Existence of natural and anti-natural solution in thermosolutal convection in a tilted porous square cavity under cross temperature and concentration gradients

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Abstract— The present study is focused on double-diffusive convection in a tilted square porous cavity under cross temperature and concentration gradients. The flow in the porous media is modeled using the Darcy law and the Boussinesq approximation. The situation, where the horizontal components of the thermal and solutal buoyancy forces are equal and opposing each other, is considered. The study is performed for a tilt angle of 45° and in this case the buoyancy ratio is equal to unity (*N*=1). Results are presented in terms of the Nusselt, Sherwood numbers and the flow intensity as functions of the thermal Rayleigh number and the Lewis number. In this study, the existence of the onset of convection is demonstrated and both natural and anti-natural flows solutions are obtained. Also, when the Lewis number is bigger than unity, subcritical flows are found to exist for the anti-natural convective solutions.

Keywords — Square tilted cavity, Multiple solutions, Thermosolutal convection, Cross gradients of temperature and concentration, Numerical study.

Résumé— La présente étude se concentre sur la convection naturelle double diffusive dans une cavité carrée poreuse inclinée soumise à des gradients opposés de température et de concentration. L'écoulement est modélisé par la loi de Darcy et l'approximation de Boussinesq. La situation, où les composantes horizontales des forces de volume thermiques et solutales égales et opposées, est prise en considération. Dans le régime diffusif, une solution état de repos est possible, mais devient instable au-delà d'un seuil critique. L'étude est réalisée pour un angle d'inclinaison de 45° et dans ce cas le rapport des forces de volume est égal à l'unité (*N* = 1). Les résultats sont présentés en termes de nombres adimensionnels de Nusselt, et de Sherwood ainsi que l'intensité du flux en fonction du nombre de Rayleigh thermique et du nombre de Lewis. Dans cette étude, l'existence de la convection est démontrée et que les deux solutions naturelles et antinaturelles des écoulements sont obtenues. En outre, lorsque le nombre de Lewis est plus grand que l'unité, les écoulements de type souscritiques existent pour les solutions antinaturelles.

Mots clefs — Cavité carrée inclinée, Solutions multiples, Convections thermosolutales, Gradients opposés de température et de concentration, Etude numérique

1. Introduction

Natural thermosolutal convection in porous media is of a growing interest owing to its presence in many and diverse engineering problems such as ground dispersion of chemical or radio-active contaminants, the exploitation of continental geothermal reservoir, the migration of moisture through fibrous insulation, grains formation in metallurgy, electrochemistry batteries, geophysics, etc. A comprehensive review on the phenomena related to heat and mass transfer and convection in porous media could be found in the book by Nield and Bejan [1].

Concerning the present study, Mamou *et al*. [2] examined the flow in a square cavity subjected to horizontal fluxes of heat and mass. In case where the volume forces are in opposite direction and same order of magnitude, the existence of multiple solutions was demonstrated. The existence of multiple solutions depended on the thermal Rayleigh and Lewis numbers. Trevisan and Bejan [3] used a numerical method and scale analysis to study double diffusion convection in a porous square cavity, with vertical walls maintained at constant temperatures and concentrations. It was found that the fluid flow was possible beyond a certain number of the critical Rayleigh when Le≠1. However, the fluid motion disappeared completely for the Le = 1 and N= -1. The results of this analysis were found in agreement with numerical study.

