The combined effects of couple stresses and the pressure dependent viscosity of lubricant on the non-linear dynamic response of an unbalanced flexible rotor supported by coated journal bearings

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Abstract: On the basis of the V. K. Stokes micro-continuum theory and the Barus' law, the combined effects of couple stresses due to the presence of polymer additives in lubricant and the pressure dependent viscosity on the nonlinear dynamic response of the unbalanced Jeffcott's flexible rotor supported by coated hydrodynamic journal bearings are investigated in this work. A nonlinear transient modified Reynolds' equation is derived and discretized by the finite element method to obtain the fluid-film pressure field as well as the film thickness by means of the implicit Euler method. The nonlinear orbits of the rotor center are determined by solving the nonlinear rotor-dynamic equations with the explicit Euler's scheme taking into account the flexibility of rotor. According to the obtained results, the combined effects of couple stresses and the piezo-viscosity of lubricant on the nonlinear dynamic response of the rotor-bearing system are significant and cannot be overlooked. As expected, these effects are more noticeable for oils blended with polymers characterized by higher length molecular chains.

Keywords: couple-stress piezo-viscous fluid, flexible rotor, elastic bearing, finite element method.

1. Introduction

Rotating machines such as turbo-machines are the most important class of machinery that are extensively used in diverse engineering applications such as power stations, aircraft propulsion systems, etc. These machines include at least one rotor generally supported by oil-lubricated journal bearings which must not be considered to be passive elements but elements which sensitively affect the dynamic behavior and stability of the rotor. The design trend of such mechanical systems in modern engineering is towards lower weight and operating at super critical speeds.

The role of rotor is to transmit or to transform the mechanical power. It is often of very much complex realization and includes bladed disks or impellers, gears, for example.

The correct prediction of dynamic behavior is extremely essential when the rotor rotating at high speeds is made very flexible. Further, the dynamic behavior of a rotor-bearing system largely depends on the nonlinear dynamic characteristics of oil-lubricated bearings which may be a source of self-induced vibrations popularly known as oil whirl and oil whip phenomena which occur especially when the rotor-bearing system is lightly loaded or operates at low values of eccentricity ratio. This vibratory motion can cause considerable mechanical problems, like rubbing between shaft and bearing, blades and stator in turbo-machines, or more generally vibrations of the whole rotating machinery. In 1924, Newkirk and Taylor [1] first demonstrated that the oil whirl is caused by the dynamic oil film forces in the bearings. Since, a number of researchers have focused their theoretical and experimental studies on the stability, bifurcation, and chaos of rotor-bearing systems, and a significant progress in rotor-dynamics field has been made [2-21].

Of the many published works, the most extensive portion of the literature on rotor dynamics is concerned with determining critical speeds, natural whirl frequencies, instability thresholds, and imbalance response. In these works generally based on the linearized theory, the authors used in their analysis several assumptions among them the rotor and bearings are assumed to be rigid. Moreover, the linear stability analysis predicts the stability limits only under small disturbances, it does not give any information about the transient phenomena of a rotor-bearing system under large disturbances. Those phenomena should be studied by means of a nonlinear stability theory.

On the other hand, the rheological behavior of mineral oils used as lubricants in industrial machinery is influenced by the presence of additives such as Viscosity Index (VI) improver polymers which are characterized by long-chains. Thus, Oils containing VI additives such as multi-grade engine oils must be considered as non-Newtonian fluids and can be modeled as polar fluids. Experimentally, it was found that the presence of dissolved polymer in the lubricant increases the load carrying capacity of the lubricating film and decreases the friction coefficient [22-23].

In order to better describe the rheological behaviour of this kind of non-Newtonian lubricant so-called couple stress fluid or polar fluid, different micro-continuum theories have been developed [24-25]. The *Stokes* micro-continuum theory [26] is the simplest theory of fluids proposed in the technical literature, which allows the polar effects such as the presence of couple-stresses and body couples in addition to the body forces and surface forces.

Rheologically, such fluids are characterized by two constants, namely μ and η whereas only one parameter appears for a Newtonian iso-volume fluid which is the dynamic viscosity μ. The new material constant η is responsible for couple-stress property. According to the papers published in the technical literature, the effects of couple-stresses on the behaviour of journal bearings are studied by defining the dimensionless couple-stress parameter $\ell^* = \dfrac{\ell}{C}$ where $\ell = \sqrt{\dfrac{\eta}{\mu}}$ which has the dimension of length and can be thought of as a fluid property depending on the size of the high polymer molecule.

