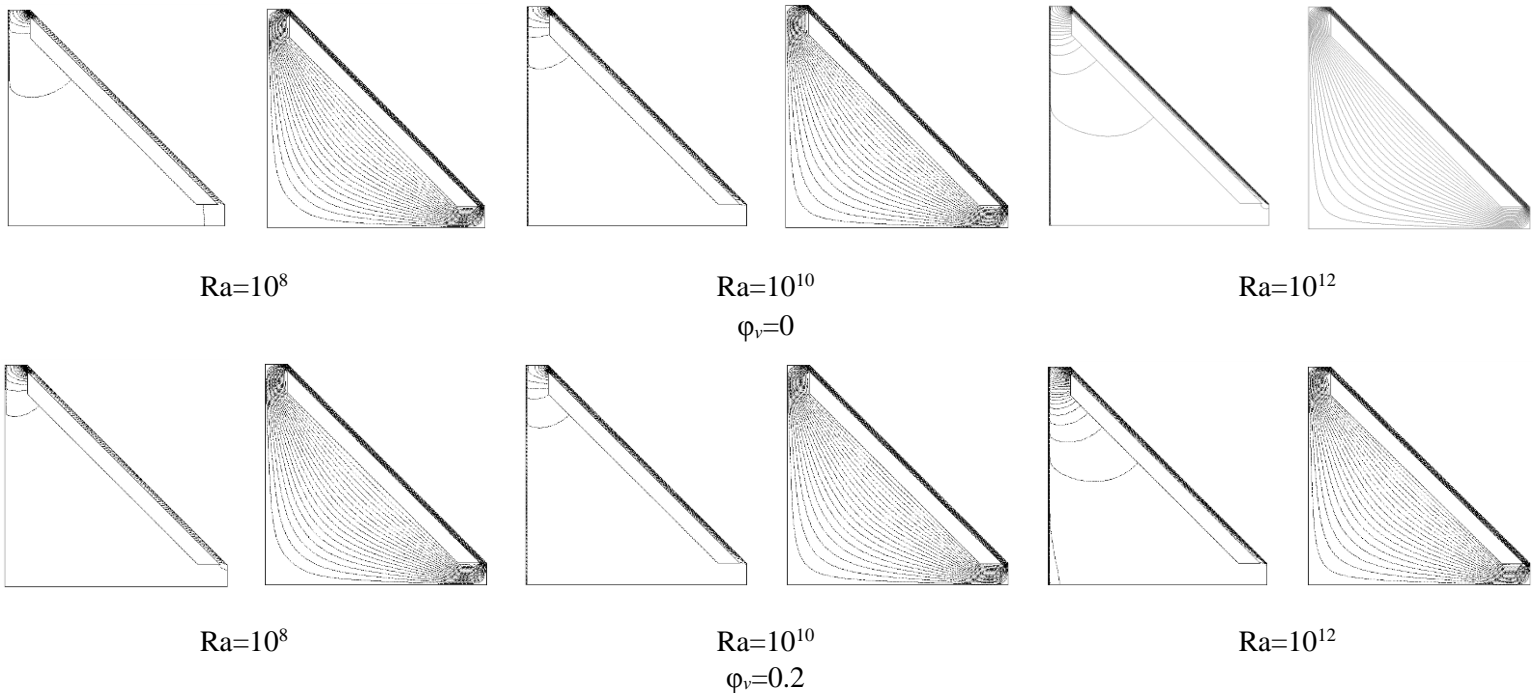


FIGURE 5. Isotherms (left) and streamlines (right)

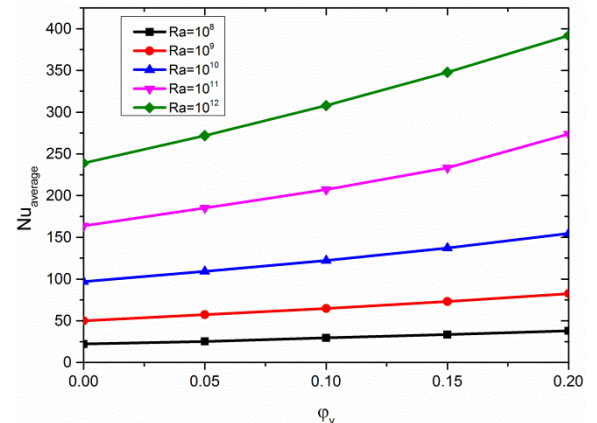
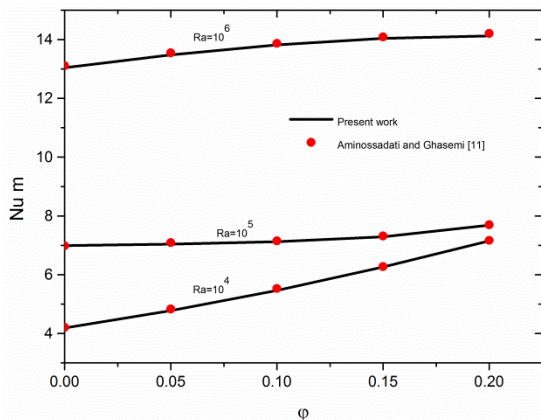


FIGURE 4. Comparison of average Nusselt number with the study of Aminossadati [11]

the cold wall of the tank, the cold fluid is transported to the bottom of the solar collector and the heating cycle is repeated again. It is noted that for each value of Ra, the isotherms are almost identical for  $\phi_v$  varied from 0 to 0.20, and we noted that with the increasing Rayleigh number for a given value of  $\phi_v$  the thermal stratification increases at the solar collector outlet. There is also an increase of fluid temperature at this position.

#### 5-2. Dynamic fields

The numerical results show that for all values of Ra or  $\phi_v$ , the streamlines remain almost identical.

#### 5-3. Heat transfer

The evolution of the average Nusselt number in function of the Rayleigh number for different values of  $\phi_v$  is shown in Figure 6, from which we can observe that the average Nusselt number increases with increasing  $\phi_v$  and Ra.

FIGURE 6. Average Nusselt number

## 6. CONCLUSIONS

In this numerical study we modeled the heat transfer by turbulent natural convection in a solar thermal water heater enclosure to study the improvement of heat transfer with the use of a water-copper nanofluid instead of a pure fluid (water). The results obtained clearly show that the use of nanofluids can influence greatly on the heat transfer in this geometry at the turbulent regime. The use of a water-copper nanofluid as coolant with volume fraction equal 0.20 increases the heat transfer by 64.02% compared to the use of pure water. The results also show that for a water-copper nanofluid  $\phi_v = 0.20$  is the value of the volume fraction which allows the maximum heat through the hot wall in this solar thermal water heater enclosure.

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