Mansour *et al*. [4] studied numerically the Soret effect on multiple solutions in a square cavity. The authors concluded that the Soret parameter might have a strong effect on the convective flow. One, two or three solutions were possible. Khanafer *et al*. [5] presented a numerical study of mixed-convection heat and mass transport in a square enclosure filled with a non-Darcian fluid-saturated porous medium. The two vertical walls of the enclosure were insulated, while the horizontal walls were kept at constant but different temperatures and concentrations with the top surface moving at a constant speed. The results of this investigation indicated that the buoyancy ratio, Darcy, Lewis and Richardson numbers have profound effects on the double-diffusive phenomenon. Mohamad and Bennacer [6] obtained numerical results, on the basis of two- and three-dimensional flows, on heat and mass transfer in a horizontal enclosure with an aspect ratio of two filled with a saturated porous medium. The enclosure was heated differentially and a stably stratified species concentration was imposed vertically. It was found that the difference in the rates of heat and mass transfer predicted by the two models was not significant. Bennacer *et al.* [7] studied both numerically and analytically the natural convection with Soret effect in a binary fluid saturating a shallow horizontal porous layer. The vertical walls of the enclosure are heated and cooled by uniform heat fluxes and a solutal gradient was imposed vertically The authors used the Darcy model. It was concluded that in the presence of a vertical destabilizing concentration gradient, the existence of both natural and anti-natural flows was demonstrated. When the vertical concentration gradient was stabilizing, multiple solutions are possible, which depended on the Soret effect.

Mansour *et al.* [8] studied numerically the Soret effect on fluid flow and heat and mass transfer induced by double diffusive natural convection in a square porous cavity submitted to cross gradients of temperature and concentration. They concluded that the Soret effect might affect considerably the heat and mass transfer as it led to an enhancement or to a reduction of the mass transfer, depending on the flow structure and the sign and magnitude of the Soret coefficient. Bourich *et al.* [9] studied analytically and numerically the Soret effect on thermal natural convection within a horizontal porous enclosure uniformly heated from below by a

Nomenclature

A	cavity aspect ratio, L'/H'	**Greek symbols**	
D	mass diffusivity of species	α	thermal diffusivity, $k/(\rho C)_f$
H'	height of the layer	β_S	concentration expansion coefficient
j'	constant mass flux per unit area	β_T	thermal expansion coefficient
K	permeability of the porous medium	Θ	angle of inclination of the cavity relative to the horizontal plane
k	thermal conductivity of the fluid saturated porous Medium	ε	normalized porosity of the porous medium,

L'	thickness of the enclosure		ϕ/σ
Le	Lewis number, α/D	ν	kinematic viscosity of the fluid
N	buoyancy ratio, $\beta_S \Delta S'/\beta_T \Delta T'$	μ	dynamic viscosity of fluid
Nu	Nusselt number	ρ	density of the fluid
q'	constant heat flux per unit area	$(\rho C)_f$	heat capacity of fluid
R_T	thermal Darcy Rayleigh number, $g\beta_T K H' \Delta T'/\alpha\nu$	$(\rho C)_p$	heat capacity of saturated porous medium
		σ	heat capacity ratio $(\rho C)_p/(\rho C)_f$
S	dimensionless concentration, $(S'-S'_0)/\Delta S'$	ϕ	porosity of the porous medium
Sh	Sherwood number	Ψ	dimensionless stream function, Ψ'/α
S'_0	reference concentration at x'=0, y'=0	Ψ_C	stream function value at center of the enclosure
$\Delta S'$	characteristic concentration, $j'H'/D$		
ΔS	dimensionless wall-to-wall concentration difference	**Superscript**	
		$'$	dimensional variable
T	dimensionless temperature, $(T'-T'_0)/\Delta T'$	sub	subcritical
t	dimensionless time, $t'\alpha/\sigma H'^2$	sup	supercritical
	reference temperature at $x'=0$, $y'=0$		
$\Delta T'$	characteristic temperature, $q'H'/k$	**Subscripts**	
ΔT	dimensionless wall-to-wall temperature difference	C	critical value
u	dimensionless velocity in x-direction, $u'H'/\alpha$	M	average value
v	dimensionless velocity in y-direction, $v'H'/\alpha$	max	maximum value
x	dimensionless coordinate axis, x'/H'	min	minimum value
y	dimensionless coordinate axis, y'/H'	$\circ$	reference state

constant heat flux using the Brinkman extended Darcy model. It was found that the Soret separation parameter had a strong effect on the thresholds of instabilities and on the heat and mass transfer characteristics.

Saied [10] studied the problem of natural convection in a two-dimensional square porous cavity with the temperature of the hot (left) wall oscillating in time. He finds that during the heat transfer process the hot wall temperature dropped which resulted, at some locations inside the cavity, with a temperature higher than the hot wall temperature. Also, it was observed that the average Nusselt number had a peak value at the non-dimensional frequency of 450 in the range considered (1–2000) for Rayleigh number 103, as the convection currents are stronger than those at other frequencies.