Owing to its relative mathematical simplicity, the couple-stress fluid model has been widely applied to analyse various hydrodynamic lubrication problems [27-33]. The theoretical results obtained showed that the presence of the couple-stress provides an enhancement in the load carrying capacity, improve the dynamic performance characteristics and the stability of journal bearings, and lengthens the response time of the squeeze film action of the system as compared to the Newtonian lubricant case. It was also shown that the effects of couple stress are more pronounced for high values of the couple-stress parameter ℓ.

As far as we know, very few research works have been devoted to analyze the nonlinear dynamic behavior of flexible rotors mounted in coated journal bearings lubricated with complex non-Newtonian fluids taking into account the combined effects of fluid couple-stresses, pressure-viscosity dependency, and compliance of the bearing structure.

The main objective of the present research is to theoretically investigate the nonlinear dynamic response of a flexible rotor supported by coated bearings using piezo-viscous couple-stress fluids as lubricants. The elastic deformation of the liner due to the fluid film pressure is calculated using a simplified elasticity model known as the "thin liner model". So, we assume that the radial deformation at the fluid-liner interface is proportional to the hydrodynamic pressure. On the other hand, the Barus law is used in order to take into account the viscosity-pressure dependency at constant temperature:

$$\mu(p) = \mu_0 e^{\alpha p} \tag{1}$$

where μ_0 is the atmospheric dynamic viscosity, and α is the pressure-viscosity coefficient which can be obtained by plotting the natural logarithm of dynamic viscosity μ versus pressure p. The slope of the graph corresponds to the value of α. The pressure-viscosity coefficient is a function of the molecular structure of the lubricant and its physical characteristics. If α=0, the viscosity is then constant and the fluid is considered as iso-viscous. There are various formulae available to calculate the pressure-viscosity coefficient. One of the early ones was derived by Wooster [34]:

$$\alpha = \left(0.6 + 0.965 log_{10}(\mu_0)\right) \times 10^{-8} \quad (2)$$

Where α is the pressure-viscosity coefficient in [Pa^{-1}], and μ_0 is the atmospheric dynamic viscosity in [cP] or [$mPa.s$]. An accurate value of this coefficient can be determined experimentally. The piezo-viscosity effect varies between oils, and it is more considerable for naphthenic oils than paraffinic oils. Water, by contrast shows only a small rise, almost negligible, in viscosity with pressure.

As a first approximation, we consider the simplest flexible rotor model popularly known as the Jeffcott model (1919). In spite of its age of over 90 years, the Jeffcott rotor is still widely used.

Figure 1 shows a typical Jeffcott rotor. It consists of a simply supported flexible massless shaft with a rigid disc mounted at the mid-span. The disc center of rotation, *C*, and its center of gravity, *G*, is offset by a distance, *e* which is called the unbalance eccentricity. The shaft spin speed is ω and the shaft whirls about the bearing axis with a whirl frequency ν. For present study, synchronous condition has been assumed, i.e. ν= ω. According to the beam theory, the stiffness of the shaft k_r can be expressed as

$$k_r = \frac{load}{deflection} = \frac{48 E_r I_{Gy}}{L_r^3} \quad (3)$$

where E_r is the Young's modulus of rotor, $I_{Gy} = \int_S x^2 ds = \frac{\pi D^4}{64}$ is the second moment of area of the rotor (shaft) cross-section, and L_r denotes the span of the rotor (shaft).

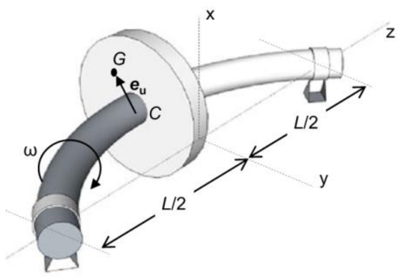


Figure 1 A typical Jeffcott rotor

The finite element method is used to approach the nonlinear transient pressure equation called in the present study the modified Reynolds' equation derived from momentum and mass conserving principles for an incompressible and piezo-viscous couple stress fluid using the V. K. Stokes micro-continuum theory [26]. The first order differential equations system resulting from the discretization of the pressure equation is solved by the implicit Euler's scheme and the relaxed substitution iterative method in order to determine the instantaneous hydrodynamic forces acting on the rotor (shaft), i.e. at each time step. The nonlinear trajectories of the shaft centre are obtained by solving the rotor-dynamics equations with a direct integration procedure, namely the explicit Euler's scheme.

In the parametric study, there are three key parameters dominating the combined effects of the pressure-viscosity dependency, the presence of couple-stresses, and the compliance of the bearing-liner which are the dimensionless viscosity–pressure coefficient $\tilde{\alpha}$, the dimensionless couple stress parameter $\tilde{\ell}$, and the normalized elasticity parameter C, respectively.