The transient free convection in a two-dimensional square cavity filled with a porous medium was considered by Saeid and Pop [11]. The flow was driven by considering the case when one of the cavity vertical walls is suddenly heated and the other vertical one was suddenly cooled, while the horizontal walls were adiabatic. The results were obtained for the initial transient state up to the steady state, and for Rayleigh number values of 10^2–10^4. It was observed that the average Nusselt number showed an undershoot during the transient period and that the time required to reach the steady state is longer for low Rayleigh number and shorter for high Rayleigh number.

Finally, Mansour, *et al.* [12] studied the transient MHD natural convection in an inclined cavity filled with a fluid saturated porous medium by including the effects of both of an inclined magnetic field and heat source in the solid phase. The flow was driven by considering the case where one of the cavity vertical walls was suddenly heated and the other one was suddenly cooled, while the horizontal walls were adiabatic.

The authors found that in general, they could increase the temperature of the fluid by increasing both of the Magnetic field force and the inclination angle.

In the present study, a numerical investigation was performed to examine the effect of cross fluxed of heat and solute on the heat and mass transfer rates within a tilted square porous cavity.

The case, where the horizontal components of the thermal and solutal buoyancy forces are equal and opposing each other, was considered. The Darcy model was used to simulate the convection flow inside the cavity. The existence of natural and anti-natural flows was demonstrated and the threshold of convective instability was obtained as function of the Lewis number.

2. Mathematical Formulation

The configuration considered in this work is an inclined porous square cavity. The origin of the coordinate system is located at the center of the cavity. As shown in Fig. 1, the solutal and thermal gradients were induced by imposing constant fluxes of heat, *q*', and solute, *j*', on the cavity walls. The fluid saturating the porous matrix was assumed to be Newtonian fluid and obeying the Boussinesq approximation, which state that the fluid is incompressible with constant properties except for the density which is supposed to vary linearly with the temperature, and the viscous dissipation and the pressure work are negligible.

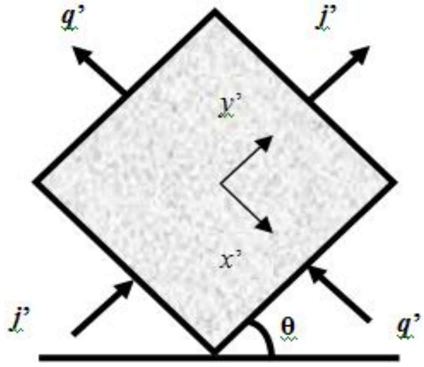


Figure 1: Flow configuration and coordinates system.

The governing equations that describe the double-diffusive convection are expressed in terms of the stream-function, temperature and concentration, in dimensionless form:

$$\nabla^2\Psi = -R_T F(T + NS) \tag{1}$$

$$\nabla^2 T = \frac{\partial T}{\partial t} + \frac{\partial(uT)}{\partial x} + \frac{\partial(vT)}{\partial y} \tag{2}$$

$$\nabla^2 S = Le\left(\varepsilon\frac{\partial S}{\partial t} + \frac{\partial(uS)}{\partial x} + \frac{\partial(vS)}{\partial y}\right) \tag{3}$$

Where *F* is an operator defined by:

$$F(f) = \frac{\partial(f)}{\partial x}\sin\theta + \frac{\partial(f)}{\partial x}\cos\theta$$

where Ψ is the dimensionless stream function, *T* and *S* the dimensionless temperature and concentration, *u* and *v* the dimensionless velocity components, *t* the dimensionless time, *x* and *y* are the dimensionless coordinates axes, R_T is the thermal Rayleigh number, *N* the buoyancy ratio, θ the inclination, *Le* the Lewis number and ε is the normalized porosity of the porous medium.

In the Darcy model, the inertial forces are negligible and the acceleration parameter is supposed to very weak (Nield and Bejan [1]), so they are omitted in the present study.