2. Governing equations

2.1. Modified transient nonlinear piezo-viscous Reynolds' Equation and boundary conditions

The transient nonlinear modified Reynolds' equation can be written as :

$$\frac{\partial}{\partial x}\left(G(h,\ell,\alpha,p)\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(G(h,\ell,\alpha,p)\frac{\partial p}{\partial z}\right)=6\mu_0\left[U_j\frac{\partial h}{\partial x}+2\frac{\partial h}{\partial t}\right] \tag{4}$$

Where $U_j=\omega R$ is the linear velocity of the journal surface and $G(h\ ,\ell\ \ ,\alpha\ \ ,p\ \ )$.

Using the non-dimensional quantities (indicated with a superscript asterisk (*))

$$\theta=x/_R,\ z^*=\frac{z}{L},\ p^*=\frac{p}{\mu_0\omega\left(R/_C\right)^2},\ h^*=\frac{h}{C},\ \lambda=\left(R/L\right)^2,\ \alpha^*=\mu_0\omega\left(R/_C\right)^2\alpha,\ \ell^*=\frac{\ell}{C},\text{and } t^*=\omega t \text{ in}$$

equation (4), the normalized transient modified Reynolds equation for a journal bearing system figure 2 is obtained in the form

$$\frac{\partial}{\partial\theta}\left(G^*(h^*,\ell^*,\alpha^*,p^*)\frac{\partial p^*}{\partial\theta}\right)+\lambda\frac{\partial}{\partial z^*}\left(G^*(h^*,\ell^*,\alpha^*,p^*)\frac{\partial p^*}{\partial z^*}\right)=6\frac{\partial h^*}{\partial\theta}+12\frac{\partial h^*}{\partial t^*} \tag{5}$$

Where $h^*(\theta,z^*,t^*)=1+X^*(t^*)cos\theta+Y^*(t^*)sin\theta+\mathrm{C}\,\tilde{p}(\theta,z^*,t^*)$ (6)

In equation (5), the term $\frac{\partial h^*}{\partial t^*}=X^{*\prime}cos\theta+Y^{*\prime}sin\theta+\mathrm{C}\,\tilde{p}'$ represents the non-dimensional squeeze velocity.

In equation (6), $(X^*,Y^*)=(X,Y)/C$ are the Cartesian instantaneous co-ordinates of the journal centre and the parameter C represents the compliance factor of the elastic bearing-liner which is defined as

$$\mathrm{C}=\sigma_0 t_h^* C_d^* \tag{7}$$

Where $\sigma_0=\frac{(1+\sigma)(1-2\sigma)}{1-\sigma}$, $t_h^*={t_h}/_R$ is the relative thickness of the bearing-liner, and $C_d^*=\dfrac{\mu_0\omega\left(R/_C\right)^3}{E}$ is the deformation coefficient, E and σ are the Young's modulus and Poisson's ratio of bearing-liner material, respectively.

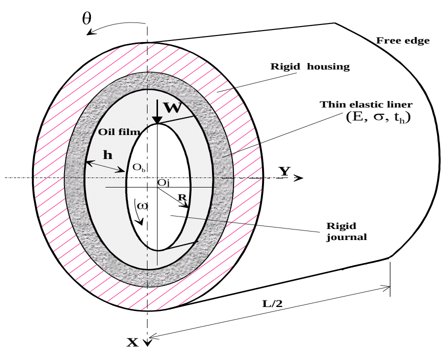


Figure 2 Schematic representation of a coated journal bearing with the coordinate system.

The normalized pressure field must satisfy the modified Reynolds' equation (5) in $D^* = (0,2\pi) \times \left(-{}^{1}/_{2}, {}^{1}/_{2}\right)$ and the following boundary conditions on ∂D^*:

$p^*\left(\theta, z^* = \pm \frac{1}{2}, t^*\right) = 0$, $p^*\left(\theta, z^*, t^*\right) = p^*\left(\theta + 2\pi, z^*, t^*\right)$ (8), which is the periodicity condition.

In addition, the Gümbel's condition also known as the half-Sommerfeld condition is used to take into account the film rupture occurring in the divergent region of the bearing. This condition neglects the sub-ambient pressure completely when calculating bearing performance and it is frequently used in the numerical simulations of hydrodynamic lubrication problems owing to its simplicity.

2.2. Rotor-dynamic equations

When the external load acting on the rotor or the journal varies both in direction and magnitude, the journal center describes a trajectory within the bearing. The determination of this trajectory requires the solution of the rotor-dynamic equations coupled with those governing the hydrodynamic behavior of lubricating oil films. Generally, the external loads acting on the rotor are:

- the weight of the rotor $2W_0=2mg$;
- the dynamic load components $W_X(t)$ and $W_Y(t)$ due an unbalance mass characterized by its eccentricity e ;
- the hydrodynamic forces F_X and F_Y due to the presence of the lubricating oil film.

The rotor-bearings system is modeled as a flexible Jeffcott rotor symmetrically supported by two identical layered hydrodynamics bearings ignoring the gyroscopic effects as depicted in figure 1. We assume that the rotor mass is lumped at the midpoint with massless shaft, central load of $2m$, rotor damping of $2b_r$, and rotor stiffness of $2k_r$. For each bearing are attributed a mass m of the rotor, a static applied load $W_0=mg$, and a synchronous dynamic excitation due to an unbalance mass $\left|\vec{W}(t)\right| = me\omega^2$.

The application of the dynamic fundamental principle gives

$$\begin{cases} m\ddot{X} + b_r\dot{X} + k_r X = W_0 + me\omega^2 \cos\omega t + F_X(X, Y, \dot{X}, \dot{Y}) \\ m\ddot{Y} + b_r\dot{Y} + k_r Y = me\omega^2 \sin\omega t + F_Y(X, Y, \dot{X}, \dot{Y}) \end{cases} \tag{9}$$

Where

$$\begin{Bmatrix} F_X(t) \\ F_Y(t) \end{Bmatrix} = \int_{-L/2}^{L/2} \int_0^{2\pi} p(\theta, z, t) \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} R\,d\theta\,dz \tag{10}$$

F_X and F_Y are the hydrodynamic forces in X and Y directions that are nonlinear functions of the displacement components (X, Y) and the velocities $(\dot{X}, \dot{Y})$ of the journal center O_j. They are calculated by integrating the hydrodynamic pressure over the bearing surface. This latter is obtained by solving the transient modified Reynolds' equation by finite element method.

In dimensionless variables, equation (9) is written as

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \{U''\} + \begin{bmatrix} B_r & 0 \\ 0 & B_r \end{bmatrix} \{U'\} + \begin{bmatrix} K_r & 0 \\ 0 & K_r \end{bmatrix} \{U\} = \begin{Bmatrix} W_0^* + \varepsilon\cos(t^*) + \hat{F}_X(X^*, Y^*, X^{*\prime}, Y^{*\prime}) \\ \varepsilon\sin(t^*) + \hat{F}_Y(X^*, Y^*, X^{*\prime}, Y^{*\prime}) \end{Bmatrix} \tag{11}$$

where

$$B_r = \frac{b_r}{m\omega}\ ,K_r = \frac{k_r}{m\omega^2}\ ,W_0^* = \frac{W_0}{mC\omega^2} = \frac{g}{C\omega^2}, \varepsilon = \frac{e}{C}\ ,\left(\hat{F}_X, \hat{F}_Y\right) = \frac{\mu_0 \omega RL\left(R/_C\right)^2}{mC\omega^2}\ (F_X^*, F_Y^*),$$

$$\{U\} = \begin{Bmatrix} X^* \\ Y^* \end{Bmatrix}, \qquad \{U'\} = \begin{Bmatrix} X^{*\prime} \\ Y^{*\prime} \end{Bmatrix}\ ,\{U''\} = \begin{Bmatrix} X^{*\prime\prime} \\ Y^{*\prime\prime} \end{Bmatrix}, and\ (^\circ)' = \frac{d}{dt^*}(^\circ) = \frac{d(^\circ)}{dt}\frac{dt}{dt^*} = \frac{1}{\omega}\frac{d}{dt}(^\circ)$$

3. Computation procedure of the journal bearing nonlinear dynamic response

Solving the normalized equations of motion (11) ahead of time numerically using the direct integration methods of second order differential equations such as the explicit Euler's method, the transient motion of the journal center O_j for balanced and unbalanced shafts is determined figure 3.