The stream function Ψ is defined as:

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x} \tag{4}$$

The dimensionless boundary conditions are given by:

$$y = \pm\frac{1}{2} : \Psi = 0, \quad \frac{\partial T}{\partial y} = 0 \ and \ \frac{\partial S}{\partial y} = -1 \tag{5}$$

$$x = \pm\frac{1}{2} : \Psi = 0, \quad \frac{\partial T}{\partial x} = 1 \text{ and } \frac{\partial S}{\partial x} = 0 \tag{6}$$

3. Numerical Solution

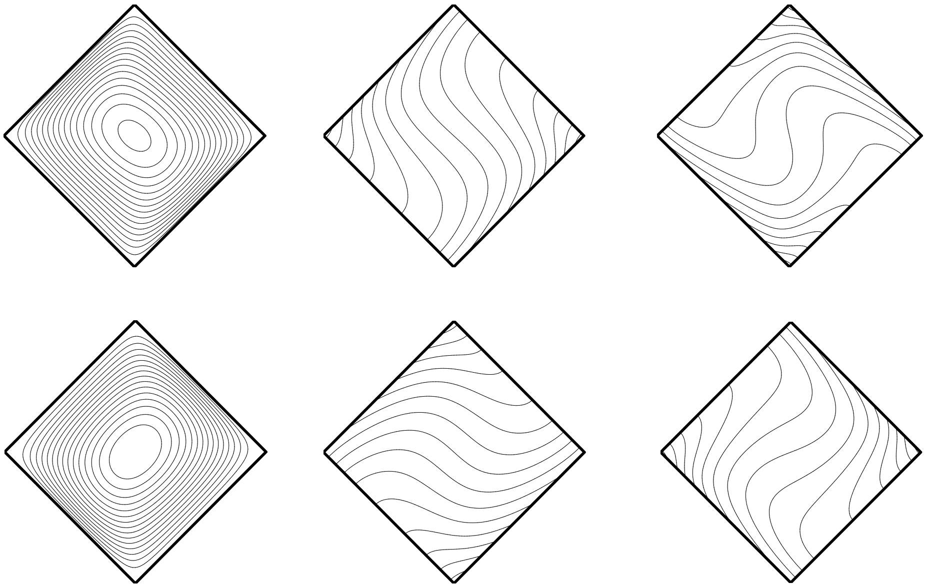
The numerical solution of the fully governing equations (1)-(3) with boundary conditions (5)-(6) is obtained using a finite-difference scheme. The entire domain, as shown in Fig. 1, was discretized with a uniform mesh size (101 × 101). The solution consists of the stream function, temperature and concentration fields. Central-difference scheme with second-order accuracy is used to transform the governing equations into a set of finite difference equations. At the boundaries, forward or backward difference also with second-order accuracy is considered. The energy and concentration equations, after be written in conservative form, are solved using the alternating direction implicit (ADI) method, while the stream function field is obtained from the discretized momentum equation using the successive over-relaxation method (SOR). Finally, the integrals in the expressions of the Nusselt and Sherwood numbers were computed numerically by using the Simpson scheme.

4. Results and Discussion

The present investigation is limited to the equilibrium state where the horizontal components of the thermal and solutal buoyancy forces are equal and opposing each other. For any inclination angle, θ, of the cavity, the equilibrium state can be reached only when N=tang θ. In this study, the inclination angle is fixed to $\theta = 45°$, thus the buoyancy ratio at equilibrium is N=1. At this condition, a rest state solution is possible, where the fluid density gradient is vertical (opposing the gravity). For this situation, convective flows are possible only when the Rayleigh number and there exists a threshold for the onset of convective flows. The effect of the Rayleigh and the Lewis numbers on the flow behavior and the heat and mass transfer rates are considered and the thresholds for convective flow instabilities are determined.

Figures 2 show the stream function, the temperature and concentration contours obtained for both natural (clockwise circulation) and anti-natural (counterclockwise circulation) with the same values RT=100 and Le=2. As displayed in the figures, the two solutions are asymmetric, however, they become anti–symmetric when Le=1.