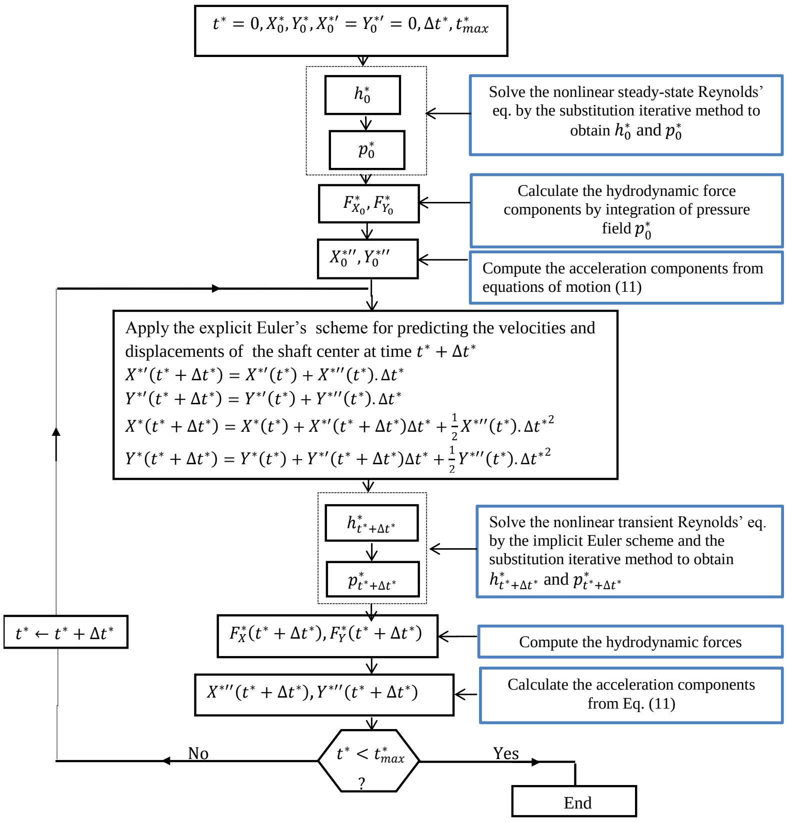


Figure 3 Flow chart of the computational procedure.

4. Results and discussions

4.1. Transient solution vs. Steady-state solution

The nonlinear dynamic analysis, described above, has been incorporated into a computer program. In order to check the correctness of the results obtained from the nonlinear transient analysis of hydrodynamic journal bearings, a separate 2-D finite difference computer program was developed to calculate the steady-state position by solving the inverse hydrodynamic lubrication by means of the relaxed iterative Newton-Raphson method. For (2-D) finite difference analysis, which in the present work paper will be referred to as the exact solution, a grid size of (61×16) nodes is used in θ and z directions, respectively. The number of nodes is chosen in order to ensure accurate results with minimum CPU time. The rotor and bearing parameter values used in the calculations as well as the finite element meshing characteristics and data of calculations are reported in tables 1 and 2. In this section, the calculations were made for a balanced layered journal bearing lubricated with piezo-viscous couple stress lubricant : $W_0=2720\ kN$ $(S=0.0225,\ W_0^*=14.)$, $\alpha=17\times10^{-9}\ Pa^{-1}$, $\varepsilon=0.$, $\ell^*=0.3$, $t_h=10\times10^{-3}\ m$, $E=0.9\ GPa$ and $\sigma=0.35$.

The geometrical characteristics such as the bearing length, the bearing radius and the radial clearance as well as the rotational velocity are given in table 1.

Figure 4 depicts the shaft center trajectory calculated with the transient analysis, i.e. when considering the compliance of the bearing-liner and the lubricant as a non-Newtonian couple stress fluid. The path of shaft centre gradually approaches the static equilibrium position whose the co-ordinates are (X_0^*=*0.3143*, Y_0^*=*0.3268*). The values predicted by the transient analysis are (X^*=0.3272, Y^*=0.3136). It is clearly seen that the discrepancy between the two results is very small.

4.2. Parametric study

4.2.1. Flexible rotor with large unbalance mass

Figure 5 represents the trajectories of the shaft centre in given running conditions:

W_0=340 kN, $t_h=10^{-2}$ m, E=0.9 GPa, σ=0.35, b_r=0., k_r=5 MN/m, N=3000 rpm or $\omega=100\times\pi$ rad/s, and ε=0.80. Note that for the rigid bearing-liner, the Young's modulus of liner E tends to infinity and the deformation coefficient C_d is then equal to zero.

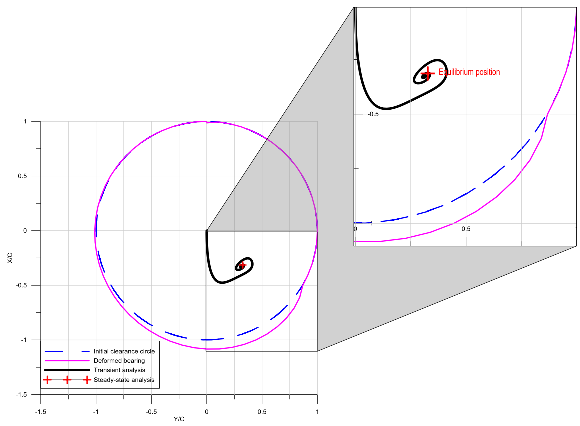
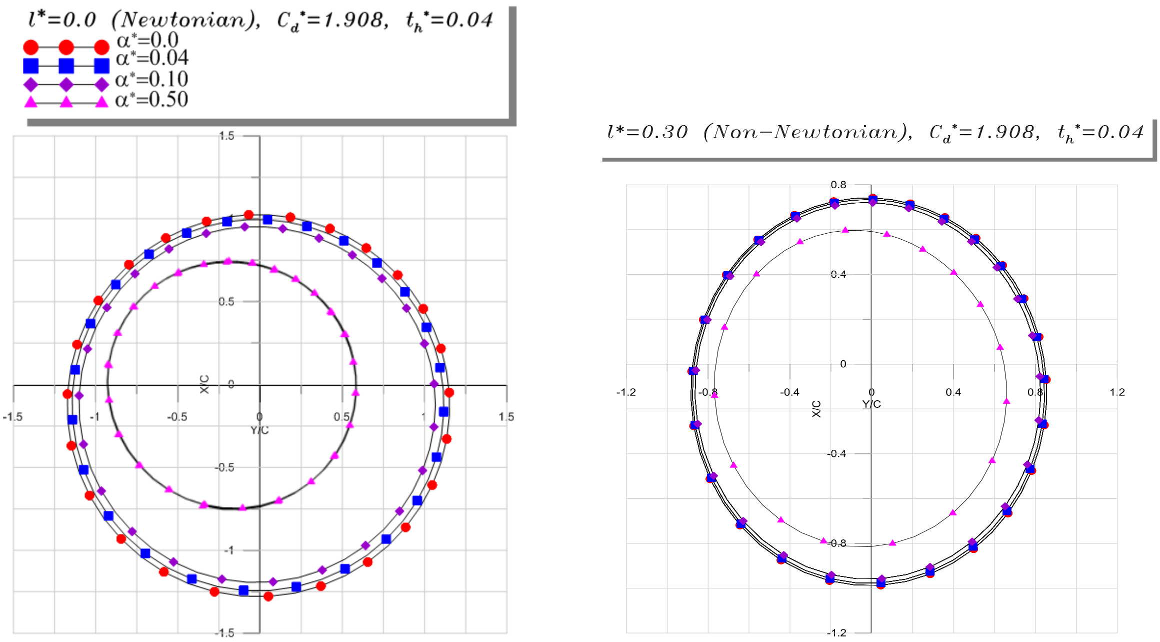


Figure 4 Journal center trajectory and final position

(W_0=2720 kN (W_0^*=14.14), S=0.0225, $\alpha=17\times10^{-9}$ Pa^{-1}, ε =0., ℓ^*=0.3, $t_h=10\times10^{-3}$ m, E=0.9 GPa, σ=0.35).

a) Compliant bearing liner



b) Rigid bearing liner

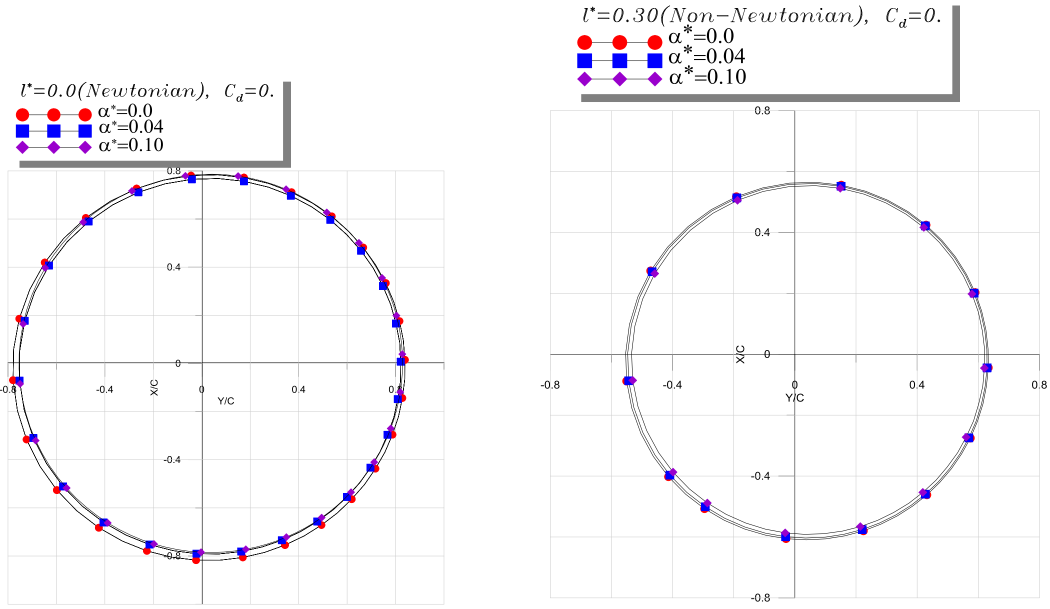


Figure 5 Comparison of nonlinear unbalanced shaft centre trajectories for flexible rotor and large unbalance mass (ε=0.80)