The natural convection is usually the preferable solution when initiating the numerical code with the rest state solution. The anti-natural solution is obtained by imposing a likewise solution as initial condition. As the mass diffusivity is higher than the thermal one (Le=2), the mass transfer rate is bigger than the heat transfer rate. It can be noticed that the flow intensity of the natural-solution is stronger than that of the anti-natural solution.



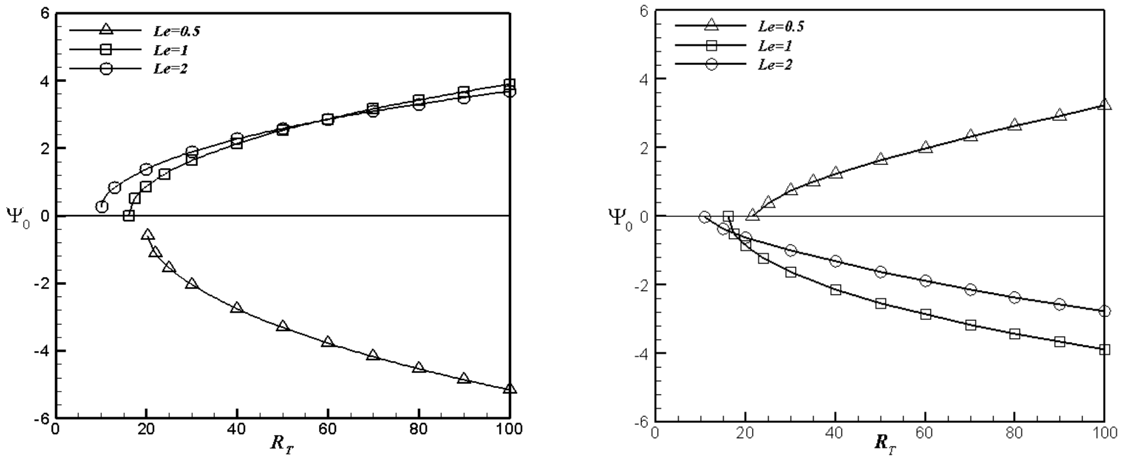
a)

b)

Ψ *T* *S*

Figure 2: Stream function, temperature and concentration contours obtained for R_T=100 and Le=2: a) Ψ_{min}=0.00, Ψ_{max}= 3.68, Nu_m= 3.21 *and* Sh_m=4.16. *b)* Ψ_{min}=-2.79, Ψ_{max}= 0.00, Nu_m= 2.19 *and* Sh_m=4.19.

The effect of the Rayleigh number on the flow intensity, $\boldsymbol{\Psi_0}$**,** and the heat and mass transfer rates, *Nu* and *Sh***,** are presented in figures 3 for various values of *Le*. As expected, there exists a threshold for the onset of convection, which depends strongly on the Lewis number**.** Three values of the Lewis number are considered, namely *Le*=0.5, 1 and 2.



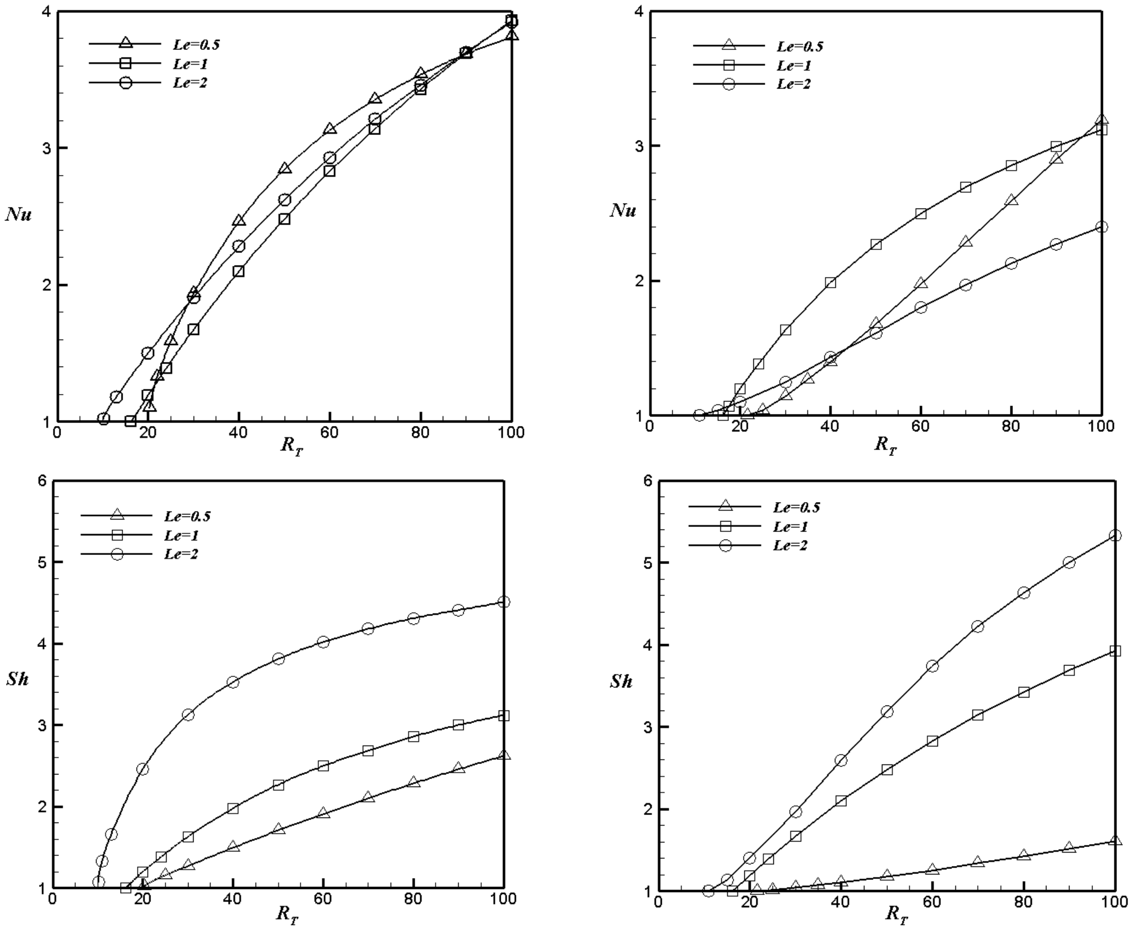


Figure 3: Natural (left) and anti-natural (right) solution bifurcation diagrams: flow intensity, Ψ_0, heat and mass transfer rates, Nu and Sh, as functions of the Rayleigh number, R_T, for various Lewis number values, Le.

The corresponding thresholds of the convection instability is obtained as R_{TC}=21.57, 16.21 and 10.78 for *Le*=0.5, 1 and 2, respectively. The computational technique to obtain the thresholds, where exchange of stability occurs, is discussed hereafter. **The natural convective solution is discussed first.** Starting from the threshold (critical Rayleigh number), as displayed in Fig. 4-6, the heat and mass transfer rates and the flow intensity increase monotonically with R_T. It is observed that, when R_T is relatively small, the flow intensity increases with the Lewis number, however it looks decreasing for higher R_T, as the curves are crossing each other (see, for instance, the curves obtained for Le=1 and 2). Same trend is observed for the Nusselt number. However, the Sherwood number is seen to increases monotonically with the Lewis number.

Figures 3. exemplifies similar results for the anti-natural solution at the same values of the governing parameters. For this situation the flow circulation is counterclockwise (Ψ_0<0). As for the natural solution, the flow intensity, $|\Psi_0|$, is seen to increase monotonically with R_T. However, above criticality, the flow intensity is seen to increase with decreasing of the Lewis number. Same trend is observed with the Nusselt number. For the Sherwood number, a monotonic increase with both R_T and *Le* is observed as displayed.

This technique is novel as it uses the full governing equations rather the linear stability equations. The technique straight forward and it is valid only for the determination of the supercritical Rayleigh number, where exchange of stationary instability occurs. Another classical technique but tedious is used in parallel to determine the threshold of subcritical convection when it does exist. To calculate the critical value of the Rayleigh we consider two values of *Le* (1 and 2).