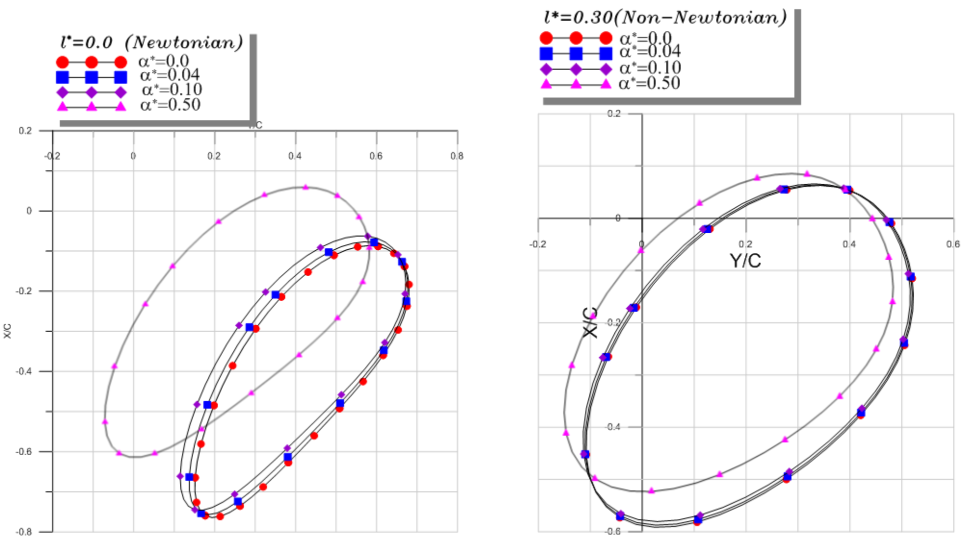
The relative unbalance eccentricity $\varepsilon=0.80$ corresponding to $e=280$ *microns* generates a dynamic load $W=me\omega^2=940\ kN$ at synchronous frequency $(\nu/\omega=1)$. For this operating condition, the value of W is greater than the static load W_0 $(W/W_0 \approx 3)$ and is representative of some emergency conditions in turbomachinery when a blade is lost for example.

It is noted that only the final form of shaft center orbits will be presented in the following, i.e., the results corresponding to the transient numerical effect due to initial conditions will be omitted.

The trajectories of the shaft center within the compliant bearing ($C_d^* = 1.908, \sigma = 0.35$ and $t_h^* = 0.04$) are given in figure 5(a) for two values of dimensionless couple stress parameter l^*=*0.30* (Newtonian case) and l^*=*0.0* (non-Newtonian couple stress fluid), and different values of the dimensionless piezo-viscosity coefficient ranging from 0.0 (iso-viscous case) to 0.50 (piezo-viscous case). In these operating conditions, the rotor has a very large amplitude of circular or pseudo-circular motion and the nonlinear dynamic behaviour appears clearly. This is due to the fact that the dynamic loading due to a large unbalance mass is very important compared to the static one as afore-mentioned above.

On the other hand, we observe in the Newtonian case l^* =0 that increasing the pressure coefficient α^* shorten the shaft trajectories. This is due to the pressure rise leading to a higher load carrying capacity which reacts and reduces the trajectories size. We can notice that the operating eccentricity of the journal bearing can be greater than the radial clearance in this case where the unbalance mass is large. Furthermore, the orbits described by the shaft center are widely modified by the bearing compliance especially for the non-Newtonian case as clearly illustrated in figure 5 (a). They exactly follow the shape of the deformed bearing.