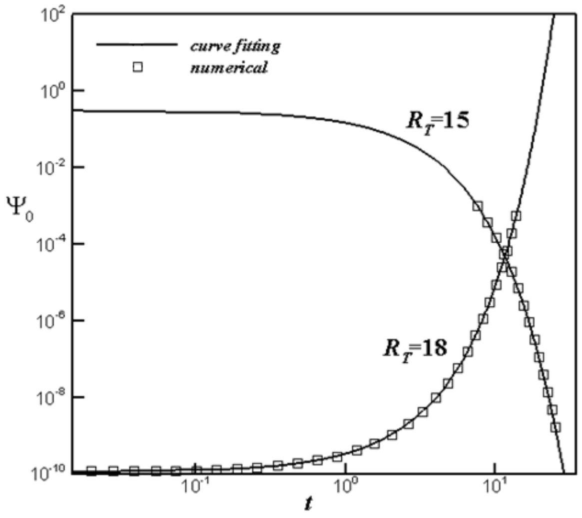


Figure 4: Flow intensity time histories below and above the threshold of supercritical convection for Le=1.

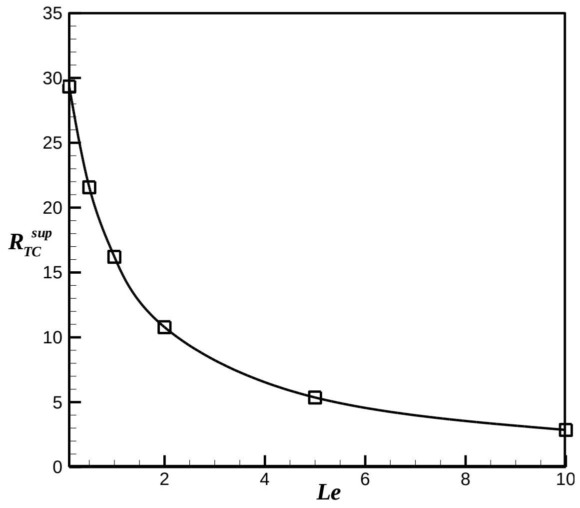


Figure 5: Supercritical Rayleigh number as function of the Lewis number.

As known, for infinitesimal amplitude convection, the time evolution of the flow intensity is exponential, according to the linear stability analysis, and could be correlated by:

$$\Psi_{max}(t) = \phi_0 e^{pt}$$

Where Ψ_{max} is the convective flow amplitude and ϕ_0. From the stability analysis point of view, ϕ_0 must be very small. In the present analysis, we considered $\phi_0 < 10^{-2}$. To avoid any numerical contamination of the signal amplitude when it becomes very small, we consider only the data of ϕ_0 within a reasonable range of $10^{-8} < \phi_0 < 10^{-2}$. The parameter *p* represents the amplitude growth rate. When *p*<0 the flow is decaying and when *p*>0 the flow is amplified. Then, *p*<0 below the threshold of convection and *p*>0 above the threshold. By performing two quick simulations for two Rayleigh number numbers; one below and the other one above the threshold, the growth rate parameter can be computed numerically using a simple interpolation technique. It is well known that, usually, the growth rate parameter varies linearly with R_T near the point of exchange of instability, thus a linear interpolation is far enough to obtain an accurate value. Using the two computed values for the growth rate, he threshold of convection can be determined by a linear interpolation for *p*=0. To be more practical let`s give two examples.

From the previous numerical simulations, Fig. 3, the thresholds of instabilities can be localized easily. For example, for *Le*=1, we can estimate that the critical value for the onset of convection is between 15 and 18. Thus two flow simulations are required to determine the growth rate of the perturbation. Using the numerical code which solves the full governing equations, the parameter p is computed as follows. Starting from the rest state solution, a numerical simulation is performed for R_T=18. For this situation, the flow is amplifying until it reaches a convective steady state solution.