a) Compliant bearing liner



b) Rigid bearing liner

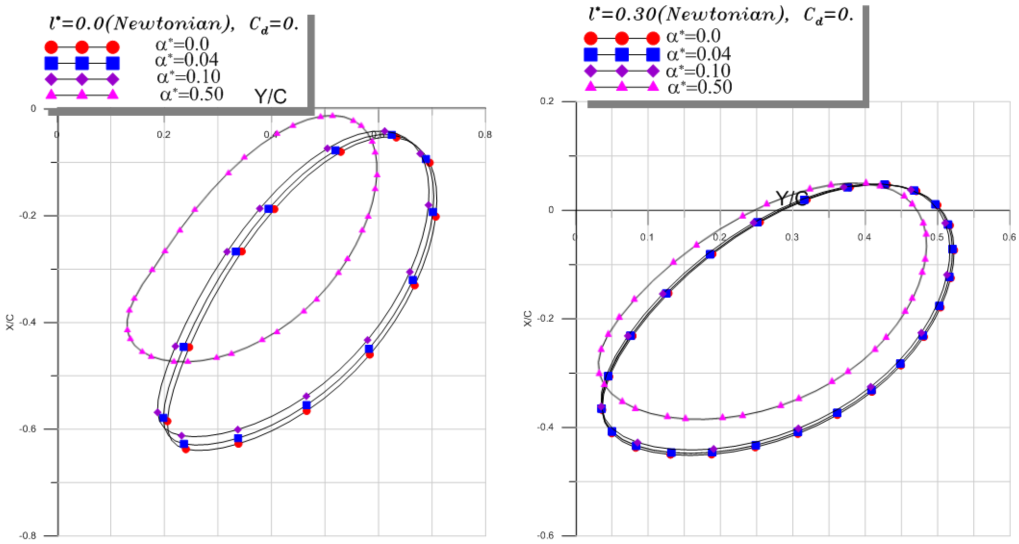


Figure 6 Comparison of nonlinear unbalanced shaft centre trajectories for flexible rotor and small unbalance mass (ε=0.20)

Figure 5 (b) represents the case of rigid bearing-liner. It is seen that the influence of pressure viscosity α^* on the orbits is weak in both Newtonian and non-Newtonian cases. That means in this case the distortion plays a major role in the bearing response. Moreover, the non-Newtonian orbits are smaller than those obtained in the Newtonian case, i.e. when the couple stress parameter $\boldsymbol{l^*}$of the lubricating fluid increases.

4.2.2. Flexible rotor with small unbalance mass

The relative unbalance eccentricity ε=0.20 corresponding to *e=70 microns* generates a dynamic load $W=me\omega^2=235$ *kN* at synchronous frequency *($\nu/\omega=1$)* which corresponds to *0.70* times the static load. This defect may be attributed to a large residual unbalance which could exist in the shaft. The trajectories of the shaft centre within the rigid and compliant bearings are given in figure 6 for the same values of the couple stress parameter l^* and the piezo-viscosity coefficient. As expected, the shaft centre moves around the equilibrium position in both cases since the dynamic load is smaller than the static one. Moreover, the co-ordinates of the equilibrium position change when increasing the piezo-viscosity coefficient especially for higher values of this coefficient in both Newtonian and non-Newtonian cases. So, it results a shift of orbits towards the bearing centre as depicted in the figure.

For small unbalance value as the pressure magnitude is lower the impact of bush distortion is less pronounced than for large unbalance case. For the rigid case, we observe in figure 6 (b) the same tendencies with smaller size trajectories.

5. Conclusions

In this work, a transient nonlinear analysis was proposed to investigate the nonlinear dynamic response of an unbalanced Jeffcott flexible rotor mounted in coated journal bearings lubricated with a piezo-viscous polar fluid. The analysis is based on the V. K. Stokes micro-continuum theory for describing the flow of piezo-viscous polar lubricants blended with additives. Using the classical assumptions of hydrodynamic lubrication, a transient and nonlinear modified Reynolds equation was derived in order to take into consideration the combined effects of pressure dependent viscosity and couple stresses resulting from the presence of polymer additives in the base oil. The trajectories of the shaft centre were obtained numerically by solving the rotordynamics equations with the explicit Euler's scheme.

The two dominant parameters in the present analysis are the viscosity-pressure coefficient and the couple stress parameter. Such parameters must be considered as key parameters to design adequately bearings of rotating machineries. The obtained results have been compared with the iso-viscous and Newtonian case.

According to the obtained results, the combined effects of couple stresses due to the presence of polymer additives in lubricant and the pressure dependent viscosity provide lift load enhancement and produce higher oil-film thickness and more contracted trajectories of the shaft centre even at severe running conditions (e.g. higher loading, large unbalance mass, …). In these circumstances, the destructive metal-to-metal contact between the shaft and bearing surfaces may also be avoided because of the elasticity of the bearing. Qualitatively, these results agree very well with those obtained by the same authors [33] in the case of internal combustion engine connecting-rod bearings lubricated with an iso-viscous polar fluid.

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