The flow intensity as a function of time is depicted in Fig. 4. For stability analysis purposes, we consider $10^{-8} < \phi_0 < 10^{-2}$. By performing a curve fitting exercise, an excellent exponential curve fit is obtained and the growth rate parameter is obtained as *p*=1.117. Now, for R_T=15, using the converged solution as initial conditions, the flow simulation is performed again and as can be seen from Fig. 4 also, the flow decays towards the pure conductive state. Focusing only on the infinitesimal curve branch, $10^{-8} < \Psi_{max} < 10^{-2}$, exponential curve fitting lead to a growth rate of p=-0.756. Considering the two couples of values (R_T=15, p=-0.756) and (R_T=18, p=1.117), the threshold of marginal stability is obtained by a linear interpolation as R_{TC}^{sup}=16.21.

A same exercise can be repeated for various Lewis number values as depicted in Table 1. A complete curve showing the effect of the *Le* on R_{TC} is exemplified in Fig. 5. It can be noticed that R_{TC}^{sup} is decreasing monotonically with *Le* increase. Using a quick data analysis and curve fitting, it is found that R_{TC} is a function of the *Le* number according to the following relationship:

$$R_{TC}^{sup} = \frac{32.37}{Le+1}$$

TABLE 1: Critical values of R_{TC} and type of bifurcations.

Le	R_{TC}^{sup}	R_{TC}^{sub}	***Bifurcation***
2	10.78	10.11	subcritical
1	16.21	/	supercritical
0.5	21.57	/	supercritical

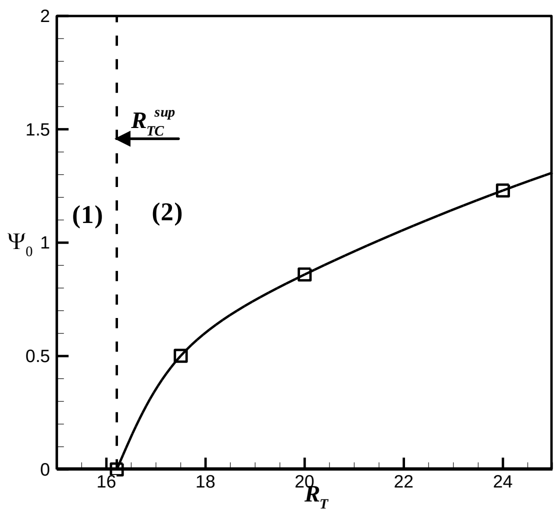


Figure 6: Bifurcation diagrams in terms of Ψ_0 Versus R_T for the natural solution with Le=1.

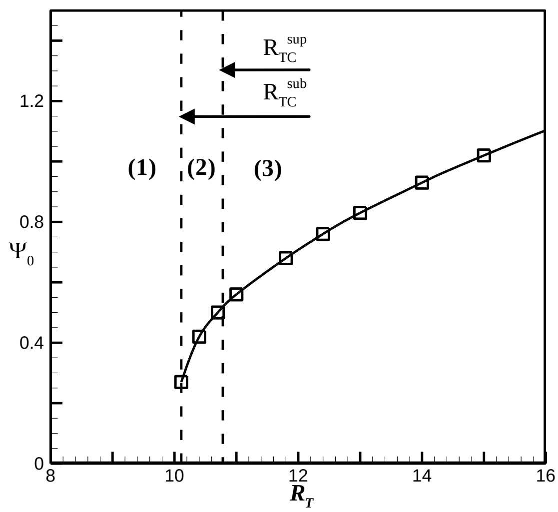


Figure 7: Bifurcation diagrams in terms of Ψ_0 Versus R_T for the natural solution with Le=2.

5. Conclusion

In the presentation investigation, thermosolutal convection in a tilted porous square cavity subjected to cross-flow heat and mass is studied numerically. The cavity was inclined at an angle of 45°. The conditions where the horizontal components of the thermal and solutal buoyancies are equal (N=1) and opposing each other is considered. For this situation, the existence of two stable convective solutions for the same governing parameters values is demonstrated. For *Le*=1, the two solutions (natural and anti-natural), are anti-symmetric, however they become asymmetric when Le≠1. The flow intensity increases monotonically with R_T. Both the two solution displayed the similar trend with the Rayleigh number. The existence of exchange stability is demonstrated and the thresholds for onset of supercritical and subcritical convection are obtained.

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